Computer Methods of Analysis of Offshore Structures
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Module - 03
Lecture - 05
Stochastic Modelling (Part - 2)

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Now, let us assume N experiments have taken place, what are experiments in this case?

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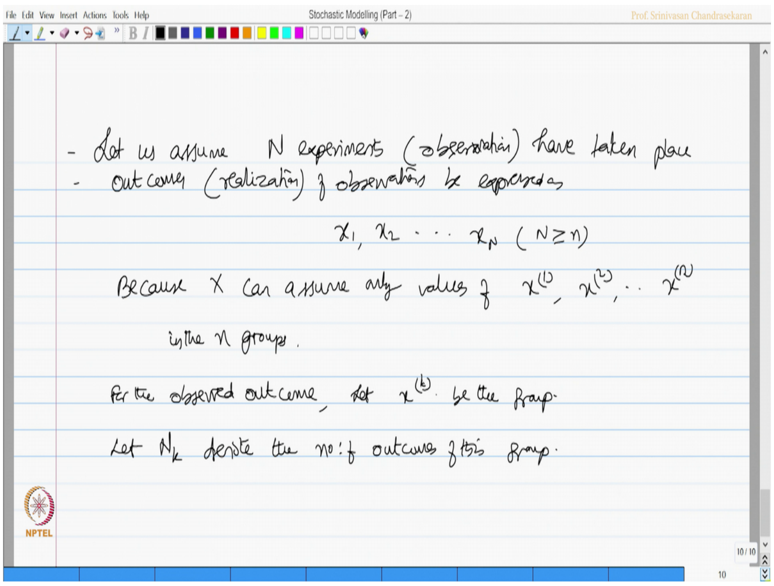

Observations of a value for realization have taken place and the outcomes which is nothing but realized values or realization of observations be expressed as the x 1, x 2 and x N where N is greater than equal to n. It means you pick up the threshold value and only observe when, the values exceeding here, because X can assume only values of let us say x 1, x 2, x n in the n groups. So, for the observed outcome let x k be the group value. Let N k denote the number of outcomes of this group.

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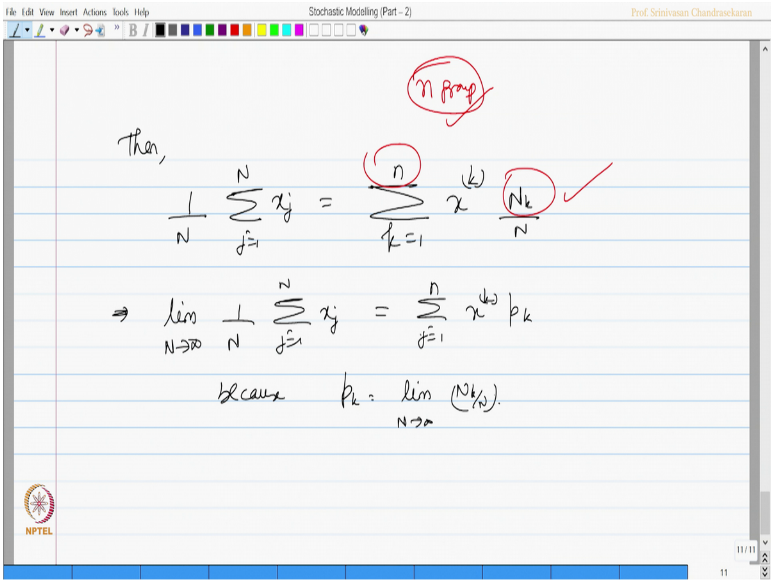

Then 1 by N summation of j equals 1 to N x j which is the mean of that value can be now said as k equals 1 to n, because I am looking for only the n group and the realized values are x k and I am going to say I am going to check only within the group and find the mean value. So, the mean is now redefined slightly in a different manner you can always find the mean for a given variable, if you are looking only for the realized values within a specific group.

So, this can also be further extended as limit N tends to infinity 1 by n summation of j equals 1 to N x j can be summation of j equals 1 to small n x k p k. We already know this is true, because p k is actually as we saw is given by this expression N k by N. So, probability of that values over the given data therefore, in general the expected value of the random variable.

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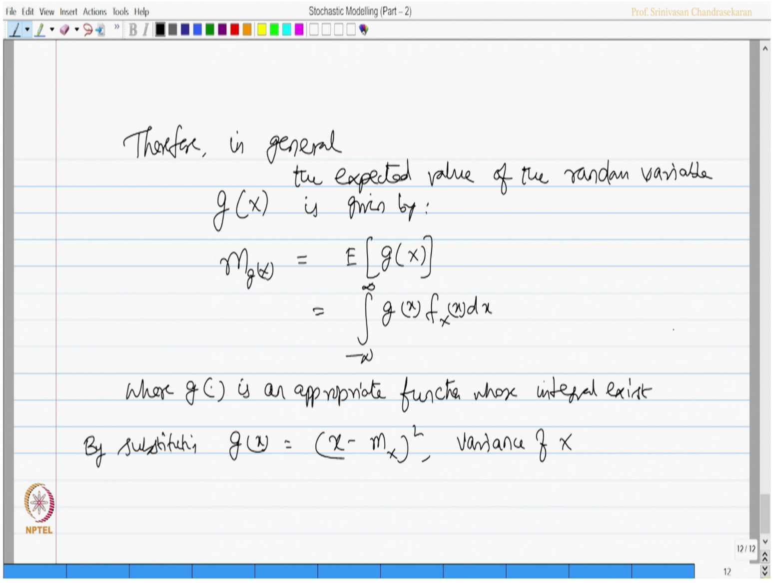

Let us say g of x is given by we are going for a generic expression now m g of x is expected value of g of x.

Which can be given as minus to plus infinity g of x f of x x d x where g is an appropriate function, whose integral exist; that is very important otherwise; you cannot be estimate or evaluate this integral. Now by substituting g of x equals x minus m x square; now I can also find the variance of X which can be given by variance of X are also expressed as sigma X square is expected value of X minus M x square which is again expressed as minus 2 plus integrity x minus M x square f of x x d x.

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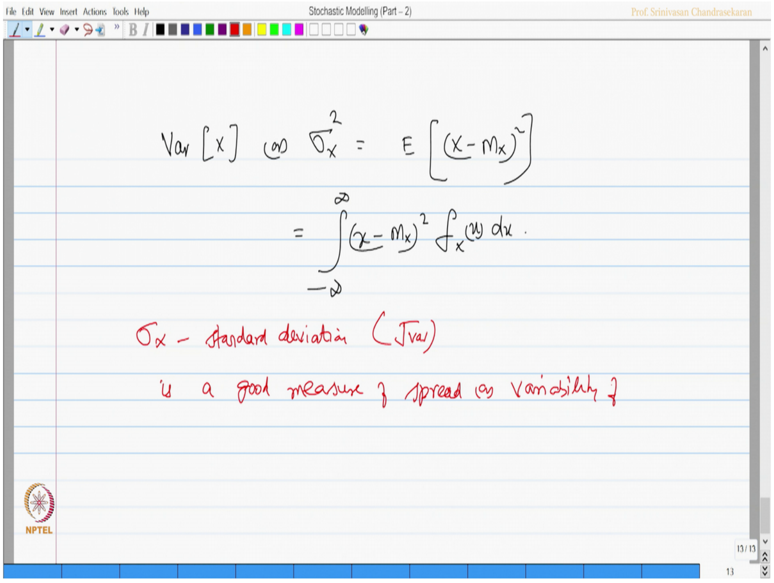

And we know that sigma X which is a standard deviation, which is actually a square root of variance is a good measure of spread or variability of outcomes of X. Further, we can also say sigma X square is again expected value of X minus M x square which can be said as expected value of X square minus 2 m x x plus M x square which can be said as expected value of X square minus M x square.

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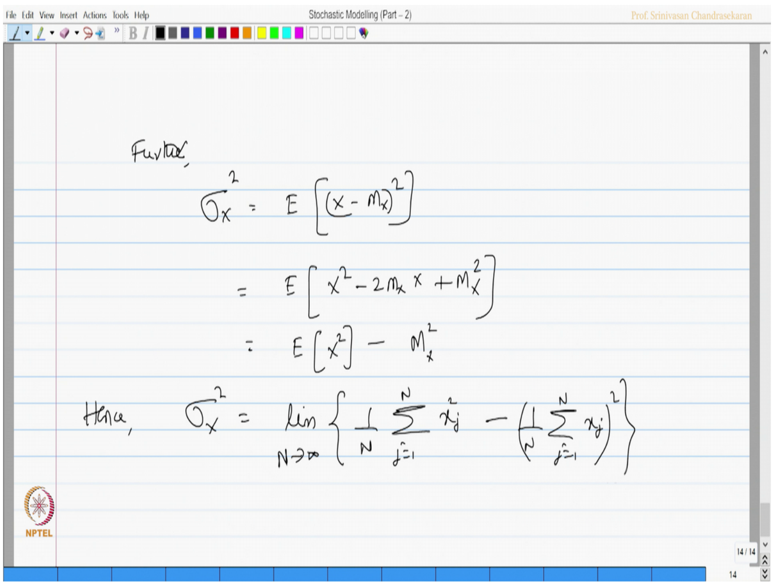

And hence sigma x square is limit N tends to infinity 1 by N of summation of j equals 1 to N of x j square minus 1 by N of j equals 1 to N of x j the whole square extending this logic.

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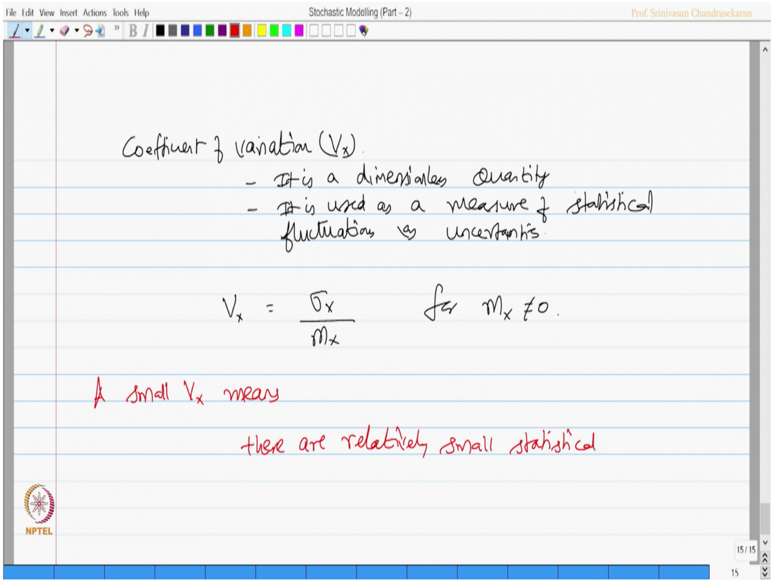

One can also find the coefficient of variation. Actually this is expressed as V x it is actually a dimensionless quantity, this is used as a measure of statistical fluctuations or what other ways we call as uncertainties is given by sigma x by M x for non zero mean process.

So, interestingly a small V x, that is coefficient of variation means; there are relatively small statistical fluctuations around the mean value. So, that is what we understand by expressing or determining V of x. So, friends let us reiterate this statement which we already made.

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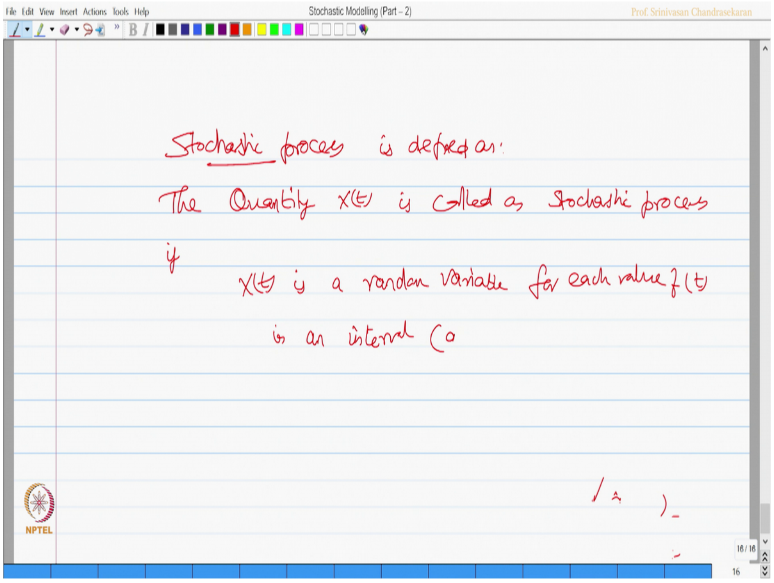

So, stochastic process is defined as the quantity X of t is called as stochastic process if X of t is a random variable for each value of t in an interval which is designated say a and b. Let us quickly take an example of a stochastic process.

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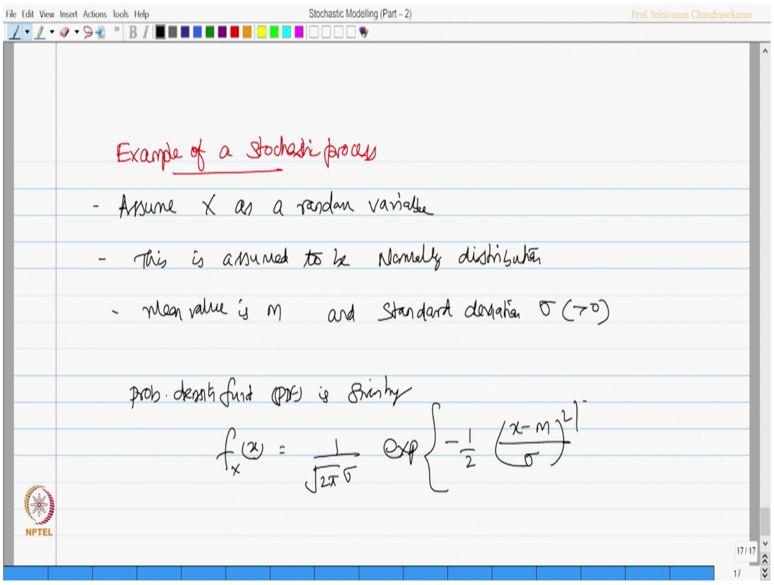

Assume X as a random variable, this is assumed to be normally distributed there are different forms of distribution available in statistics.

Mean value of the variable is m and standard deviation is greater than 0 the probability density function pdf is given by for your normal variate we know this equation, but still let us write this 1 by root pi sigma exponential minus half x minus m by sigma the whole square.

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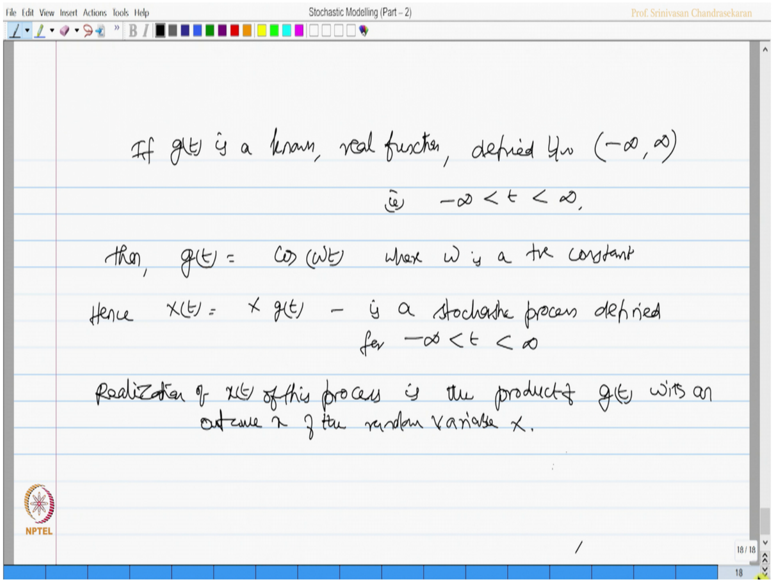

If g of t is a known real function which is defined between the interval minus infinity to plus infinity that is exist between minus infinity to plus infinity, then g of t cos omega t where omega is a positive constant. Hence, X of t is also true as X g of t which is again a stochastic process defined for minus infinity to plus infinity. Now realization X of t of this process is actually given by

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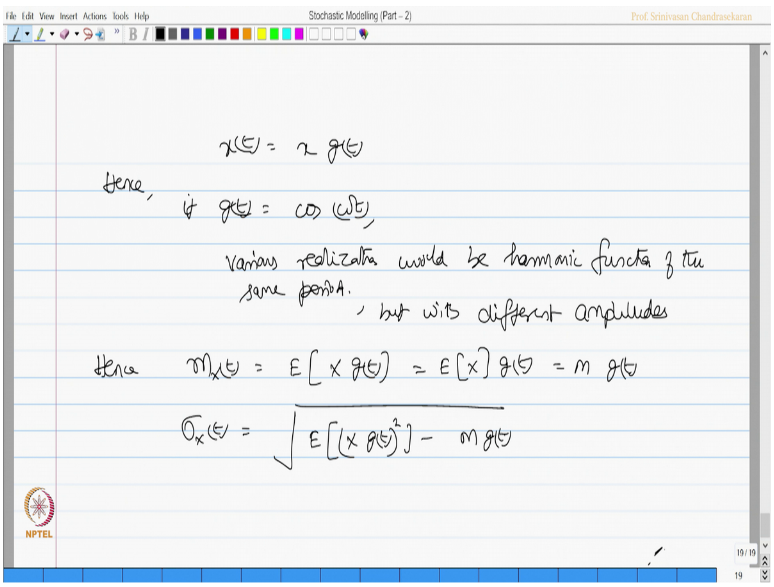

The product of g of t with an outcome X of the random variable X mathematically X of t will be X g of t.

Hence, if g of t is cos omega t, then various realization would also be harmonic function of the same period, but with different amplitudes hence the mean of value this expected value of X g of t is expected value of X g of t is nothing but m g of t and sigma X of t a square root of expected value of X g of t square minus M g of t square which can be expected value of X square g of t square minus m square.

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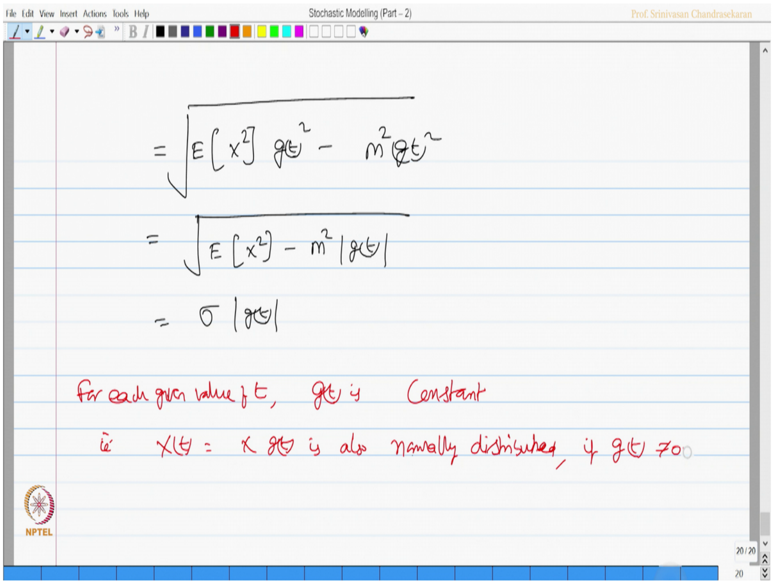

G of t square which can be said as square root of expected value of X square minus m square g of t which is actually equal to sigma g of t therefore, for each given value of t g of t is constant that is X of t is X g of t is also normally distributed, because we have assumed y normal variate this is true if g of t is not zero and hence,

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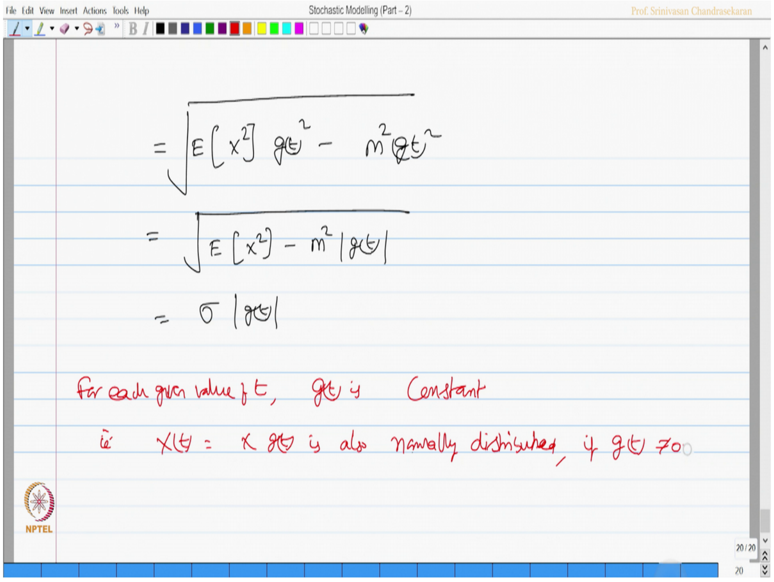

The pdf of X of t is given by f of X t X which is 1 by root 2 pi sigma g of t mod the exponential of minus half X minus M g of t by sigma g of t the whole square.

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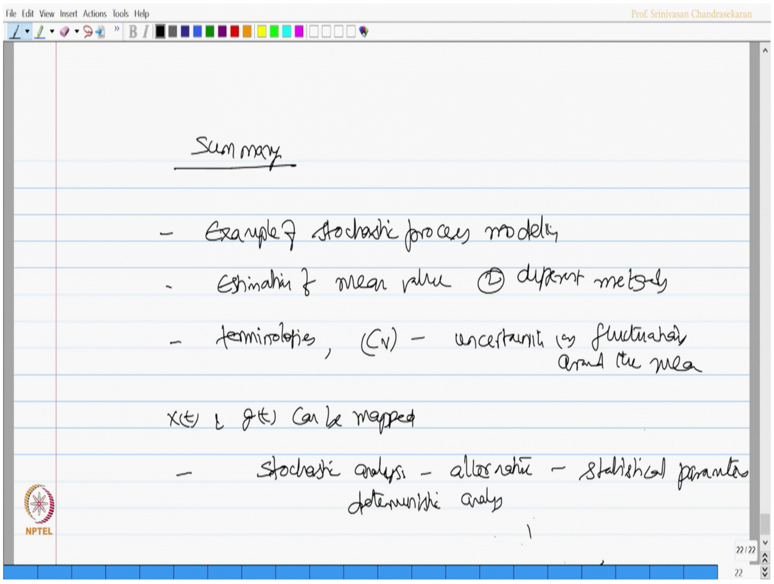

So, friends in this lecture we understood the example of stochastic process. Modelling we understood estimating of or estimation of mean value by two different methods. We have also understood certain terminologies one classical and interesting terminology C v coefficient of variation which is very interesting for us to know the uncertainty or fluctuations statistical fluctuations around the mean. We have also understood that how X of t and g of t can be mapped and we estimate the statistical parameters which are important. So, why we are looking for statistical parameters, because stochastic analysis is an alternative which uses statistical parameters for analyzing instead of deterministic analysis.

In the next lecture we will take up an example of fatigue prediction, because is an important application in offshore structures. We will talk about fatigue prediction in couple of lectures and give lot of example problems and also coding the mathematical coding to estimate fatigue predictions.

Thank you very much.