

**Computer Methods of Analysis of Offshore Structures**  
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**Module - 03**  
**Lecture - 04**  
**Return Period & Stochastic Process (Part - 1)**

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When you talk about computer methods of structural analysis applied to offshore structures, we all now understand that offshore structures encounter loads which belong to are originate from a stochastic process or a random process. In the previous few lectures we discussed about salient characteristics of random process importantly, one designer is interested to know what would be the period within which or after which a load amplitude will reoccur on a given structure, this is what we call as return period, Return period and stochastic process have a very close association. So, in lecture four now we will talk about return period and more details about stochastic process.

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Module 3

Lecture 4: Return period and Stochastic process

Let  $Z$  be a random variable

$$p = \text{Prob}(Z > z) = 1 - F_z(z) \quad (1)$$

Assume that we can make series of observations of  $Z$ .

mean value of the observations to the first time observed or measured value of  $Z$  exceeds  $z$  is called as Return period.

Let us say  $Z$  be a random variable,  $p$  be the probability of  $z$  exceeding  $z$  which can be simply given by  $1 - F_z(z)$ . I call this as equation 1.

Assume that we can make series of observations of  $z$ , now once you make series of observations mean value of the observations to the first time observed or maybe measured value of  $z$  exceeds  $z$  is called as return period.

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Return period is indicated as  $\bar{R}(z)$

$$\bar{R}(z) = \frac{1}{p} = \frac{1}{1 - F_z(z)} \quad (2)$$

The above Eqn can also understood as

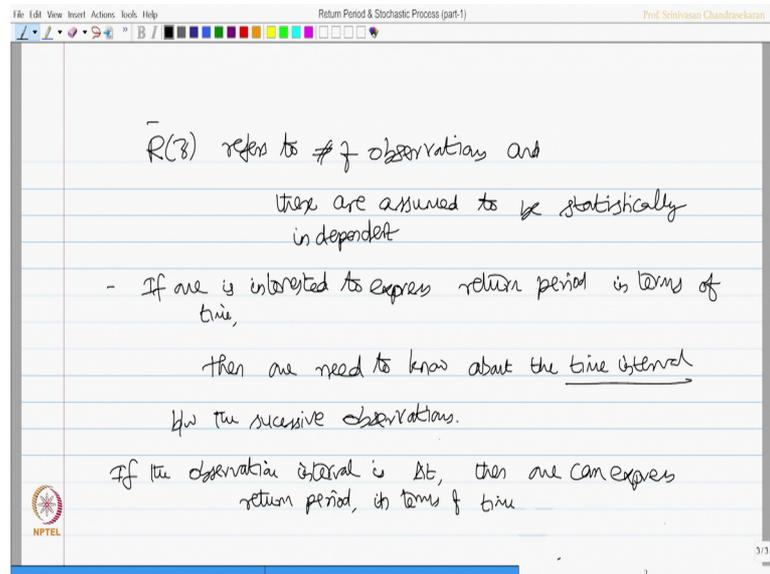
an average of  $\frac{1}{p}$  trials of an event.

which should be conducted before an event of probability  $p$  occur

Let us elaborate this more in detail, return period is indicated as  $\bar{R}$  of small  $z$  which is given by  $1/p$  by probability which is  $1 / (1 - F_z(z))$  equation number 2.

Now, the above equation can also be understood as an average of 1 over  $p$  trials of an event, this should be conducted before an event of probability  $p$  occurs.

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The screenshot shows a presentation slide with a white background and blue horizontal lines. The text is handwritten in black ink. At the top, there is a toolbar with various icons and a title bar that reads "Return Period & Stochastic Process (part-1)" and "Prof. Srinivasan Chandrasekaran". The main text on the slide is as follows:

$\bar{R}(z)$  refers to # of observations and  
these are assumed to be statistically  
independent

- If one is interested to express return period in terms of  
time,  
then one need to know about the time interval  
b/w the successive observations.

If the observation interval is  $\Delta t$ , then one can express  
return period, in terms of time

In the bottom left corner, there is a small logo for NPTEL. In the bottom right corner, there is a small number "3" and a small icon.

Now,  $\bar{R}$  of  $z$  refers to the number of observations and these are assumed to be statistically independent that is an important statement.

Suppose if one is interested to express return period in terms of time, then one need to know about the time interval between the successive observations. If the observation interval is  $\Delta t$  then one can express return period in terms of time which can be given by  $R_z$  is  $\Delta t \bar{R}_z$  which I call equation 3.

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$R(z) = \Delta t \bar{R}(z) \quad (3)$

The observation interval must be chosen sufficiently long because individual observations should be approximately independent.

For example, a design load has probability of  $10^{-2}$  of being exceeded during one year.

Interestingly the observation interval must be chosen sufficiently long that is an important statement why because, individual observations should be approximately independent. Let us apply this for an example case and see what happens. Let us take for example; a design load has probability of 10 power minus 2 of being exceeded during one year.

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Prob of exceedance of the design load in 1 year } =  $10^{-2}$

(a commonly used scenario is analysis of offshore structures)

If we could define  $F(z)$  as the relevant load process considered is the design provision, and  $z$  denotes the corresponding load level, then

So, probability of exceedance of the design load in one year is taken as 10 power minus 2, which is a commonly used scenario in analysis of offshore structures. Suppose if we

could define  $F$  of  $t$  as the relevant load process, which is considered in the design provision and  $z$  denotes the corresponding load level.

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The image shows a digital whiteboard with handwritten mathematical notes. The text is as follows:

$$\text{Prob}(Z > \xi) = 0.01 \text{ - mathematically}$$
 where  $Z = \max(F(t), 0 \leq t) \leq \underline{1 \text{ year}}$

Here, Return period, of exceedance of  $\xi$  then becomes

$$\bar{R}(z) = \frac{1}{\text{prob}(z > \xi)} = \frac{1}{0.01} = 100 \text{ yo}$$

Reference period, in the above example is One year

Return period of exceedance is 100 yo

The whiteboard interface includes a menu bar (File, Edit, View, Insert, Actions, Tools, Help), a toolbar with drawing tools, and a title bar that reads "Return Period & Stochastic Process (part-1)" and "Prof. Srinivasan Chandrasekaran". An NPTEL logo is visible in the bottom left corner.

Then probability of  $z$  exceeding  $\xi$  is 0.01 mathematically is it not where  $z$  is now the maximum of  $F$  of  $t$ ,  $0 \leq t \leq 1$  year.

Because the probability of exceedance of this value of  $10^{-2}$  is for one year; hence return period of exceedance of  $z$  then becomes  $\bar{R}(z) = 1 / \text{probability of } z \text{ exceeding } \xi = 1 / 0.01 = 100$  years. So, the reference period in the above example is one year and return period of exceedance is 100 years.

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It is important to note the following.

- (1) Time-varying loads (wave loads, wind loads) cannot be considered as stationary, over an extended period of time
- (2) This implies that Quantities such as yearly maxima must be calculated using long-term statistics

Friends it is important to note the following time varying loads like wave loads wind loads etcetera cannot be considered as stationary over an extended period of time therefore, this implies that quantities such as yearly maxima must be calculated using what we call long term statistics.

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(a) Return period, calculated based on prob of exceedance

(b) Return period, can also be estimated based on the Risk associated.

Example: Earthquake event

→	{	DBE: 10% risk @ occurrence of 50 yrs
	}	MCE: 2% risk @ occ of 50 yrs.

$$R = 1 - \left(1 - \frac{1}{T}\right)^n$$

0.1 =  $1 - \left(1 - \frac{1}{T}\right)^{50}$ , T, return period = 475 yrs ✓

0.02 =  $1 - \left(1 - \frac{1}{T}\right)^{50}$ , T = 2500 yrs ✓

One can also estimate return period can also be estimated based on the risk associated that is a second component. What you have learnt in the first component was return period calculated based on probability of exceedance. Let us now see how to estimate

return period based on risk associated, we will take an example of an earthquake event. Let us say there are different levels of earthquake considered for design of strategic structures like offshore structures design basis earthquake, which has got 10 percent risk at occurrence of about 50 years. Maximum credible earthquake which is 2 percent risk at occurrence of 50 years.

So, now, I want to estimate return period based upon the risk level. So, return period can be given as  $1 - (1 - R)^T$  where  $R$  is the risk and  $T$  is the return period. So, let us substitute the risk is 0.1 that is 10 percent  $1 - (1 - 0.1)^T = 0.1$  which amounts  $T$  the return period as 475 years. Similarly for 2 percent risk  $1 - (1 - 0.02)^T = 0.02$  which gives me  $T$  as 2500 years. So, friends these example simply illustrates depending upon the risk admittance in a given design return period can be as close or as far away as 2500 years.