

Computer Methods of Analysis of Offshore Structures
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Module - 01

Lecture - 07

Stiffness Method of Analysis of Plane, Orthogonal Structures (Part – 2)

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- Stiffness method
- Planar orthogonal structure
- Joint load vector
- Fixed end moments due to load and displacements

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Fixed End Moments

$M_{AB}^F = \frac{Pl}{8}$ $M_{BA}^F = -\frac{Pl}{8}$

$M_{AB}^F = \frac{wl^2}{12}$ $M_{BA}^F = -\frac{wl^2}{12}$

$M_{AB}^F = \frac{pab^2}{l^2}$ $M_{BA}^F = -\frac{pba^2}{l^2}$

M = fixed moment
 F = fixed moment
 BA = nomenclature end, where moment is computed other end of the beam
 End moments are positive

So, step number 4 rather should be to estimate the joint load vector $J L u$ and $J L r$. In fact, we should say joint load vector $J L$ which is actually derived based on the fixed end moments; generated from the applied loads on each member. Now the question comes how do I know the values of fixed end moments for different forms of loads. So, let us quickly **add** the summary of fixed end moments for different load cases which is available in all standard textbooks, but still for completion **sake**, let us write down that.

Let us take both ends fixed for all the cases that is our standard beam subjected to central concentrated load p and length of the member be l . So, I say $M F A B$ this is end A, this is end B and I say $M F B A$. So, this nomenclature stands for M stands for the fixed moment, F stands for the fixed moment $B A$ is the nomenclature where the first notation is the end where moment is computed and the other end is the end of the beam.

So, for this case it is $p l$ by 8 these also $p l$ by 8. Now for doing any numeric analysis, we need to have sign convention, we already said anti clockwise moments end moments or positive. So, I should say this is plus this is minus. Now I have fixed beam end A B span l subjected to uniform distributed load w . So, this will give me $M F A B$ and $M F B A$; this will be plus $w l$ square by 12 this will be minus $w l$ square by 12.

Similarly, this is also $M F A B$, this is $M F B A$ fixed beam under the concentrated load, but x entry. So, p this a and this is b . So, this will be again $M F A B$ this will be $M F B A$; $M F A B$ will be plus $p a b$ square by l square this will be minus $p b a$ square by l square; if you have a fixed beam with varying load of intensity w_0 . So, this will be again $M F A B$; this will be $M F B A$ and $M F A B$ and $M F B A$ or given as $w_0 l$ square by 20 plus minus $w_0 l$ square by 30.

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Note Title 10/20/2017

$$+ \frac{w_0 l^2}{20} \quad - \frac{w_0 l^2}{30}$$

$$+ \frac{5}{96} w_0 l^2 \quad - \frac{5}{96} w_0 l^2$$

$$+ \frac{M b (b-2a)}{l^2} \quad + \frac{M a (2b-a)}{l^2}$$

If you have a fixed beam with triangular loading of intensity w this will be M_{FAB} ; this will be M_{FBA} . So, this will be plus $\frac{5}{96} w l^2$; this will be minus $\frac{w l^2}{96}$; if I have a beam which is subjected to some anti clockwise moment M at A distance A and B where this span is l then this becomes M_{FAB} and this becomes M_{FBA} ; please note the sign both will be anti clockwise in all other cases reverse.

So, this going to be plus M by l square into B minus $2A$ this is also going to be plus M A by l square $2B$ minus A . So, these all some standard loading which have very common.

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Note Title 10/20/2017

$$M_{AB}^F = \frac{M}{l^2} \left(\frac{k}{2}\right) \left(\frac{l}{2} - 2\frac{l}{2}\right)$$

$$= -\frac{M}{4}$$

$$M_{BA}^F = \frac{M}{l^2} \left(\frac{k}{2}\right) \cdot (l - l/2)$$

$$= +\frac{M}{4}$$

Joint load - reversal of the fixed end moments

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Let us have one more fixed beam apply with the same concept I apply a clockwise moment M at equal distance. So, M F A B will be of the same order clockwise and clockwise. So, it should be this is this way. So, M by l square you can substitute here M by l square B is l by 2.

Then B minus 2 A, there is l by 2 minus 2 of l by 2 which gives me minus M by 4. So, this minus indicates it is become clockwise similarly M of B A will be M by l square A that is l by 2 l minus l by 2 which will give me plus M by 4 which is anti clockwise. So, that is what we can easily for. So, for different loading pattern one can estimate the fixed end moments let us quickly explain; how the joint load can be calculated for a given problem joint load will be simply reversal of the fixed end moments let us do this let us remember this.

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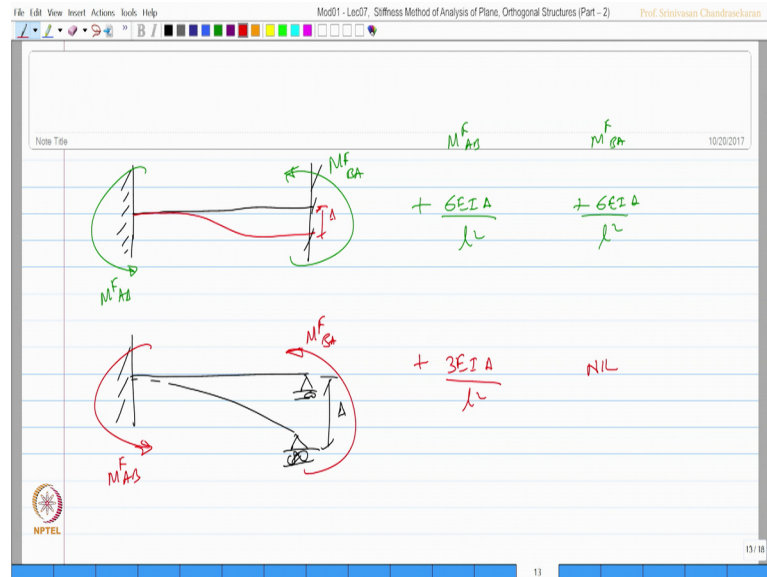
End moments due to displacements

Diagram Description	M_{AB}	M_{BA}
Beam with rotation θ_A at left end	$+\frac{4EI\theta_A}{l}$	$+\frac{2EI\theta_A}{l}$
Beam with rotation θ_B at right end	$+\frac{2EI\theta_B}{l}$	$+\frac{4EI\theta_B}{l}$
Beam with rotation θ_A at left end and roller support at right end	$+\frac{3EI\theta_A}{l}$	NIL

Joint load is reversal of fixed end moments one can also find end moments due to displacements because the earlier where end moments due to loads . So, let us take a beam this way give a rotation which will be theta a then this will be M F A B and this will be of the same order M F B A. So, M F A B and M F B A will be plus 4 E I theta a by l this is going to be plus 2 E I theta a by l this is the l.

Suppose we have a beam fixed at the left and roller I want to give displacement by rotation at the end. So, this is going to be theta B then this will be M F B A this will be M F A B and M F A B will be B A will be plus 4 E I theta B by l; this is plus to E I theta B by l; if I have a simply supported beam where I want to give unit rotation and the A end or some rotation theta A; then I should say this is going to be M F A B and this is going to be M F B A and M F A B will be plus 3 E I theta A by l and this will be 0.

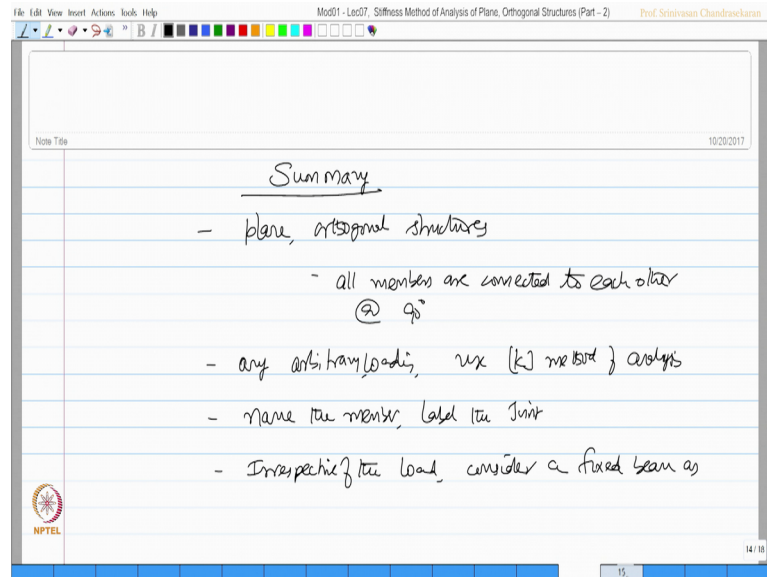
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Suppose we have a fixed beam and this is settlement of support this is the original axes of the beam and the beam settles by some amount which is delta, then this is M F B A this is M F A B; M F A B and M F B A are given by plus 6 E I delta by l square and plus 6 E I delta by l square if I have a fixed beam on one end and roller on the other and the beam settles by delta, then there will be a moment of this order this will be M F B A and this will be M F A B.

M F A B will be plus 3 E I delta by l square and this will be 0. So, these are some of the standard formats based on which one can calculate the end moments either **caused** by the load on the beam or by displacements of the joints or rotations of the joint by reversing this we will able to get the joint loads once I get the joint loads I can compute the understand displacements of the problem that is the general solution what we have.

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The image shows a screenshot of a presentation slide. The slide has a title bar at the top with the text "Mod01 - Lec07, Stiffness Method of Analysis of Plane, Orthogonal Structures (Part - 2)" and "Prof. Srinivasan Chandrasekaran". Below the title bar is a toolbar with various icons. The main content area of the slide is a white background with a blue border. The word "Summary" is written in the center in a cursive font. Below it, there are four bullet points, also written in cursive: "- plane, orthogonal structures", "- all members are connected to each other @ 90°", "- any arbitrary loads, use [k] method of analysis", and "- Name the member, Label the Joint". At the bottom left of the slide, there is a small logo for NPTEL. At the bottom right, there is a small number "14/18".

Let us quickly look at the summary; we have been discussing about **planar** orthogonal structure where all members are connected to each other at ninety degrees; that means, orthogonal.

May be subjected any arbitrary loading we can use stiffness method of analysis. So, we need to name the member label the joint irrespective of the load irrespective of the load consider a fixed beam as the basic element and derive the stiffness matrix for each member identify unrestrained and restrained displacements; it can be both translational rotational label them in specific sequence.

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basic element, and derive $[k]$ for each member

- Identify unrestrained and restrained displacements (translational rotational)
- Label them in a specific sequence
 - group the matrix
 - cross-partitioning
- $[k_{uu}] [\Delta u] = \{F_{uu}\}, [\Delta u]$ ✓
- Fixed end moment @ each node/joint - (reverse K) $\{F_u\}$

So, that you can group the matrix and do cross partitioning once it is there we said that s u u sorry; k u u which a sub matrix multiplied by Δu will be actually equal to k u u into Δu will be equal to the joint load which is also a partition matrix from this I can find the Δu provided I know how to estimate the joint load vector to estimate joint load vector find the fixed end moments at each node for the standard loading than reverse them to find the joint load vector and then solve the problem. So, friends we should apply this principle to couple of example problems and understand; how to use them and how we conveniently solve the problem without using any transformation matrix for an orthogonal structure.

Thank you very much.