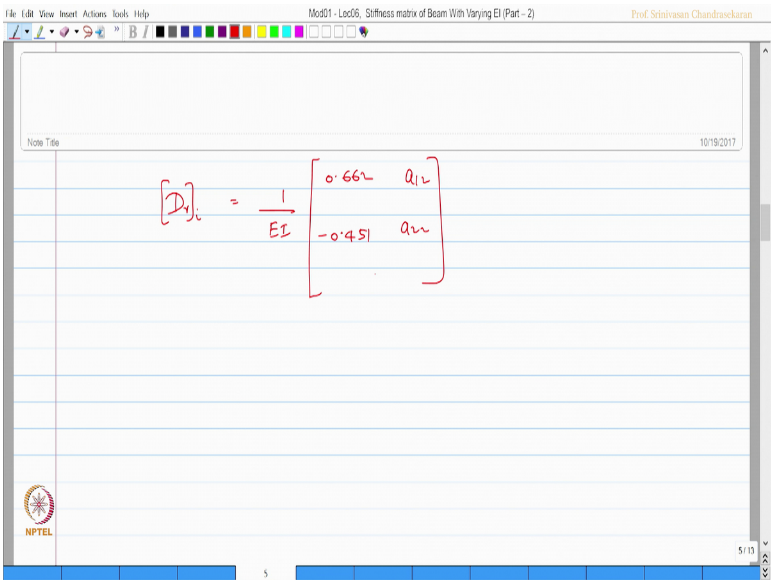
Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module – 01
Lecture – 06
Stiffness matrix of Beam With Varying EI (Part - 2)

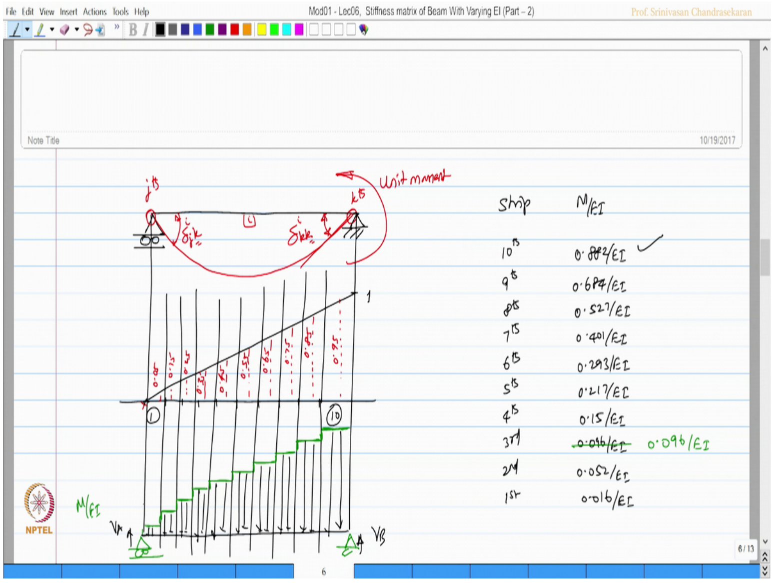
(Refer Slide Time: 00:17)

(Refer Slide Time: 00:22)



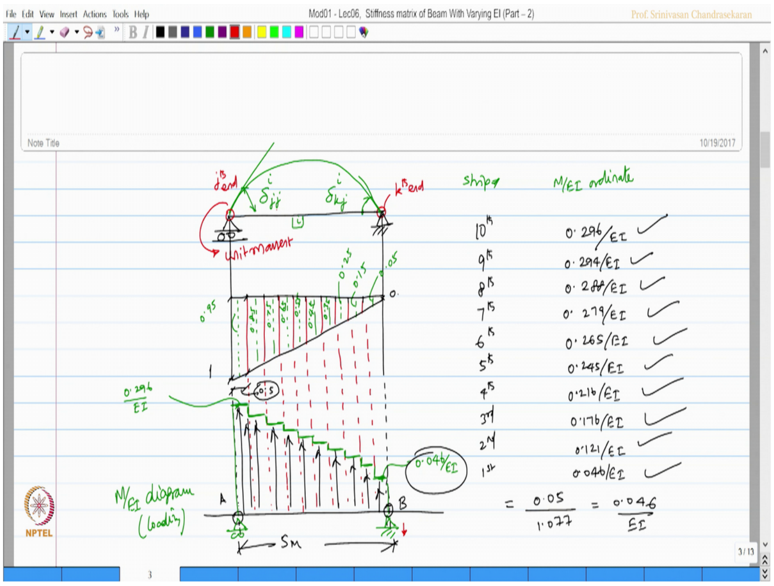
So, friends; now I have got the flexibility matrix of the ith element which is 1 by E I of 0.662 that is what V A is and V B is 0.451 down. So, minus 0.451 the first column. Now you want to get the second column that is a 2 1, a 1 2 and a 2; I should get this. So, to get this, I should again do the same exercise by applying unit moment at the kth end.

(Refer Slide Time: 01:13)



So, let us do that. So, this is by mean; I want to apply unit moment anti clockwise here. So, the beam will deflect like this say this is my value, which I am going to get is going to be my delta kk of ith beam this is delta j k of ith of beam if you look carefully.

(Refer Slide Time: 02:01)



This was the second subscript is j because the unit moment was given at the jth end in the same logic; the second subscript will be k because unit moment is given at the kth end. So, this is my jth end this is my k th end this is my ith beam.

Now, I am interested in working out these values. So, what should I do again divide them into 10 strips the moment here is 1 and here it is 0. So, it is going to very linearly, I will divided into 10 pieces; 10 pieces, let us draw this lines, I can now find the average values at these middle of the strips; as we did in the last case, we can do this, let us write down these values; this is 0.95, this is 0.85, this is 0.75, 0.65, 0.55, 0.45, 0.35, 0.25, 0.15, 0.05 and this is 0.

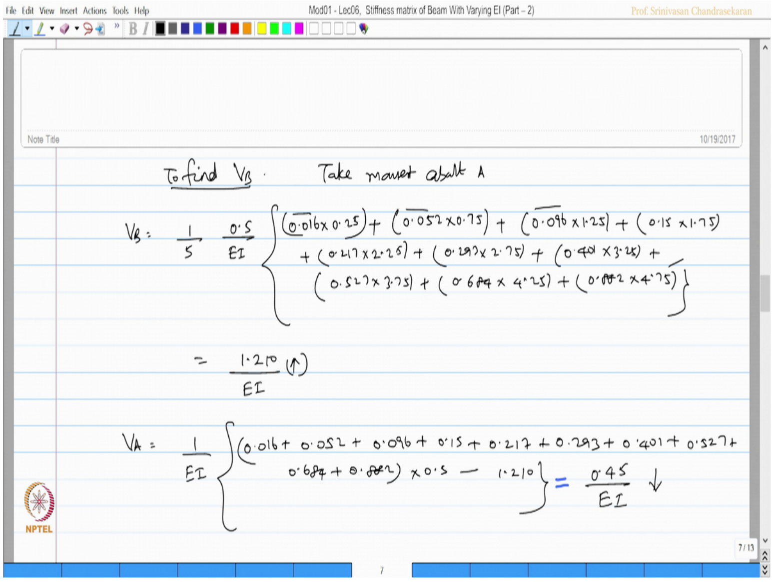
So, now, we can now calculate; similarly what we have done here we have done the M by E I ordinate for all the 10 strips; we can also do here the strip and M by E I ordinate; let us say this is my first strip, this is my first strip and this is my tenth strip let us do this. So, 10, 9, 8, 7th, 6^{th}, 5^{th}, 4^{th}, third, second and first strip; the 10 strips; so, let us talk about the first strip; this value will be the same manner as we have done here; we have done the same given a; for example, we want to find this value I can always say the moment divided by the corresponding I value.

Similarly, here this value will be 0.882 by E I, I will leave this an homework to you to calculate, this is very simple to do it 0.684 by E I, 0.527 by E I, 0.401 by E I, 0.293 by E I, 0.217 by E I, 0.15 by E I, 0.096 by E I, 0.052 by E I and 0.016 by E I.

This is my first strip, this is wrong, this is my first strip, this is my 10th strip; 10th strip has got the maximum here, let us convert this it is M by a diagram by plotting this, let us do that. So, this will have 0.882, then this will have 0.684, then 0.527, then 0.401, then 2 9 3 2 1 7 1 5 0 1 6 0 5 2 and 0 1 6 this is 0 9 6; let me very clear let me write it again.

This is 0 9 6 by E I. So, this; my M by E I diagram this is the loading diagram for this beam. So, this becomes my beam and this becomes by loading diagram for the beam I want to find now the reactions which is V B and which is V A.

(Refer Slide Time: 07:55)



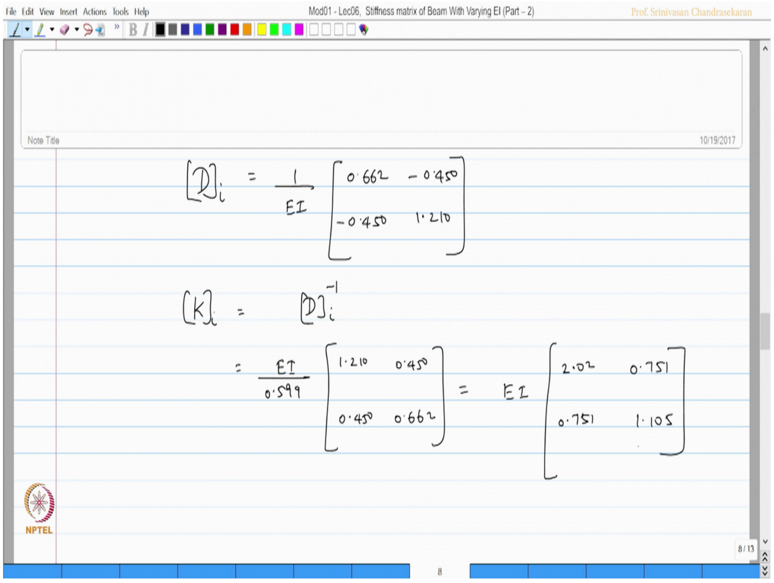
Now, to find V B, take moment about C A. So, V B will be equal to 1 by 5 that is the stand of the beam 0.5 is the width of the strip E I is common.

So, let us start from this value 0.016 and that distance will be 0.25; the next one will have a distance from here which will be added with 0.5 and So on. So, this will be 0.016 into 0.25 plus. So, we got this let us; then 0.052 into 0.75 plus 0.096 into 1.25 plus 0.15 into 1.75 plus 0.217 into 2.25 plus 0.293 into 2.75 plus 0.401 into 3.25 plus 0.527 into 3.75 plus 0.684 into 4.25 plus 0.882 into 4.75.

Which is very obvious this distance from here will be 0.25, therefore, this distance from here will be 4.75 which is checked here; this will give me V as 1.210 by E I of the vertical value. Now V A can be computed as 1 by E I of as usual take up only the ordinates of this diagram 0.016, 0.052, 0.096, 0.15, 0.217 2 9 3 4 0 1 5 2 7 6 8 4 8 8 2.

The width of the strip is 0.5 and subtract this 1.21 from here to get V A. So, friends which is now equal to point four 5 by E I downward.

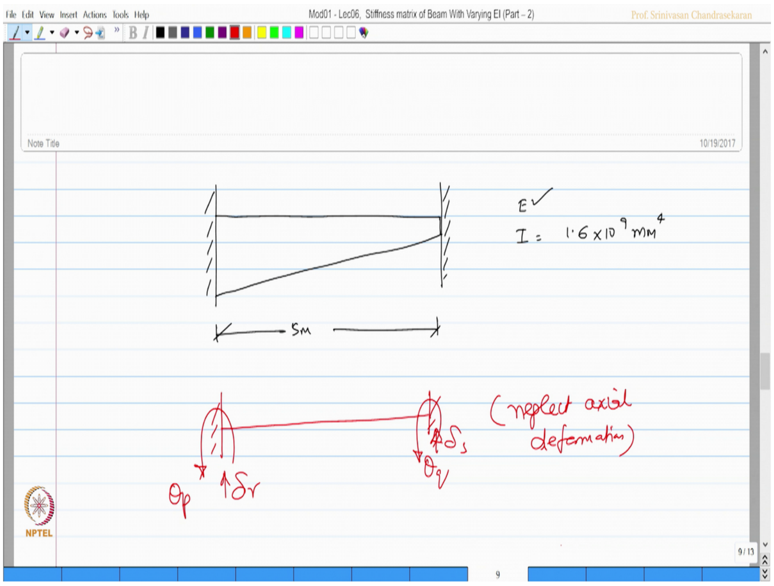
(Refer Slide Time: 11:33)



Therefore my flexibility matrix for the ith beam will be 1 by E I; we already have these data with us; these 2 with us; we borrow that again 0.662 minus 0.45. Now we have the new value a is 0.45 and this 1.21. So, minus 0.45 and 1.210.

Now, my stiffness matrix for this ith beam the flexibility matrix of this inverse which will be E I by 0.599 that is I am doing matrix inversion by the conventional method. So, 1.210, 0.662, 0.450; 0.450 which will give me k as E I times of 2.02, 0.751; 0.751, 1.105.

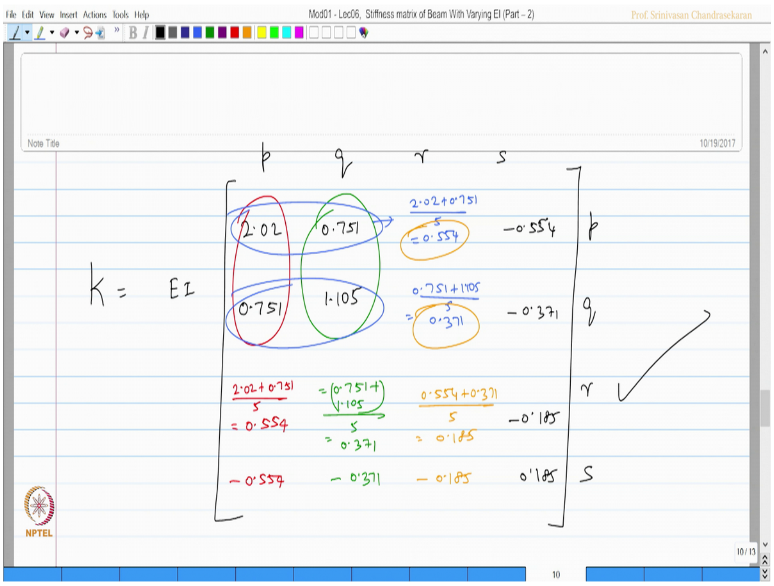
(Refer Slide Time: 12:53)



Now, friends let us say I have a beam which is fixed at both the ends; fixed at both the ends.

And it has got varying cross section. So, the length of the beam is 5 meters E is the material property and I is already known to us you see here I is known to me one point is 10 power 9 1.6 into 10 power 9 mm to the power 4. Now let us mark the degrees, let us say at this end, I have theta p theta q delta r and delta S, I neglect the axial deformation, I can now write the stiffness matrix is readily by knowing the coefficients of these 2 because now, if you look at the stiffness matrix we can straight away say k will be actually equal to E I times of this.

(Refer Slide Time: 14:16)

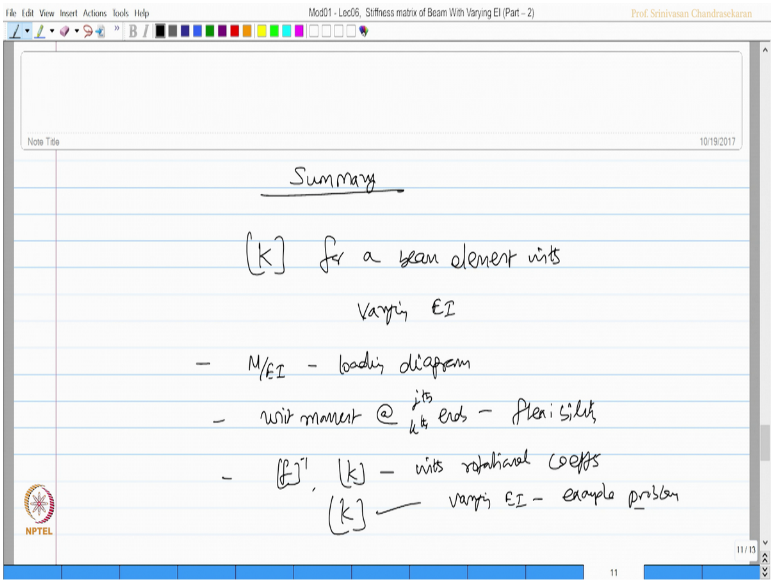


So, we already have this value with us let us borrow this and write it here I should say p q r S; p q r S. So, 2.02, 0.751 rotational coefficients 0.751, 1.105; once I know this, I can always find this value as some of these 2. So, let us say this is my; let us say this value will be some of these 2 by l which will be 2.02 plus 0.5751 by 5 which will be 0.554 and this value will be minus of that.

similarly this value the sum of these 2 by 5 which will be which will be 0.751 plus 1.105 by 5 which comes to be 0.371 and this value is minus 0.37. Once I get this, I can find this value as some of these 2 by 5 that is 2.02 plus 0.751 by 5 which will be 0.554 and this value will be 0.751 plus 1.105 by 5 which will be 0.371.

And this value will be some of these 2 that is 0.554 plus 0.371 by 5; again which will be 0.185 and this will be minus 0.185 of course, the last column is very simple; it is actually negative of r column. So, minus 0.554 minus 0.371 minus 0.185 and 0.185 I get the stiffness matrix.

(Refer Slide Time: 17:06)



So, friends in this lecture we learnt how to find the stiffness matrix for beam element with varying E I, we actually marked the M by E I diagram as the loading diagram M by E I was formed by giving unit moment at jth and kth ends.

So, I get flexibility coefficients. So, by inverting the flexibility matrix I get stiffness matrix with rotational coefficients; once I get that the full matrix can be derived by a simple algorithm shown in this screen. So, full stiffness matrix for varying E I with an example problem is now presented; I hope you have understood this and now we are very clear; how to derive stiffness matrix for a beam element with constant E I and varying E I by neglecting axial deformation.

Now, let us apply this concept to planar orthogonal structures in the next lecture and derive the unknown values of the structural system that is the moments, forces, shear forces, rotations and displacements depending upon the boundary condition and support condition of the problem.

Thank you very much.