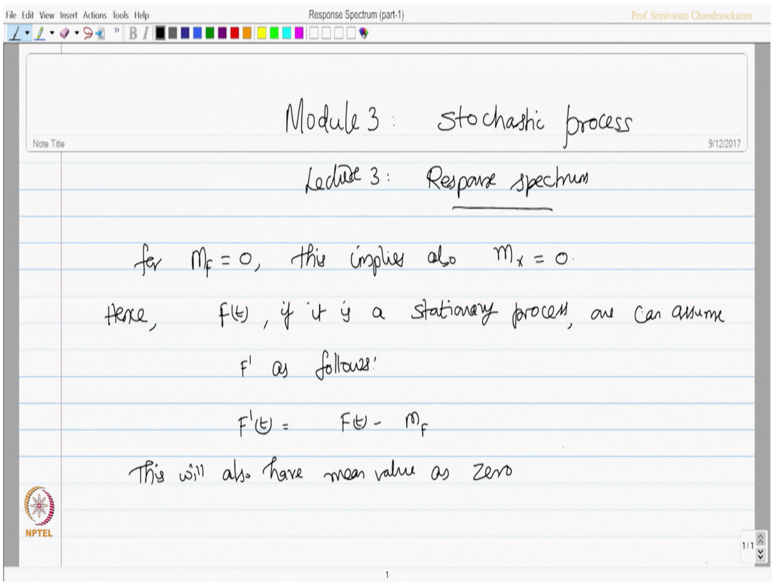
Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture - 03
Response Spectrum (Part – 1)

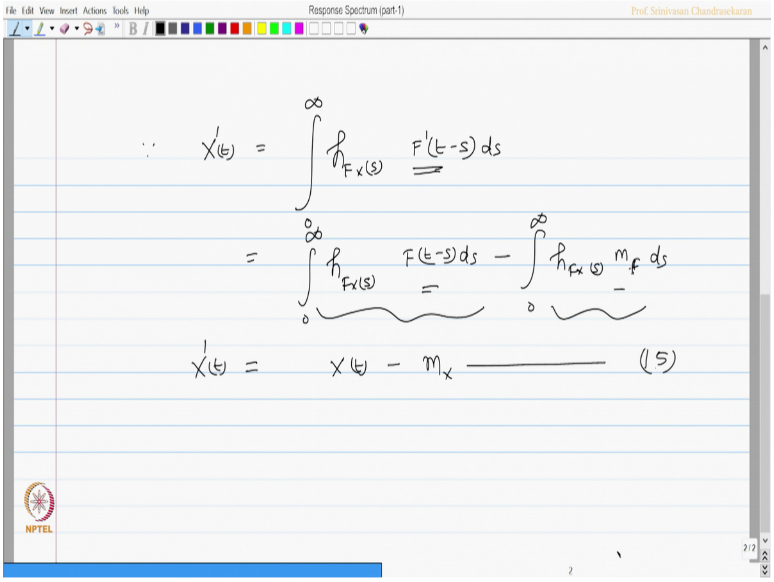
Let us continue with the discussion what we had in the last lecture. This lecture we will discuss more about response spectrum in at a stochastic process.

(Refer Slide Time: 00:33)



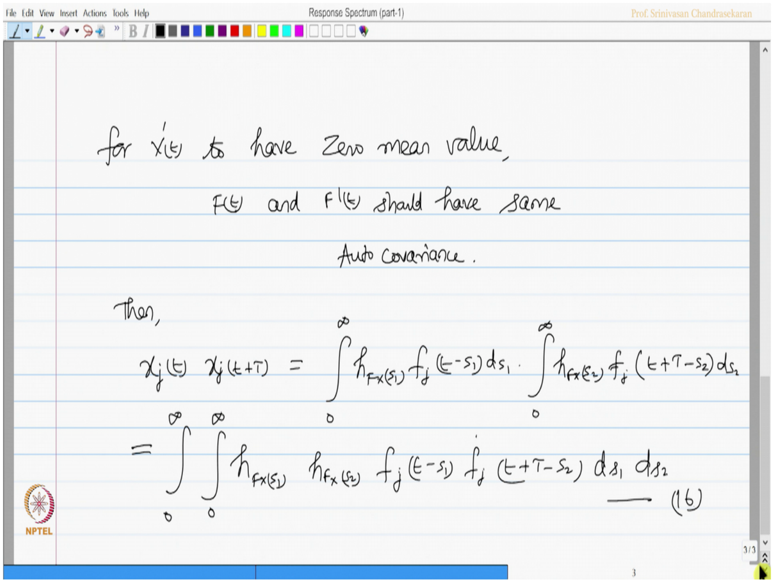
We already said that the mean value of the force realization is 0. This implies also that the mean value of the response process is also 0. Hence force realization set F of t, if it is a stationary process, one can assume F dash as follows F dash of t can be F of t minus m F in that case this will also have mean value as 0 X dash of t can be said as integral 0 to infinity the transfer function F dash of t minus s ds following the same algorithm.

(Refer Slide Time: 01:39)



What we discuss in the last lectures, this will amount to the integral of the transfer function minus integral of the transfer function ds because F dash is actually a process containing this and this which now I can say as this, of course, will give me X of t and this of course, will give me m X. So, now, I can say X dash of t is given by this equation, I will continue the same numbering what we had in the last lecture.

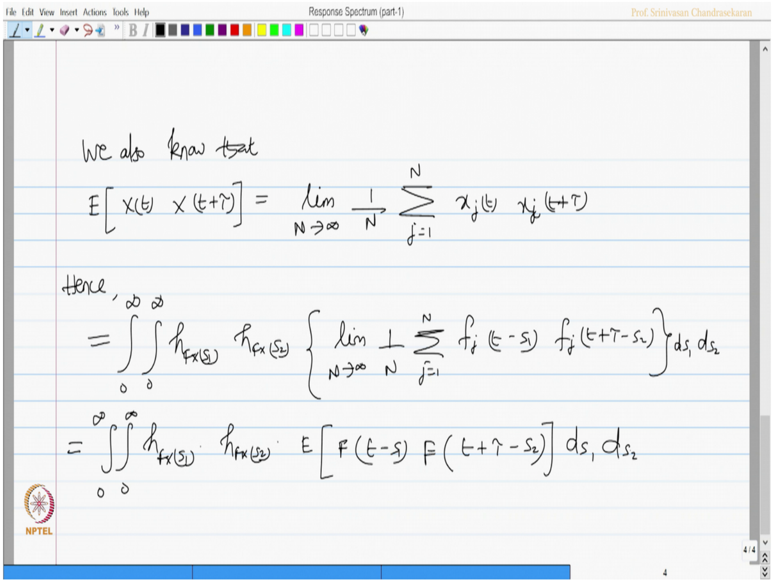
(Refer Slide Time: 03:03)



For X dash of t to have a 0 mean value F of t and F dash of t should have same auto covariance that is statistical requirement to establishes fact.

Having said this then xj of t and xj of t plus tau small interval can be expressed as 0 to infinity h F x s 1 F j t minus s 1 ds 1 multiplied by the other integral which is h F x s 2 fj t plus tau minus s 2 of ds 2 which now can be written as double integral h F x s 1; h F x s 2 F of j e minus s 1 F of j t plus tau minus s 2 of ds 1 ds 2, I call this equation number 16.

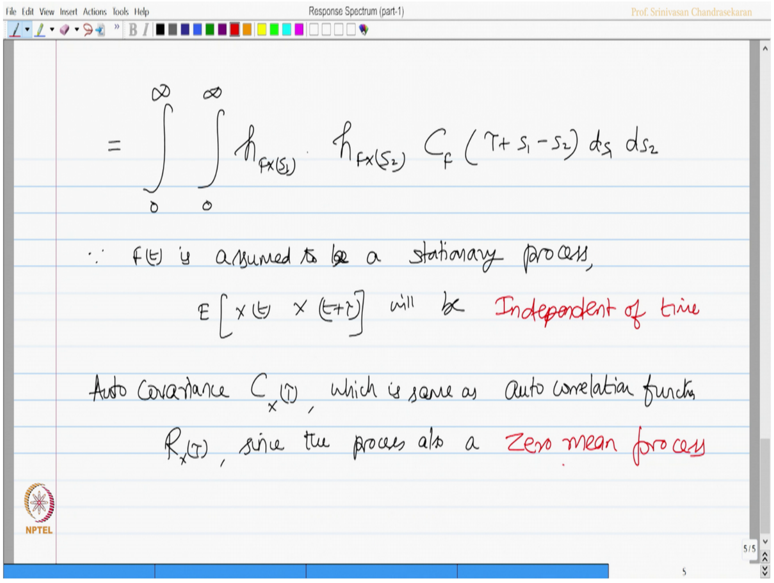
(Refer Slide Time: 05:10)



We also know that expected value of X of t and X of t plus tau can be expressed as limit n tends to infinity 1 by n of summation of j equals one to n xj of t xj of t plus tau.

Hence, the double integral which we shown in the last slide which is h F x s 1; h F x s 2 can be expressed as limit n tends to infinity; 1 by n of summation of j equals 1 to n F j t minus s 1 and F j t plus tau minus s 2 d s 1 d s 2 which can be further simplified as double integral F x s 1 h f x s 2 expected value of F of t minus s 1 F of t plus tau minus s 2 ds 1 ds 2.

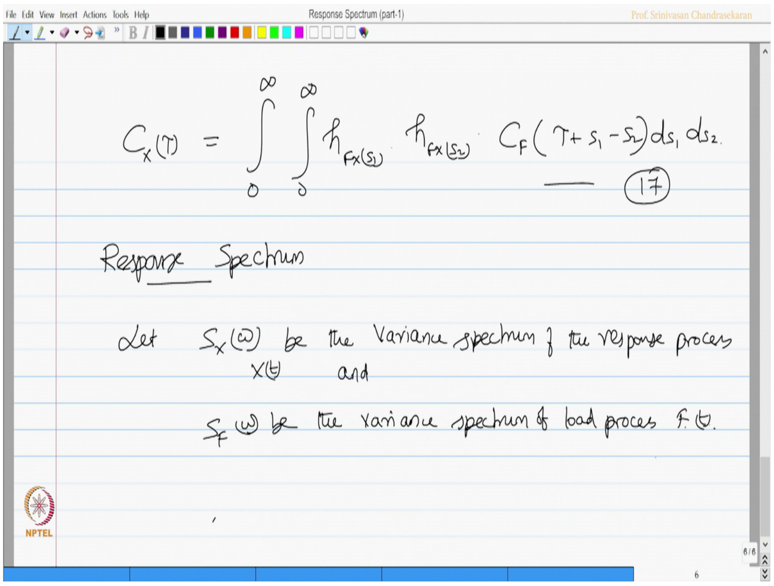
(Refer Slide Time: 07:15)



Which can be said as double integral h F x s 1 h F x s 2 auto covariance function of tau plus s 1 minus s 2 ds on ds 2.

Now, since F of t is assume to be a stationary process expected value of X of t X of t plus tau will be independent of time. Therefore, the auto covariance function C X of tau which will be as same as the auto correlation function R x of tau since the process is also a 0 mean process.

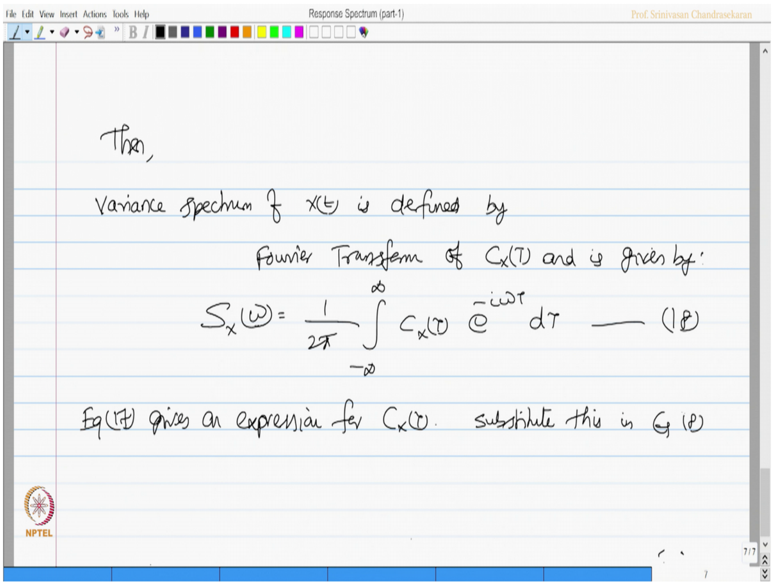
(Refer Slide Time: 08:56)



The auto covariance function are the auto correlation function can be given by the double integral of the transfer function of C F of tau plus s 1 minus s 2 ds 1 ds 2, I call this equation number 17.

Having said this, let us move towards the response spectrum let us say s of X omega be the variance spectrum of the response process X of t and s F omega the variance spectrum of load F of t.

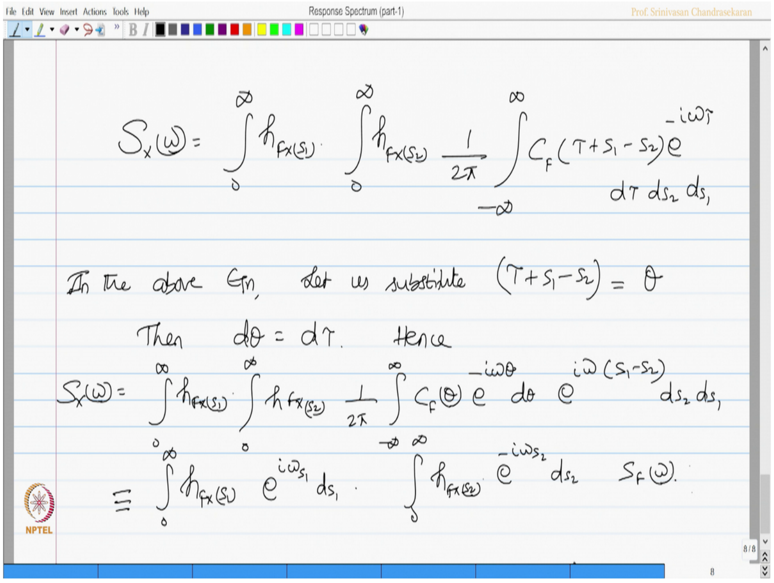
(Refer Slide Time: 10:24)



Then the variance spectrum of X of t is define by the Fourier transform of the auto correlation function which is given by S x omega is 1 by 2 pi minus to plus infinity not correlation function minus E I tau omega d tau equation 18.

From the earlier equation; equation 17 givens an expression for C x of tau, let us substitute this in equation 18.

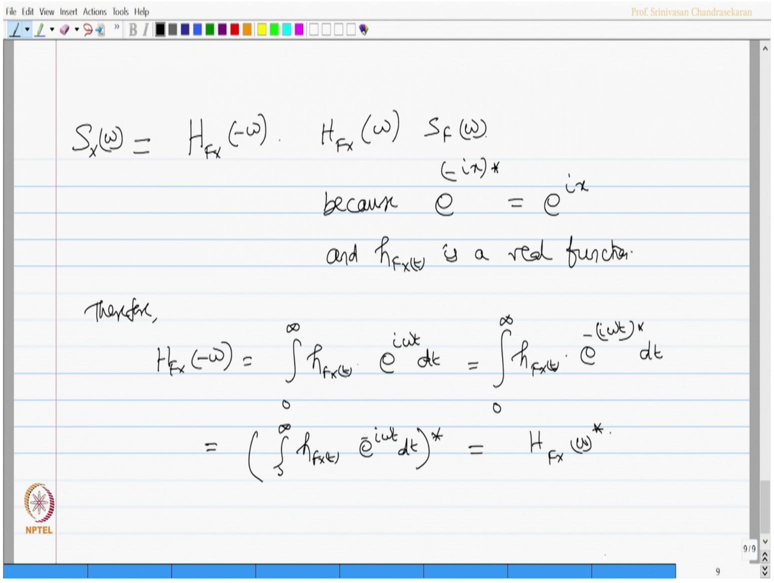
(Refer Slide Time: 11:49)



So, in that case S x omega will become 0 to infinity h F x s 1 0 to infinity h F x s 2 1 by 2 pi minus to plus infinity, the correlation function of tau plus s 1 minus s 2 e minus i omega tau d tau ds 2 ds 1.

So, in the above equation, let us substitute tau plus s 1 minus s 2 as theta, then d theta is d tau hence S x omega will be given by 0 to infinity. The transfer function 0 to infinity, the transfer function with respect to s 2 1 by 2 pi minus to plus infinity, the correlation function in terms of theta e minus i omega theta d theta e i omega s 1 minus s 2 ds 2 ds 1. So, this equation can be further simplified as 0 to infinity h F x s 1 ei omega s 1 ds 1 0 to infinity h F x s 2 e minus i omega s 2 ds 2 and s of omega.

(Refer Slide Time: 14:18)



Which can be now written as h F x minus omega h F x plus omega sf omega which is S x omega is given by this equation. This is true because e of minus ix star is as same as eix and we all agree that the transfer function h F x t is a real function therefore, capital H F x of minus omega will be 0 to infinity the transfer function e i omega t dt which can be said as 0 to infinity h F x t e minus i omega t star dt which can also be said as 0 to infinity h F x of t e i omega t dt of X star which can be said simply as H F x omega star having said this.