**Computer Methods of Analysis of Offshore Structures**
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**Module – 01**
**Lecture – 07**
**Stiffness Method of Analysis of Plane, Orthogonal Structures (Part – 1)**

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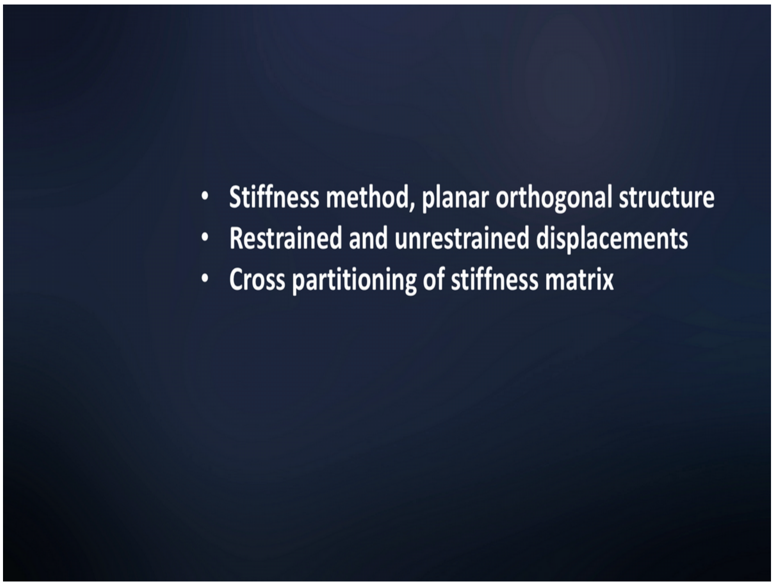


So, friends let us continue with the discussions on development of stiffness method of analysis for planar orthogonal structures. We already said that comparing to flexibility method, stiffness method in more generic; it is not problem specific, it can be easily programmable and there are lot of repetitive steps involved in development of analysis by this method; in the last lecture we already said by neglecting axial deformation.

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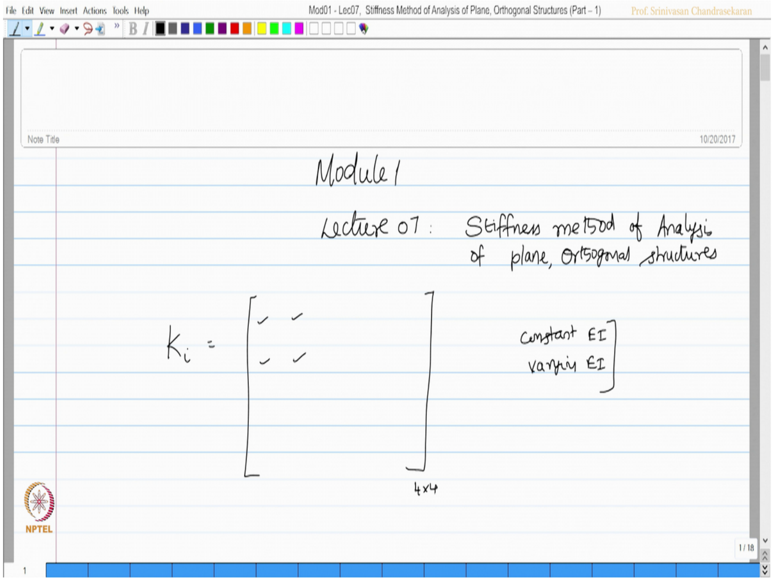


The stiffness matrix of the ith member which is essentially a fixed beam which will have 16 coefficients can be easily developed only with the rotational coefficients remaining all can be easily derived from these coefficients.

We also explained how one can generate the stiffness matrix for a fixed beam with constant E I and with varying E I; let us now apply this method to a planar orthogonal structure.

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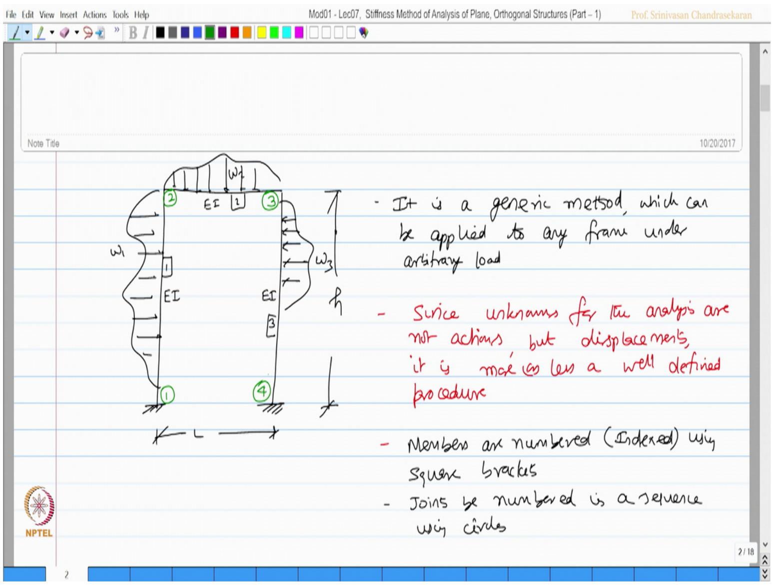

So, for example, let us consider the single story single bay frame as shown in the figure, both ends of the frames are fixed, it is subjected to some arbitrary loading, let us say this is w 1, maybe this is w 2 and may be w 3, the frame has certain geometric dimension which are known to us, the Young's modulus and moment of inertia of the sections are also known to us.

So, now there are basic steps which are very important to formulate a stiffness method of analysis for solving this problem. We know stiffness method has got unknowns which are displacements which can be translational as well as rotational. So, stiffness method is a generic method which can be applied to a frame under arbitrarily loading because the unknowns in this case are not actions, since unknowns for the analysis are not actions, but displacements it is more or less a well defined procedure.

So, we need to number the members let members are numbered I should say indexed using square brackets like this member is 1, this member is 2 and this member is 3; there are 3 members in this problem; let the joints b numbered in a sequence using circles; so, joint 1, joint 2, joint 3, joint 4.

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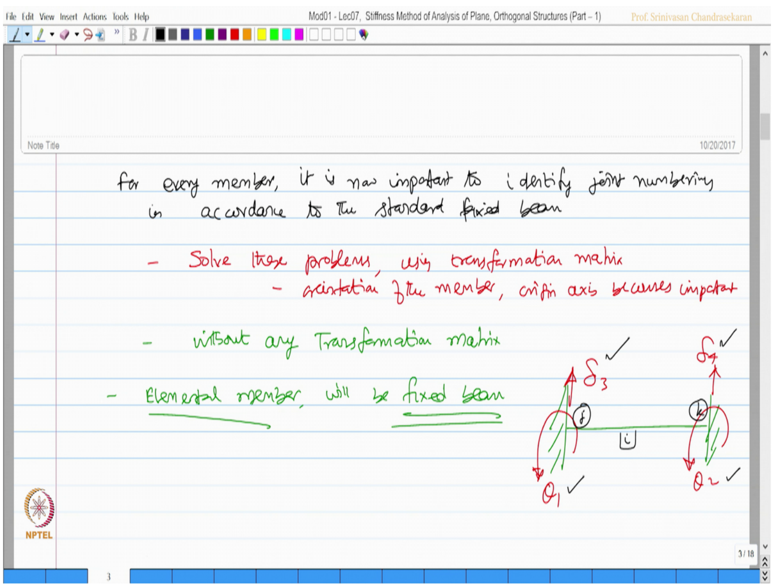


So, for every member; it is now important to identify joint numbering in accordance; to the standard fixed beam what does it mean friends that is a very important statement I wish to make here certain literature certain set of authors try to solve this problem using; what they call as transformation matrix where orientation of the member with respect to the origin axis becomes important, but we will handle this problem without any such transformation matrix.

So, our elemental member will be a fixed beam whether the ends of the beams are hinged, round rollers or free, but still our basic member for analysis will be fixed beam which has got both ends fixed which has got degrees of freedom in terms of rotations theta one theta 2 delta 3 and delta four by neglecting axial deformation . So, will follow the same order if this member is an ith member if the joint is j and k; rotation at the j th end is first rotations at the k th end is second displacement translation as the j th end is third and displacement translational at the kth end is fourth.

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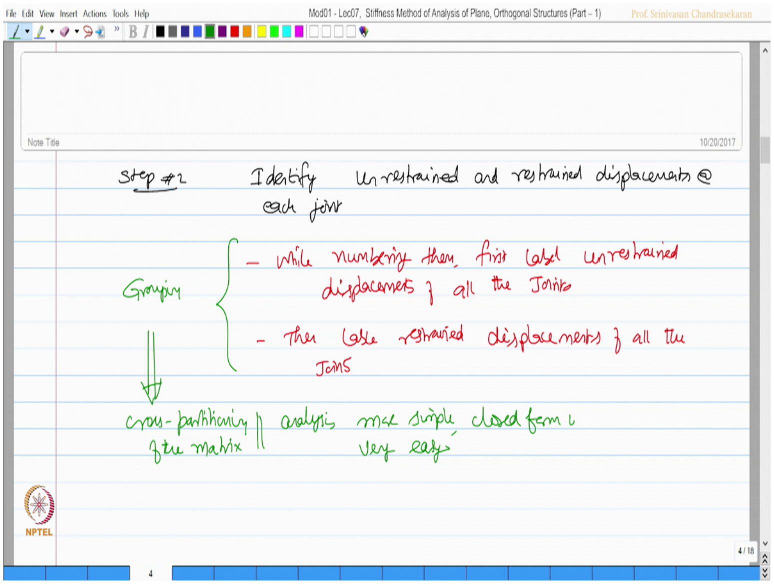


We will follow the same order for all the problems. So, for a given problem step number 2 could be identify unrestrained and restrained displacements at each joint, but while numbering them first label unrestrained displacements of all the joints then label restrained displacements of all the joints. So, we call this as grouping because this grouping will help me later on to do cross partitioning of the matrix which will make the analysis more simple closed form and very easy will see this.

Let us say; I want to demonstrate this for couple of cases let us take one frame with both ends fixed; we have given the member numbering 1, 2, 3.

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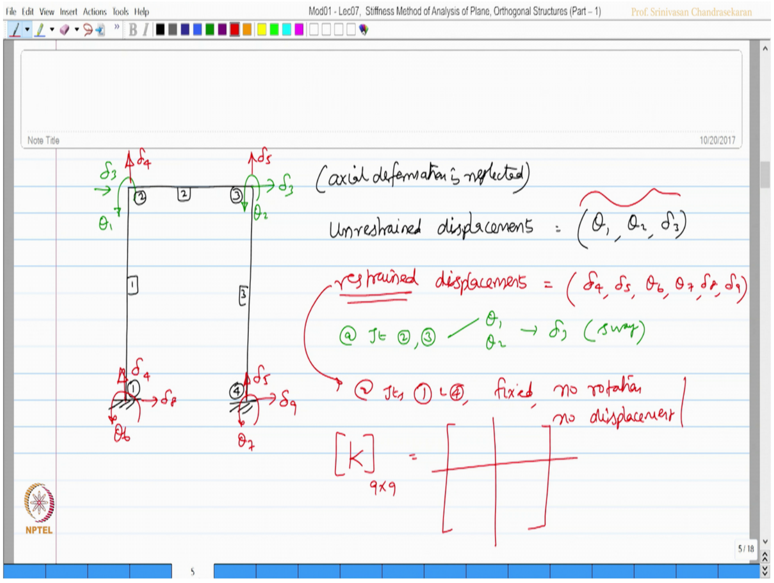


What are may be the geometric dimensions of the member, we are not concerned about the resisting moment. Now, we want to mark the unrestrained displacements we will do that in green. So, rotation at this end and displacement at this end please note at both the ends, let us say for our understanding let us name the joints 1, 2, 3 and 4.

You can note down here that the horizontal displacement both that the node 2 and 3 or delta 3; it means axial deformation is neglected. So, now, unrestrained displacements are theta 1, theta 2 and delta 3, let us mark the restrained displacement in red let us say delta four this also becomes delta 4, say delta 5, this is also becomes delta 5, theta 6, theta 7, delta 8, delta 9. So, now, restrained displacements are delta 4, delta 5, theta 6, theta 7, delta 8 and delta 9.

Let us now try to understand why these displacements are called unrestrained; unrestrained because at joint 2 and 3, they are free to rotate and they are free, displace the frame, can sway these are called restrained displacement because at joints 1 and 4 being fixed no rotation and no displacement is allowed. So, they are called restrained. So, looking into the consideration of unrestrained degrees of freedom, I can always say the matrix size for the unrestrained will be 3 by 3 and there are total 9 degrees of freedom. Therefore, the k matrix of the entire frame will be 9 by 9.

But this k matrix can be partition like this.

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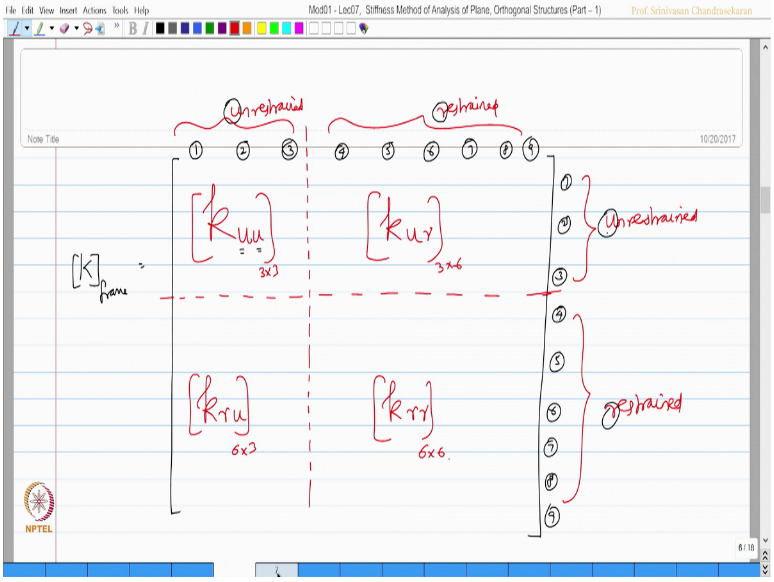


Let us see; what is this partition the k matrix of the whole frame will be 9 by 9. Let us say 1, 2, 3, 4, 5, 6, 7, 8 and 9. Similarly, 1, 2, 3, 4, 5, 6, 7, 8 and 9 out of which I am going to do a cross partitioning of this at 3 and 3; why I am doing 3 and 3 because the problem has 3 degrees of freedom restrained unrestrained and remaining 6 digress restrained.

Therefore I can called this as k u u because these are unrestrained degrees and these are restrained degrees. Similarly these are unrestrained and these are restrained. So, let us use the first word unrestrained restrained unrestrained restrained. So, row first column next. So, u and u k u u. So, this will be k u r this will be k r u and this will be k r r each one of them will be a sub matrix of a respective size. So, this will be 3 by 3, this will be 3 rows and 6 columns this will be 6 rows and 3 columns this will be 6 by 6.

So, you will; obviously, know the size of k u u depends upon; what is your unrestrained displacements.

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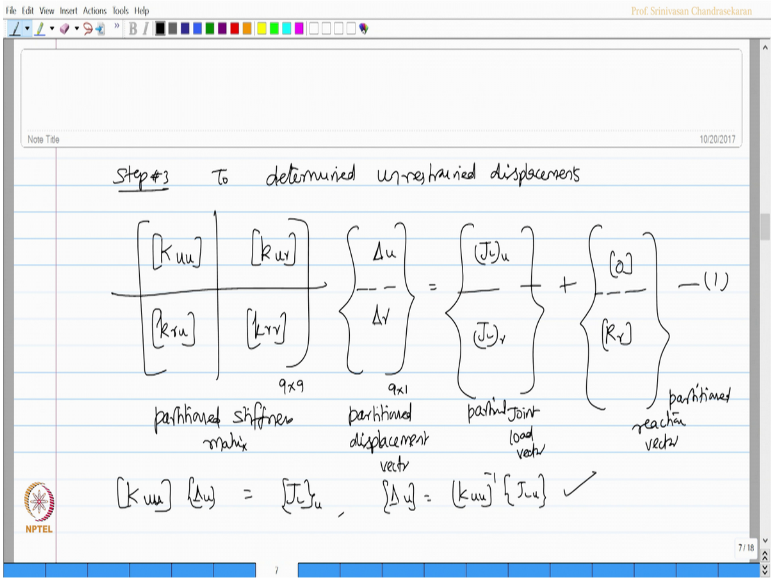


So, the second step is to mark the unrestrained and restrained displacements once we do this step number 3 could be to find out the unrestrained displacement that is to determine the unrestrained displacement; let us considered the k matrix as k u u; k u r k r u and k r r multiply this with unrestrained displacements and restrained displacements. So, this will be a matrix of 9 by 9, this will be a matrix of 9 by 1 which will land up in the joint load vector which will be again partition. So, I called this as joint load u and joint load r unrestrained restrained plus reaction vector I can called this is as partial reaction vector which will be 0 in this case and some reaction in this case. In fact, we can call this as partitioned reaction vector. So, this is partitioned stiffness matrix this is partitioned displacement vector this is partitioned joint load vector and so on calls equation number 1.

So, let us expand this equation. Now I can now write k u u into delta u will be equal to J L u. So, I can find delta u as simply inverse of this matrix multiplied by J L u that is what we wanted to know. So, delta u can be computed to compute delta u, we need to also know the joint load vector.