Computer Methods of Analysis of Offshore Structures
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Module - 01
Lecture - 06
Stiffness matrix of Beam With Varying EI (Part – 1)

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Friends in the last 2 lectures, we understood how to derive the stiffness matrix for a beam element without considering the axial deformation and we also said very clearly out of the 16 coefficients of the beam matrix; stiffness matrix, if we are able to derive only the 4 coefficients, remaining 12 can be easily derived based upon these 4 coefficients. We have learnt from the first principles; how to derive these coefficients for a beam element. We made an important assumption in that derivation saying that the beam element has uniform flexural rigidity which is uniform E I.

Now, let us take an example, if I have a beam without uniform E I that is varying E I; how do I derive a stiffness matrix. So, let us take an example of a beam which is shown in the screen. Now which has got a depth which is varying from 600 at 1 n, 600 mm to 400 mm at the other end for a length of 5 meters and let the cross section be 300 mm wide; this is 300 mm wide.

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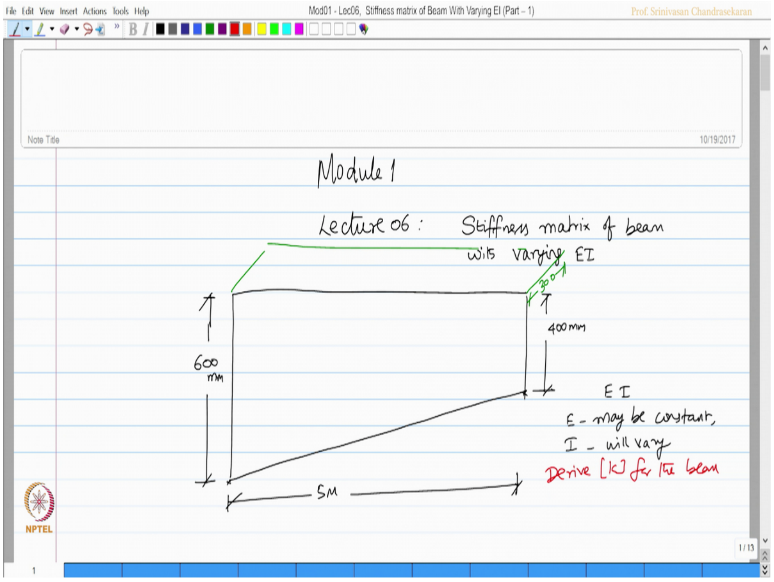

The beam is like this. So, now, the beam has got varying depth. So, the E I of the beam will now vary even though E may be constant for same material, but moment of inertia will vary continuously from this point to this point or from this section till this section; how to handle this problem. So, the question is derive the stiffness matrix for the beam element with varying E I, we also say neglect axial deformation, let us do this problem now.

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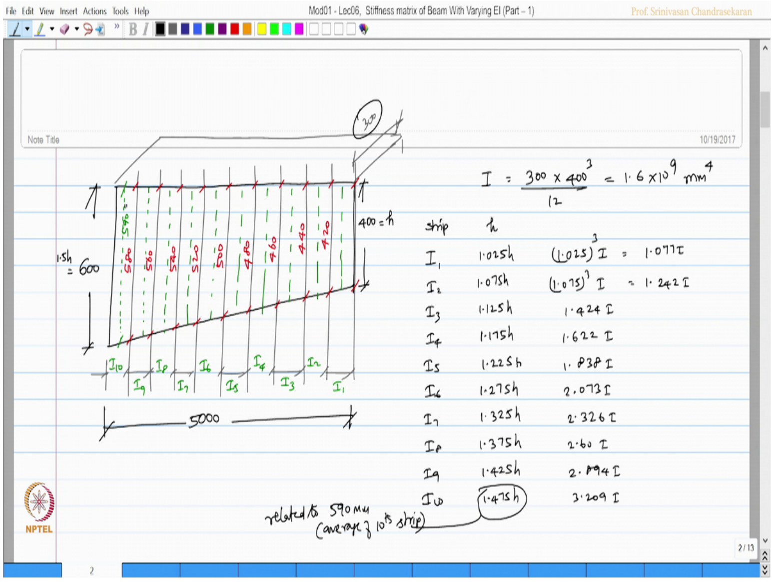

So, let us say I divide this beam into 10 parts; say this is 600 here and this is 400 here. So, what I do? I divide this into 10 parts. So, this is 600 and this is 410 parts.

So, now I mark these 10 parts with different moment of inertia we already know that the section has width of 300 width of 300; therefore, the moment of inertia, I will be actually equal to 300 into 400 cube by 12 which actually is equal to 1.6 10 power 9 mm to the power 4. Now I want to calculate the moment of inertia at different steps as indicated let us mark these steps as let us say I 1, I 2, I 3, I 4, I 5, I 6, I 7, I 8, I 9 and I 10 where I stands for the moment of inertia and 1 to 10 stands for the number of strips. So, I can keep on estimating these values; for example, I 1, I want to find I 1; I would like to know the depth here. So, if this is 600; let us say this depth will be 580, I will write down the value this is 580m this depth will be 560 because there is a gap of 10 and this total is 5000; it is 5 meters; you can see here it is 5 meters. So, each is 50. So, keep on dividing this. So, I get 560, similarly, 540, 520, 500; this depth and this will be 480 and this will be 460 and this will be 440 and this will be 420 and of course, this is 400.

We also know that the average thickness here is 60 and 580. So, this will be 590, this is 590, similarly I can find average thickness at the middle of every sector. So, I would say that if this value is h this will be 1.5 h therefore, I can convert these values into ratio of this h; for example, this will be 1.475 h. So, let me draw another figure saying that for I 1; what would be the h, this is strip value and what would be the h. So, I 2, I 3, I 4, I 5, 6, 7, 8, 9 and 10; B is same in all the cases; only h is going to vary. So, I am now working out the h for the average thickness. So, for I 10, it is going to be 1.475 h and I 9 1.425 h and I 8; 1.375 h, 1.325 h, 1.275 h, 1.225 h, 1.175 h, 1.125 h, 1.075 h and 1.025 h

Please understand these values are related to let us say this value is related to 590 mm. So, it is nothing, but the average value of the tenth strip. Similarly all these are average values, once I have these and B is same I know very well that this I 1 can be expressed in terms of I which will be 1.025 cube of I which is 1.077 I. Similarly this will be 1.075 cube of that of I which will be 1.242 I, similarly this will be 1.424 I, 1.622 I, 1.838 I, 2.073 I, 2.326 I, 2.60 I, 2.894 I and 3.209 I.

So, all moment of inertia is a different section expressed in terms of I as you see in this table and these values are easily computed based on the average thickness of the each strip once I do this then next step is to compute the loading diagram. So, let us say I have a beam which is simply supported I will not apply a unit moment to the beam.

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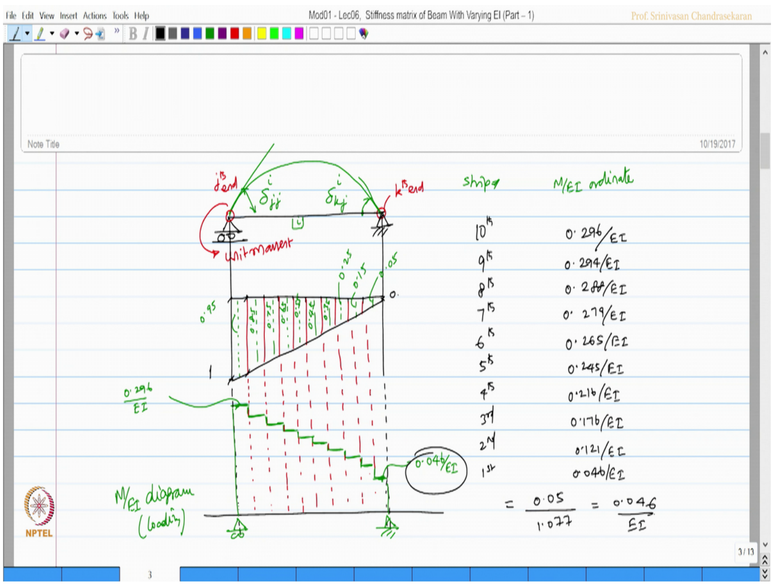

So, I am deriving the flexibility coefficient unit moment to the beam when I do that the beam will deflect like this lets say this is going to be my delta j j and this is going to be my this is of the ith beam, this is going to be my delta k j of the ith beam, please recollect this is my jth end, this is my kth end, I have given unit moment at the jth end I get delta j j and delta k j for the ith beam. So, it is my ith beam now this unit moment can be also distributed between the 10 strips I apply unit moment here this is unity and I give 0 moment here there are 10 strips .

So, let us try to find out what would be the value of these strips and the corresponding moment caused by this let us do this. So, for example, at this centre is going to be 0.95 at this centre is going to be 0.85, at this centre it is going to be 0.75, at this centre 0.65, at this centre 0.55, at this centre 0.45, at this centre 0.35, at this centre 0.25 and at this centre 0.15 and at this centre 0.05. These are the moments we have now; I have the I diagram here, I have the I values here; I have the M values here. Now I want to create the M by E I diagram for the conjugate beam. So, this is my beam which will have the loading diagram according to the strips.

Let us mark the strips, there are 10 strips 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; let us draw the M by E I diagram M is known E I, E is known and I is known for example, this value will be let us say for the tenth strip for the strip number, let us say M by E I ordinate; let us work out; this let us try to work out for the tenth for the tenth strip ninth eighth seventh sixth fifth fourth third second and first. So, now, this is unity this is 0.9. So, the average is 0.95. So, the ordinate here will be 0.296 by E I because M is 0.95 and I 10 is 3.209. So, if we apply these 2 you get 0.296 by E I and this will be 0.294 by E I this will be 0.288 by E I, this will be 0.279 by E I, 0.265 by E I, 0.245 by E I, 0.216 by E I, 0.176 by E I, 0.121 by E I and 0.046 by E I.

So, let us try to mark this here 0.296, 0.294, 0.288, 0.279, 0.265, 0.245, 0.216, 0.176, 0.121, 0.046. So, let us say this is my M by E I diagram which now become the loading diagram for my conjugate beam. So, this is my M by E I diagram which is actually the loading diagram. So, correspondingly for example, this value will be equal to 0.296 by E I and this value will be equal to 0.046 by E I and so on. So, if you want to really do the calculation 0.046 E I can be explained as 0.05 that is the moment divided by the I value is actually equal to 1.077 is it not. So, 1.077 will actually give you 0.046 by E I; similarly all of these things can be calculated

So, this becomes a loading diagram; therefore, let us apply the loads on this beam and try to find the reactions. So, what I do I call this end as let us say A, this as B to get V B, I should take moment about this point; now to get B B that is reaction and the Bth element. So, taking moment about a we get let us see what do we get I will get V B which will be actually equal to this span is 5 meters each one is 0.5 we got the total as 5 meters 10 strips. So, each one is 0.5; is it not?

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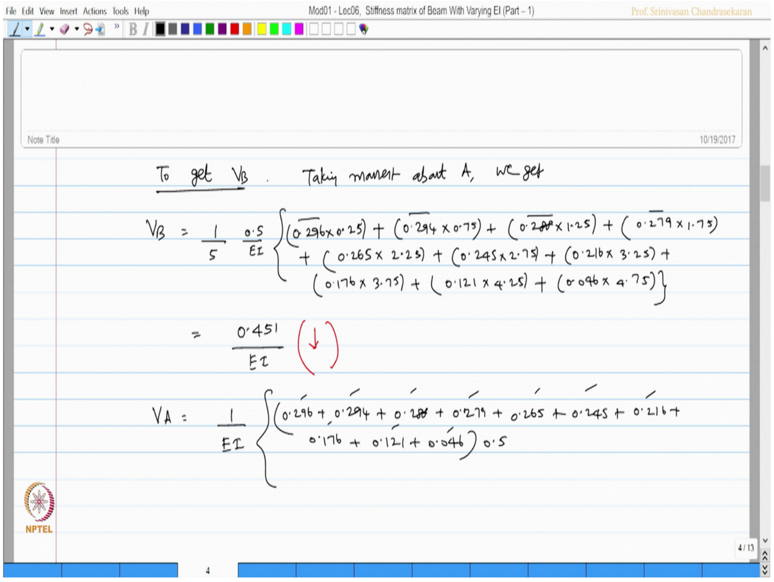

So, I should say V B will be equal to 1 by 5 of 0.5by E I which is the strip width; let us start taking from 1 by 1; the tenth strip is 0.296 and the one-tenth of this will be 0.25. So, I should say 0.296 into 0.25 plus the other one is 0.294. So, 0.294 into the one-tenth of that will be 0.5 plus 0.25 which is 0.75.

It means this will be differing by 0.5. So, plus the next one is 0.288 into 1.25 plus 0.279 into 1.75 because differ way 0.5 plus 0.265 into 2.25 plus 0.245 into 2.75 plus 0.216 into 3.25 plus 0.176 into 3.75 plus 0.121 into 4.25 plus 0.046 into 4.75 which will give me V B; you work out V B will be 0.451 by E I, but with the downward sign because you see here these will create an anticlockwise moment about A. So, V B will be reversed. So, V B is upside down.

Now, I can easily find V A. So, V A will be actually equal to 1 by E I of the total force that is 0.296 plus 0.294 plus 0.288 plus 0.279 plus 0.265; I am taking only these values plus 0.245 plus 0.216 plus 0.176 plus 0.121 plus 0, 4, 6, 10; value 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 values into 0.5 because 0.5 is the width of the strip width of the strip is 0.5 into 0.5 that is the total force I subtract 0.451 from this to get V A which will be 0.662 by E I which is upward.