

Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

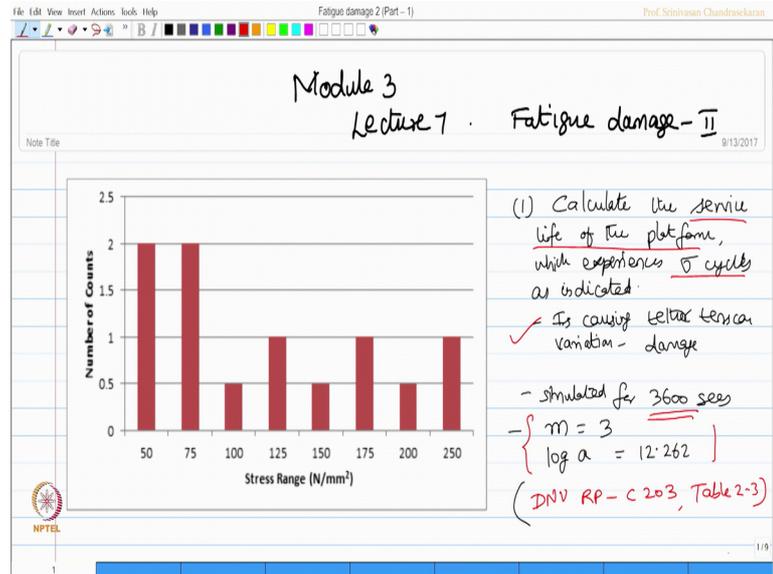
Module - 03
Lecture - 07
Fatigue Damage 2 (Part - 1)

(Refer Slide Time: 00:17)



Friends, let us continue with the next lecture. Where, we will do some problems on fatigue damage estimates for a stochastic process. What you see on the screen here is stress range versus number of counts is a typical stress bin as you see N is the number of counts.

(Refer Slide Time: 00:46)



So, what we want to do in this problem is we want to calculate the service life of the platform offshore platform which experiences stress cycles as indicated. so the stress cycle is causing tether tension variation which can result in a damage. The variation is simulated is simulated for let us say 3600 seconds for the s n curve take m value as 3 and take log a value as 12.262, This is as per the international code for offshore structures DNV Recommended Practice C203 and we have taken the values from table 2-3.

So, let us try to solve this problem. Let us understand the question first. We have to calculate the service life of the platform that is important. You have to calculate the service life of the platform. The platform undergoes stress cycles the stress range and the number of cycles are given in the figure this stress cycle amplitude reversal is crossing tether tension variation and the total duration T, is 3600 seconds. In the minus rule we need the value of m. And the DNV practice code taken for offshore structure gives me the value of m and log a directly from a specific table.

(Refer Slide Time: 02:53)

The screenshot shows a presentation slide with the following handwritten content:

$$\log N = \log a - m \log S$$

for a σ range of 50 N/mm^2 ,

$$\log_{10} N = 12.262 - 3 \log_{10} (50)$$
$$\log N = 7.165$$
$$N = 10^{7.165} = 14624801.7 \text{ cycles}$$
$$\text{Fatigue damage } D = \frac{n}{N} = \frac{2}{14624801.7} = 1.367 \times 10^{-7}$$

So, let us say we want to include the fatigue damage, we want to calculate the value n equation is $\log N$ is equal to $\log a$ minus $m \log S$. Now for a stress range of 50 let us compute for a stress range of 50 Newton per mm square let us compute this so $\log n$ is equal to 12.262 that is my $\log a$ value minus 3 is my m value and $\log 50$ there all log to the base 10 they are not Napier's logarithm that is log to the base 10 which gives me the value of $\log N$ as 7.165.

If we calculate this if you want to find N then is 10 power 7.165 which can be 1462 4801.7 cycles. Now the fatigue damage $n D$ is given by small n by capital N small n in this case is 2 so 2 by 1462 4801.7 which gives me as 1.367 10 power minus 7. So this we did for one specific case of 50, Similarly, I can do for 75, 100, 125, 150, 175, 200 and 250. And then find the cumulative damage, let us do that.

(Refer Slide Time: 04:53)

$\log N = \log a - m \log s$ ✓

Stress range	N	n	$D = 1/n$
50	1462 4801.7	2	1.3675×10^{-7}
75	4333274.59	2	4.6154×10^{-7}
100	1828100.22	0.5	2.7351×10^{-7}
125	935987.31	1.0	1.068×10^{-6}
150	541659.32	0.5	9.23×10^{-7}
175	341103.25	1.0	2.931×10^{-6}
200	228512.53	0.5	2.188×10^{-6}
250	116998.414	1.0	8.547×10^{-6}
			$\sum 1.653 \times 10^{-5}$

So, stress range let us open a table n small n and d which is n by capital N. So, let us say stress range what you see from the figure is 50, 75, 100 etcetera. Let us do that 50, 75, 100, 125, 150, 175, 200 and 250 that is what we have till 250. So, for N we just now calculated 1462 480101.7 and small n for 50 is 2. So, it is 2 and D is 1.3675 10 power minus 7.

Similarly for 75, we substitute the same equation which is $\log N$ is equal to $\log a$ minus $m \log s$ we will use the same equation calculate n for the stress range 75 which gives me 4333274. 59. Again for 75 the stress range the number of counts is 2, so this gives me D value as 4.6154 10 power minus 7. Similarly let us do for 100, I get this value as 1828100.22 and for 100 the stress value is number of counts is 0.5, so this value becomes 2.7351 10 power minus 7 for 125, I get the N value as 935987.31 and for 125 the number of counts is 1.0 so I get this as 1.068 10 power minus 6.

And for 150 I get N value as 541659.32 and the value is 0.5 and this is coming 9.23 10 power minus 7 and for 175 N value is 341103.25. I believe you are calculating using this equation in parallel and for 175, we can see the number of counts is 1.0, we have taken 1.0 and this becomes 2.931 into 10 power minus 6 and for 200 substituting this equation it is 228512.53 this is 0.5 and this is 2.188 10 power minus 6 and for 250 this value becomes 414 and the value is 1.0 and this is 8.547 into 10 power minus 6.

Now I can find the cumulative damage which is sum of this which is equal to 1.653×10^{-5} power minus 5.

(Refer Slide Time: 08:49)

Service life
 = Fatigue damage calculated for 3600 s = 1.653×10^{-5}

Fatigue damage for 1 s = $\frac{1.653 \times 10^{-5}}{3600}$

Fatigue damage in 1 year equivalent = $\frac{1.653 \times 10^{-5}}{3600} \times (60 \times 60 \times 24 \times 365)$
 = 0.14480

Fatigue damage is 0.14480 in one year. Damage is equivalent to 1 in 6.906 yr.

Service life is calculated by extrapolating this damage to Unity - service life of the platform
 service life = $\frac{1}{0.14480} = 6.906$ years

Now, I want to estimate the service life which will be we know that the fatigue damage calculated for 3600 seconds is actually 1.653×10^{-5} that is what we in the slide here. Therefore, fatigue damage for 1 second will be 1.653×10^{-5} by 3600, therefore fatigue damage in one year equivalents will be given by this is for 1 second therefore 1.653×10^{-5} divided by 3600 into 60 into 24 into 365 which comes to 0.14480.

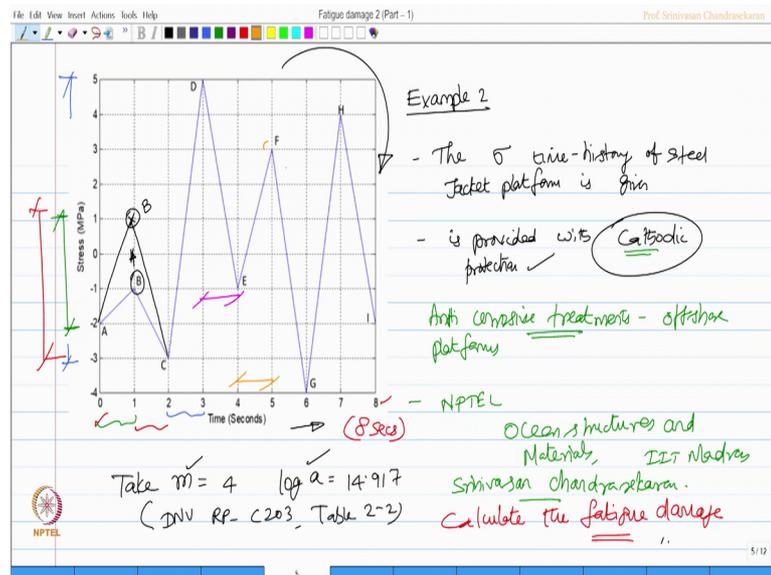
So, we know that the fatigue damage is 0.14480 in one year if you remember the hypothesis we can calculate the service life only when you equate this damage to unity whereas, this is not unity. Therefore, in simple terms service life is calculated by extrapolating this damage to unity if I do that then service life can be computed as 1 by 0.14480 which will be 6.906 year. So what we can say here is damage is equivalent to 1 in 6.906 years which would be the service life of the platform. So, that is the first problem we had the problem with the stress cycle is given to us.

The number of counts is known to me we have been asked to use a specific curve from the code, we are asked to compute the service life. So, what we did is we use this equation computed for a specific stress range computed the value of N, then found out the damage for a value. So, now we did this for all the stress ranges computed d sum

them up and found out the cumulative damage once cumulative damage is known this is for a record length of 3600 seconds. We can see here the simulated time is for 3600 seconds. So, if we did that we found out the damage for 1 second from that we formed the damage for one year we multiply it for 60 seconds 60 minutes 24 hours 365 days forward the value.

As per the hypothesis, we know that this damage should be equal to unity. So, fatigue damage is 0.14480 in one year. Therefore, equivalent damage will be extrapolating this number to unity which will get us 6.906 years which is actually the service life of the platform. So, that is a very clear illustrative example where the stress count and the stress bin are given. We will take up one more problem where the stress bin is not given, but the time history of the amplitude variation is given.

(Refer Slide Time: 13:18)



So, let us say this is my problem 2, example 2 says the stress time history how we can say time history the x axis I have time and in the y axis I have stress in Mega Pascal. So, the stress time history of a steel jacket platform is given the platform is provided with some arrangement which is called Cathodic protection.

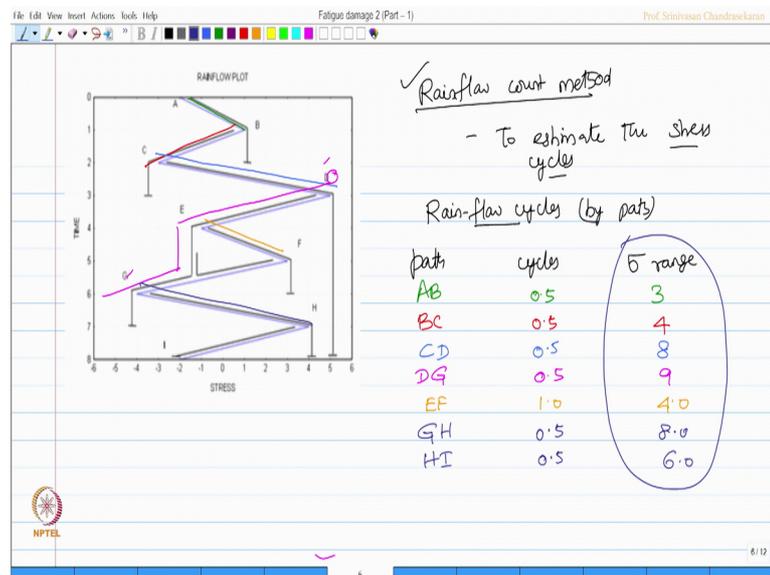
If you really wanted to know what are the anti corrosive treatments which are done for offshore platforms. I would request the readers to go through an NPTEL course on ocean structures and materials offered by iit madras I was the course coordinator. So, in that set

of lectures in module 2 we discussed very clearly; what are various treatment methods to protect offshore platforms.

Let us say the example problem has a provision of Cathodic protection and the value says take m as 4 and $\log a$ has 14.971 this is as per DNV recommended practice C203, Table 2-2. Because these values of m and $\log a$ are different because this platform has a Cathodic protection treatment enabled in the design. So, what is required is calculate the fatigue damage, one can very easily note here that the problem is subjected to or the platform is subjected to a stress range duration of about 8 seconds.

In the earlier example, it was for 3600 seconds. So, we should know also the duration of the stress cycle so it is 8 seconds is this is problem because it varies from 0 and goes till 8 so, we need to calculate the fatigue damage. So, what I did to compute the fatigue damage I am going to use a procedure called Rainflow count method. This is useful to estimate the stress cycles

(Refer Slide Time: 16:05)



In the earlier example, the stress cycles were completely and readily given to you, but in this example the stress cycle is not given, but only the stress variation and time that is stress history is given. So, we need to conclude the stress cycle I can use rain flow count method this is one of the easy methods by which you compute the stress cycle say how we are doing it.

So, what we did is I rotated this figure there is a figure available here I rotated this figure this line actually this point B is somewhere here please note that. So, this is my range I want to make an alteration here remaining all are but this point B is shifted up please note that so the problem has I changed point B is somewhere here A C everything is fine. So, I rotate this figure by 90 degree so if you do that I get a picture like this from this I want to now count the rain flow cycles path by path. Let us see how do we do it by path let us write down the path, let us then enters the cycles then also the stress range. Let us take the path AB, so path AB if you see the stress range of AB the stress range of AB is from minus 2 to 1 so it is 3.

If you look at the cycle A and B vary by a distance of 0.0 and 1 so I should say this is 0.5, similarly I do for BC which is again for C B C B and C the time variation is again an average of 0.5, but the stress value from B to C is actually from here to here by 1 and minus 3 which becomes similarly let us take the path C D C D the path C D varies from 2 to 3. So, therefore the variation is 0.5 the stress value of C and D is from 5 and minus 3. So, that becomes 8.

Now let us go to D G this is tricky D starts from here. So the rain drops cross countering to G E, it drops back again and goes to G. So, rain flow imagine a rain particle or a rain water droplet falls a D it rolls down touches G and come here.

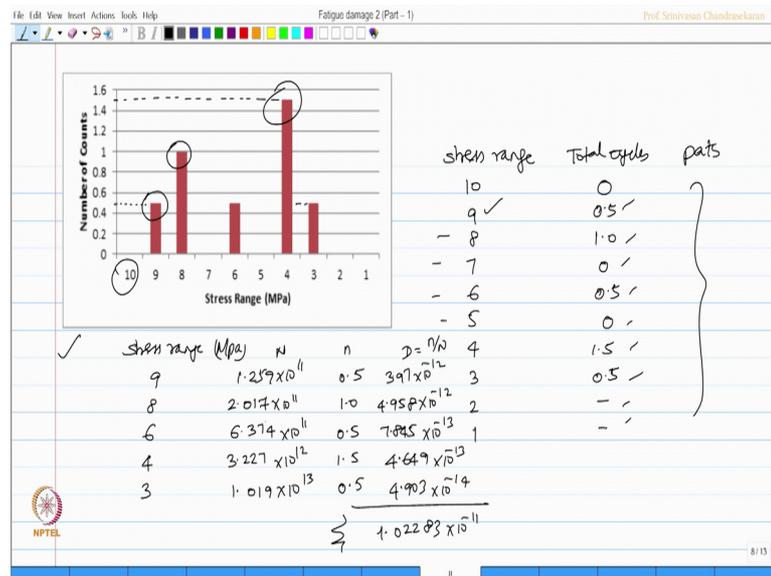
So, it is rolling from D E and then E to G so I should say the path is D G, D G, but for computing the stress cycle I will consider only D E, because after E it falls down. So D E, D E the stress cycle is only this much which is 0.5 for computing the stress range I start from D it falls in G.

So, let us go to D and come to G so minus 4 and 5 that gives me 9, then there is a path missed out here which is E F. So, I write E F, E F. Now E F will also returned back. Therefore, E F though E and F has only a gap of 1 which is 0.5 I take this as 1.0 because E F will roll back again and the value between E and F E and F for example, this and this is 1 this is 3. So, that becomes four then I go for G H which is 0.5 and G H G and H 4 and minus 4 that becomes 8.

Then H I is the last one which is again 0.5 and H I, H is at 4 and I is at minus 2 so, that becomes so we have compute this stress range, but what I wanted is the stress cycle which is called stress bin. So, from this I want to now find out the stress range and the

number of cycles let us do that. So, I take this table back again to the next cycle. So, let us say I want to compute the stress range. So stress values are anywhere varying from minimum is 3 and maximum is 9, minimum is 3 and maximum is 9.

(Refer Slide Time: 21:25)



So, let us start putting the stress range from let us say stress range total cycles and the path may not be very important let us say, stress range 10,9,8,7,6,5,4,3,2,1 let us go back here. Do we have anything on 10 here? We do not have anything on 10 therefore, let us put 10 as 0 do we have anything on 9 here we have 1 on 0.5, we have 1 on 0.5 let us say for 9 it is 0.5. Let us see that what do we have on 8, we have 8 this 1 is 8, this is also 8 so 0.5 and 0.5 so this becomes 1.0 Do we have anything on 7? We do not have anything on 7 so 0.

Do we have anything on 6? We have 1 on 6 here so which is 0.5 so let us say 0.5. Do you have anything on 5? We do not have anything on 5. So, let us say this is 0 check, we have anything on 4 yes we have we have this and we have this so, it becomes 1.5, so 1.5. See do we have anything on 3? Yes we have 1 which is 0.5.

Do we have anything on 2? No; do we have anything on 1? No. We can fill up the path I am not filling the path it is we already discuss the path here so it is only going to be reputation there so we are not interested in doing that. So, I plotted this as a histogram now. For example, you see 10 representing 0, 9 representing 0.5, this is actually 0.5, is it

not. So, 8 representing 1, 7 there is nothing, 6 representing 0.5. Again 5 is 0, 4 goes to 1.5 this is 1.5 and 2, 1, 1, 0.

So, in this previous problem you got the stress bin directly is an input for the problem whereas, in the second problem we used the rain flow common method from the stress history and computed the stress bin like this.

(Refer Slide Time: 24:04)

The image shows a digital whiteboard with the following handwritten content:

$$\log N = \log a - m \log S \quad m=4$$

$$\log a = 14.917$$

$$S = 9 \text{ N/mm}^2$$

$$\log N = 14.917 - 4 \log(9)$$

$$\log N = 11.10$$

$$N = 10^{11.10} = 1.259 \times 10^{11}$$

$$\text{Fatigue damage, } D = \frac{0.5}{1.259 \times 10^{11}} = \frac{3.97 \times 10^{-12}}{1.259 \times 10^{11}}$$

Once we found this then we can find the value of N so, we know log N will be log a minus m log S. Let us do it for one case let us take this as 9 so, S is 9 Newton per mm square because they are in Mega Pascal. So, log N you know in the problem please see m is 4 and log a is 14.917. So m is 4 and log a is 14.917 ok so let us use that. So, log N is equal to log a which is 14.917 minus 4 log 9 all these are log to the base 10. So like this you know N, you will get log N as 11.10. Therefore, N will be 10 to the power of 11.10 which we will get as 1.259 10n power 11.

So, I can now estimate the fatigue damage for this specific case D as 0.5 because that is the value you can see here 0.5 at 9 at 9 it is 0.5. So, 0.5 by 1.259 10 power 11 which gives me 3.97 into 10 power minus 12 this is for a specific value of s equals 9. So, for s equals 9 we computed. Similarly, one can do for 8, 7, 6, 5 etcetera. So, let me give the values here so stress range 9. So, I write here stress range in Mega Pascal N small n and D let us compute this.

So, 9, 8, 6, 4 and 3 we have only these values remaining all are zeros here I am not doing that for 9 we just now computed N is 1.259, let us entered that here 1.259 10 power 11 and N we know is 0.5 that is small value 0.5. Therefore, we computed D as 3.97 10 power 12 minus 12 sorry so, 3.97 10 power minus 12. Similarly for 8 you can compute the N value which comes to 2.017 10 power 11, 8 has a value of 1 so, enter 1.

So 1 by this that is this is actually equal to small n by capital N which is 4.958 into 10 power minus 12 for 6 we compute N value as 6.374 10 power 11 and for 6 the value is again 0.5 the small n D is 7.845 10 power minus 13 for 4 N is calculated as 3.227 10 power 12 and for 4 the value is 1.5. so 1.5 by this will give you 4.649 10 power minus 13 and for 3 N value is 1.019 10 power minus and for 3 the value is again 0.5. So, n by N will give you 4.903 10 power minus 14. If I make a sum the cumulative damage that comes to 1.02283 into 10 power minus 11, I want to now compute the service life.

(Refer Slide Time: 28:28)

Handwritten notes on a digital whiteboard:

- stress history = 8 sec
- fatigue damage calculated for 8 sec = 1.02283×10^{-11}
- fatigue damage for 1 sec = $\frac{1.02283 \times 10^{-11}}{8}$ ✓
- In a year, damage could be = $\frac{1.02283 \times 10^{-11}}{8} \times (60 \times 60 \times 24 \times 365)$
- = 4.03×10^{-5} in one year
- service life? - - - - -

So, the service life is very simple we already know the equation, the stress history the stress history is given for 8 seconds. Therefore, the fatigue damage calculated for 8 seconds is actually you can see here 1.02283 10 power minus 11 1.02283 10 power minus 11. So therefore, fatigue damage for 1 second is actually 1.02283 10 power minus 11 by 8 which gives me the value for 1 second.

So, in a year the damage could be 1.02283 10 power minus 11 by 8 this is for 1 second I multiply this by 60 60 24 and 365, I get the years which is approximately 4.03 10 power

minus 5 for one year. So, the fatigue damage is 4.035 in one year. If you want to find the service life equate that to 1 and find out the service life I leave this to you if you really want to find the service life.

You please find out what would be the service life in the same style what we did for the last example. So, the difference in this study or the second example with that of the earlier one is second example gave you only the time history which you see here, where as the previous example, you already had the stress cycle directly so there is a difference.

So, in this example you had this stress time history, we did the rain flow counting found out the calculation, made a table, found out the cumulative damage, plotted the stress time history, found out n , small n and damage cumulative. Once we did that we found out the damage in one year, from that one can also find the service life of the given system. So, 2 examples one stress history is given other stress time history is given stress bins are calculated from rain flow count method.