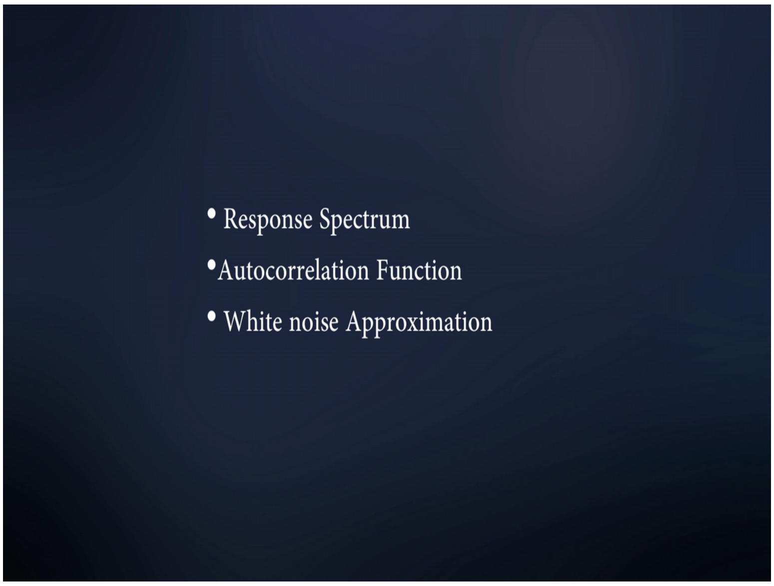
Computer Methods of Analysis of Offshore Structures

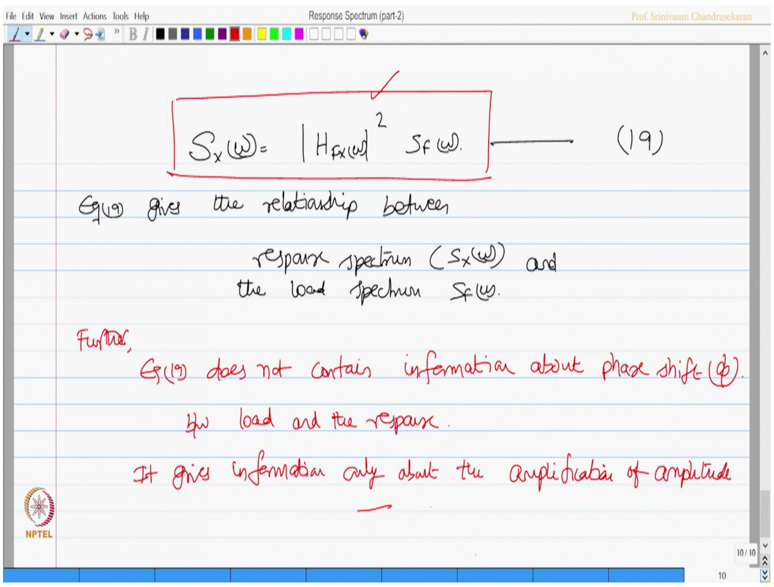
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture - 03
Response Spectrum (Part - 2)

(Refer Slide Time: 00:17)

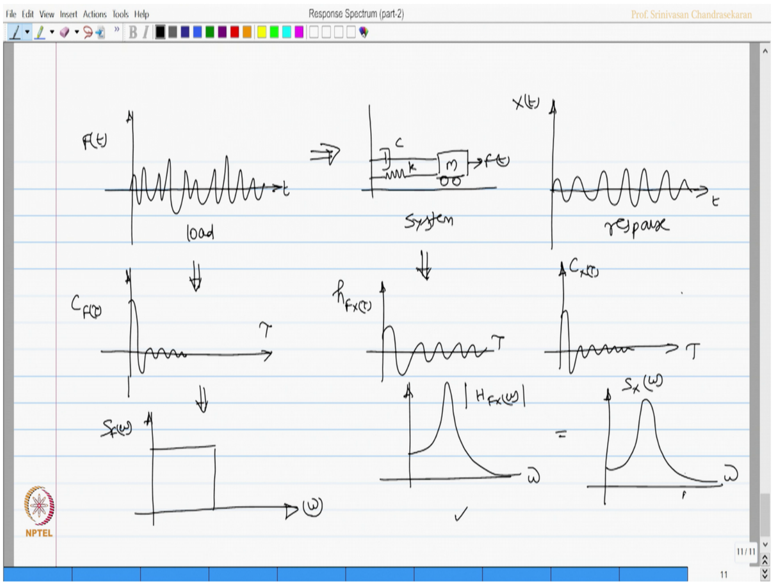


(Refer Slide Time: 00:22)



Sx omega can be HFX omega square of Sf omega; I call this as equation number 9. So, equation 19 gives the relationship between the response spectrum Sx omega and the load spectrum Sf omega. So, that is a very important relationship we arrived in this lecture further there are some interesting features about equation 19. Equation 19 does not contains information about phase shift phi between load and the response. You can see the term is square therefore, the phase shift concept are the parameter is lost. So, it gives information only about the amplification of amplitude that is very important let us try to understand this graphically let us say I have F of t which can be a signal like this on a time scale representing F of t.

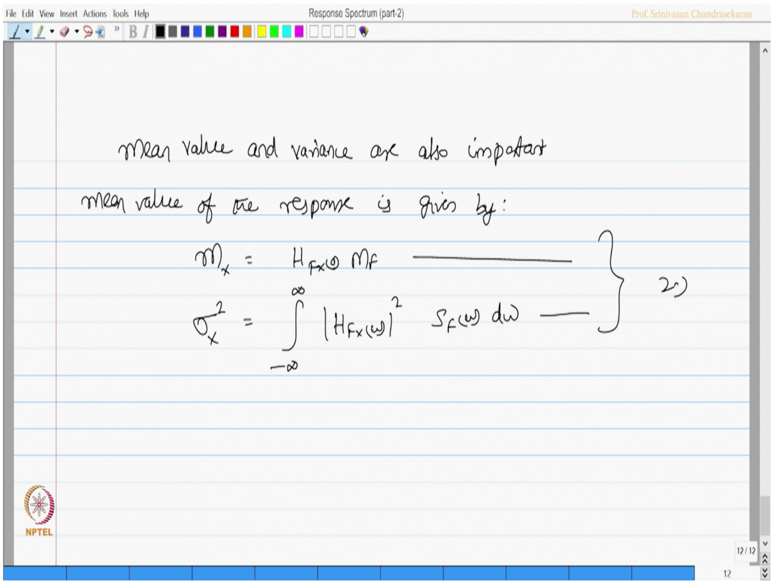
(Refer Slide Time: 01:47)



Let us say this is my load I take an autocorrelation of this, the auto correlation function typically comes like this say this is my C F of tau and ultimately this will give me the plot of the transfer function in terms of omega and S F omega which can typically look like this. If I apply this load to a system which a dynamic system which has got a mass which is got a damping force, which is got also a restoring force and external excitation is what we are applied here if this my dynamic system, whose response looks like this on a time history. The system has a property which is transfer function, which can be this way and this can be H FXtau, the system will also have the response whose autocorrelation is given by this plot ultimately the system will have a response.

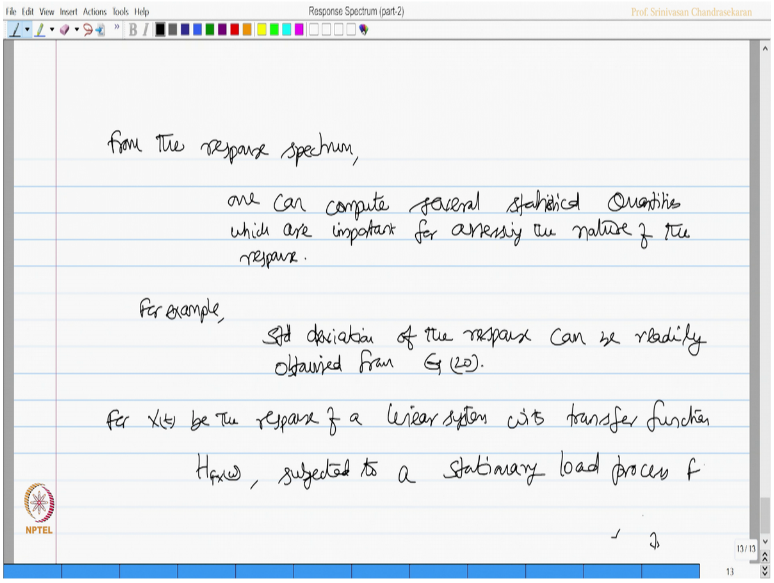
Which is get amplify for a given damping for various values of omega, which is the mod value of H FXomega. This will now lead to the response of the system in spectral ordinate which is Sx omega which we call as response spectrum. So, this becomes my load spectrum and this becomes my response spectrum and this becomes my transfer function which gives only the information about the amplitude and not about the phase shift between the response applied my response obtained and the force applied on the given system.

(Refer Slide Time: 04:17)



In addition mean value and variance are also important. So, mean value of the response is given by mx is H FX0 of mf which we already derived and the variance is minus to plus infinity H FX0 omega square Sf omega d omega I call this equation as equation twenty therefore, from the response spectrum.

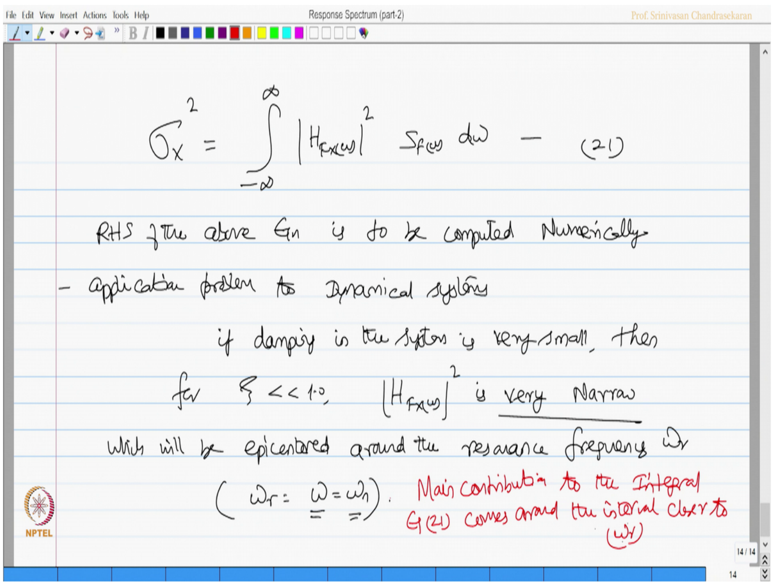
(Refer Slide Time: 04:57)



One can compute several statistical quantities which are important for assessing the response.

For example standard deviation of the response can be readily obtained from the variance that is from equation twenty for the response function be the response of a linear system transfer function H FX omega applied or.

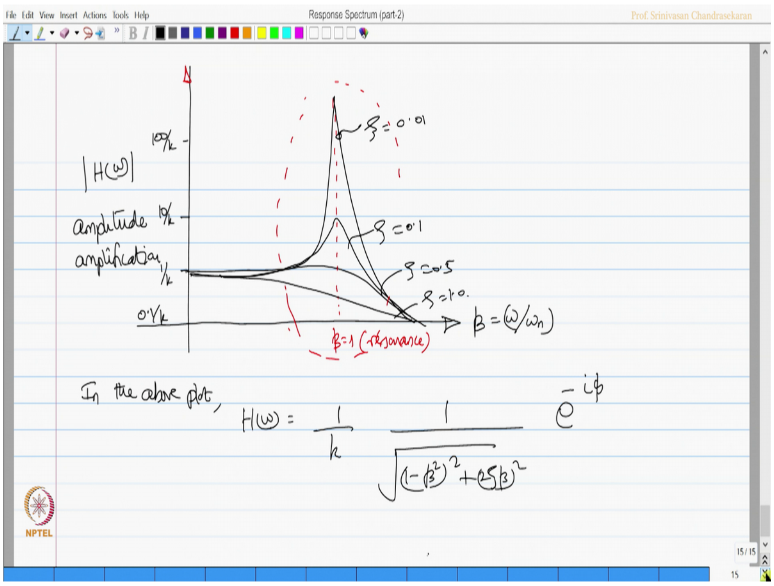
(Refer Slide Time: 05:46)



Let us say subjected to a stationary load process f of t sigma x square which we wrote earlier as Fx omega square Sf omega d omega is now valid, the right hand side of this equation is to be computed numerically. Now as an application problem to dynamical systems if damping in the system is very small then for damping to be very small than unity the transfer function is very narrow.

Which will be focused or which will be epicentered around the resonance frequency omega r where omega r is omega as omega n. There is excitation frequency and the natural frequency of the system matches; this implies a very important statement that main contribution to the integral in equation 21, comes around the interval closer to omega r that is very important statement.

(Refer Slide Time: 07:56)

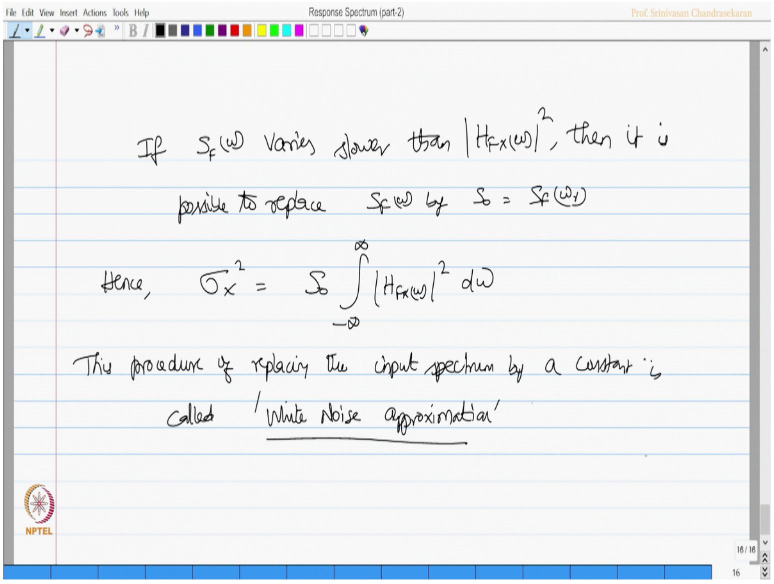


Let us try to understand this graphically, let us say I tried to plot the ratio of forcing frequency to a natural frequency, let us try to plot this closely in the range of beta equals one which is resonating, let us try to plot in the y axis h omega the absolute values we call amplitude amplification.

Say this is point 1 by k this is let is say 1 by k this is 10 by k this is 100 by k. Let us say the curve starts from one gets risen and goes to the narrow band here and then falls down steeply this is for zeta equals 0.01 typically. Then look for other zetas zeta 0.1,0.5 and zeta equals 1.0.

So, friends in the above plot you can see that there is a narrow band focus at resonance which is the main contribution of the integral and H omega is simply 1 by k the dynamic amplification of factor which we also discussed in the last lecture.

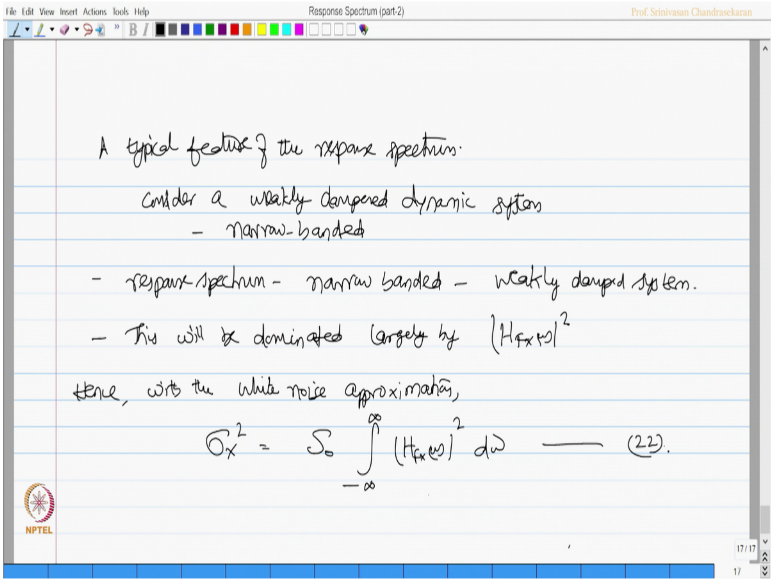
(Refer Slide Time: 10:29)



Therefore if the response spectrum varies slower than the transfer function, then it is possible to replace the response the load spectrum by S0.

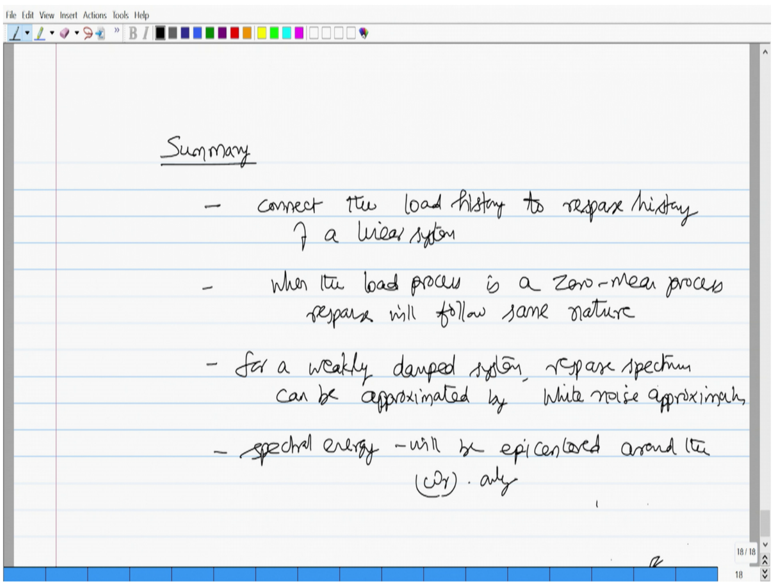
Which is s load spectrum at omega r hence sigma x square is now going to be S 0 minus to plus infinity F Fx omega square d omega, this procedure of replacing the input spectrum by a constant is called white noise approximation.

(Refer Slide Time: 12:12)



Let us look at a typical feature of the response spectrum, consider a weekly dampened dynamic system which will be; obviously, narrow banded. So, the response spectrum will become narrow banded for a weakly damped system. This will be dominated largely by H FX omega square hence with the white noise approximation sigma x square is S 0 minus to plus infinity H FX omega square d omega which is equation 22.

(Refer Slide Time: 14:01)



Let us see the summary what we learnt in this lecture.

We are able to connect the load history to the response history of a linear system, we also said when the load process is a 0 mean process, response will follow same nature further for a weakly damped dynamic systems, response spectrum can be approximated by white noise approximation, where the spectral energy will be epicentered around the resonance frequency only.

We will continue this discussion in the next lecture by explaining more about the response spectrum then we will also discuss the importance and example of stochastic process in analysis of offshore structures.

Thank you.