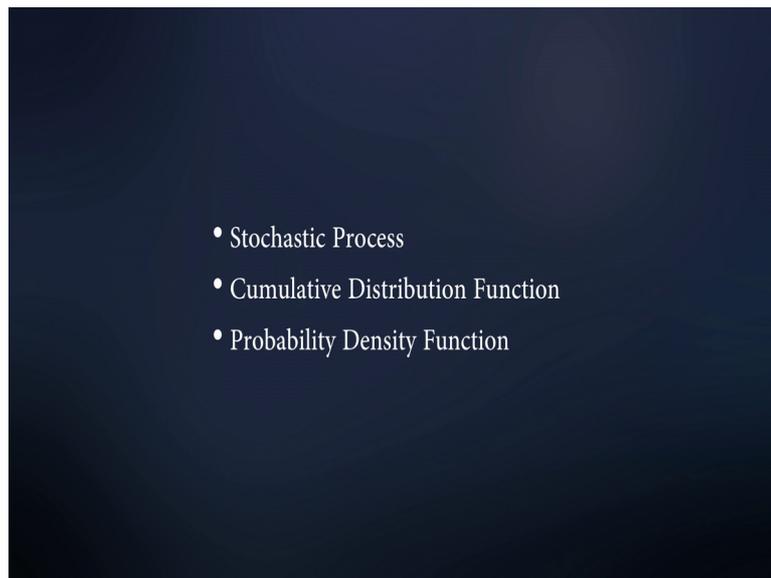


**Computer Methods of Analysis of Offshore Structures**  
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**Module - 03**  
**Lecture - 06**  
**Fatigue Damage 1 (Part - 2)**

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Now,  $E[\tilde{N}(a)]da$  is the expected no. of stress cycles with amplitude b/w  $(a)$  and  $(a+da)$ , which occurs during time  $T$

Hence, 
$$E[\tilde{N}(a)]da = \nu_x^+(a) T f_{x_p}(a) da$$

Where,  $f_{x_p}(a) da$  is the relative no. of peak with amplitude b/w  $(a)$  and  $(a+da)$

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Now, the expected value of  $N_{da}$  is actually the expected number of stress cycles with amplitude between  $a$  and  $a + da$ , which occurs during time  $t$  hence the expected value of  $n_{da}$  is now expressed as  $V \times \int_0^{\infty} f_x p_a da$  where  $f_x p_a da$  is the relative number of peaks with amplitude between  $a$  and  $a + da$ .

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$N_{x_0}^+(T)$  is Total # of peaks, which is equal to  
No. of stress cycles during the time,  $T$ .

Hence,  $D(T) = N_{x_0}^+(T) \int_0^{\infty} \frac{f_x p_a da}{N(a)}$

$= N_{x_0}^+(T) \int_0^{\infty} \frac{(2a)^m}{k} f_x p_a da$  — 5)

$V \times 0$  plus  $T$  is the total number of peaks which is equal to the number of stress cycles during the time  $T$ . Therefore,  $d$  of  $T$  is  $V \times 0 T$ , integral  $0$  to infinity  $f_x p_a da$  by  $N$  of  $a$  which expressed as  $V \times 0 T$  internal  $2 a$  to the power  $m$  by  $K f_x p_a da$  I call this as equation 5.

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for a narrow-band process, Gaussian is nature,

$$f_x(p) = \frac{a}{\sigma_x^2} \exp\left\{-\frac{a^2}{2\sigma_x^2}\right\} \quad (6)$$

Substituting eq (6) in eq (5), we get:

$$D(t) = \frac{V_x^+(t)}{(2)^{-m} K \sigma_x^2} \int_0^{\infty} a^{m+1} \exp\left\{-\frac{a^2}{2\sigma_x^2}\right\} da$$

For a narrow - band process, which is also Gaussian in nature  $f_x(p)$  of  $a$  is given by a by sigma x square exponential minus a square by 2 sigma x square now substituting that value in question 5. So, substituting equation 6 in equation 5, we get  $D$  of  $T$  is  $V \times 0$  by 2 minus  $m$   $K$  sigma x square integral 0 to infinity,  $a^{m+1}$  exponential minus a square by 2 sigma x square  $da$ . I call this as equation number 7.

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$$= V_x^+(t) \frac{(2\sqrt{2}\sigma_x)^m}{K} \Gamma\left(1 + \frac{m}{2}\right) \quad (8)$$

where  $\Gamma(x)$  denotes Gamma function, which is a standard tabulated function

For example,  $\Gamma(n+1) = n!$  for  $n = 0, 1, 2, \dots$

Which can be simplified as  $V \times 0 T 2 \text{ root } 2 \text{ sigma } x m \text{ by } K \text{ gamma function } 1 \text{ plus } m \text{ by } 2$ , where gamma x denotes a Gamma function, which is a standard tabulated function

whose values are available in the statistical table. For example, you want to find gamma n plus 1 is n factorial for n equals 0 comma 1 comma 2 and so on.

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Lifetime of the structure is estimated as below:

We need to substitute  $D(T) = \text{Unity}$

Substitute  $D(T) = 1$  in Eq (8)

$$T = \frac{k [V_{x0}^+]^{-1}}{(2\sqrt{2}\sigma_x)^m \Gamma(1 + \frac{m}{2})} \quad \text{--- (9)}$$

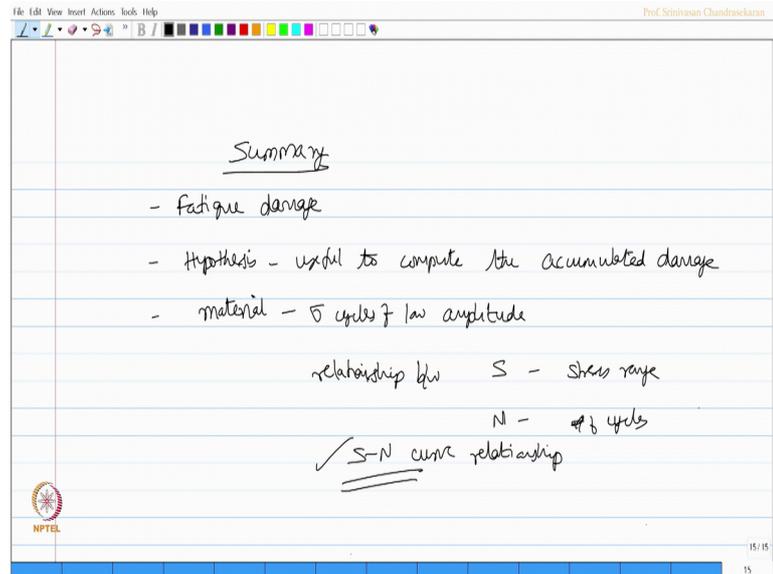
Also,  $[V_{x0}^+]^{-1} = T_z = \text{Zero-mean crossing period}$

$$T = \frac{k T_z}{(2\sqrt{2}\sigma_x)^m \Gamma(1 + \frac{m}{2})} \quad \text{--- (10)}$$

After, estimating the cumulative damage one is interested to know how to estimate the Life time of the structure it can be estimated as below, it is very simple we need to substitute D of T as unity that is the hypothesis. So, substitute D of T as one in equation 8. In equation 8, is what we see here substitute this value equals 1. So, I should say then in that case because equation 8 involves T, I am interested to find out t I substitute this equal to 1 and find T.

So, by readjusting I can write the equation for T as,  $K V x 0 T 2 \text{ root } 2 \text{ sigma } x \text{ to the power } m \text{ gamma function } 1 \text{ plus } m \text{ by } 2$  equation 9, also  $V x 0 \text{ inverse}$  there is an inverse sign here, is actually  $T z$  which is called 0 mean crossing period and now  $t$  which is the life time estimate is given by  $K T z 2 \text{ root } 2 \text{ sigma } x m \text{ gamma } 1 \text{ plus } m \text{ by } 2$ . So, this is my equation forgetting the Lifetime estimate of the structure.

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The image shows a digital whiteboard interface with a menu bar at the top (File, Edit, View, Insert, Actions, Tools, Help) and a toolbar with various drawing tools. The text on the whiteboard is handwritten in black ink. At the top center, the word "Summary" is underlined. Below it, there are three bullet points: "- Fatigue damage", "- Hypothesis - useful to compute the accumulated damage", and "- material - 5 cycles of low amplitude". In the center, there is a note "relationship b/w S - stress range" and "N - # of cycles". Below this, the text "S-N curve relationship" is written and underlined twice. In the bottom left corner, there is a small circular logo with the text "NPTEL" below it. In the bottom right corner, there is a small box containing the number "15".

So friends, let us look at the summary what we learnt in this lecture. We understood how to compute the fatigue damage, what is the hypothesis which is used to compute the accumulated damage when the material is subjected to stress cycles of low amplitude you can establish a relationship between the stress range  $S$  and the number of the cycles  $N$  which is famously called S-N curve relationship. This relationship is available for variety of marine steel in offshore structures in various International course.

So, in the next lecture we will take up an application example problem and try to estimate the fatigue damage given on different members based upon the material capacity. We will do couple of problems with an application problem in offshore structure. We will also discuss the computer code, how to estimate this damage using a simple procedure.

Thank you very much.