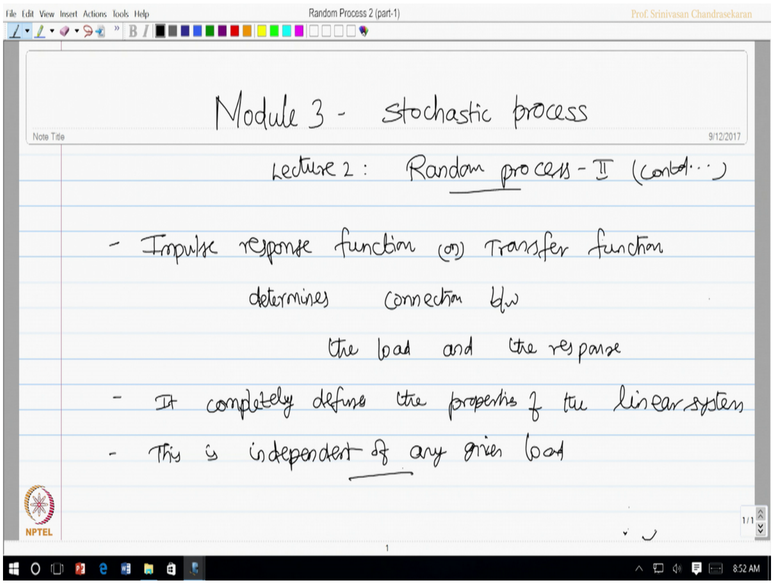
Computer Methods of Analysis of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 03
Lecture - 02
Random Process 2 (Part - 1)

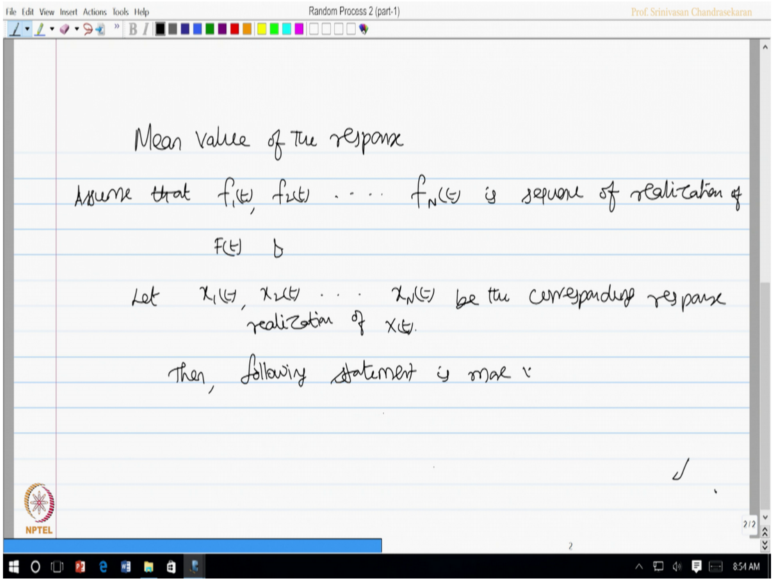
Friends, we will continue to discuss with what we left in the last lecture, we are working on lectures in module 3. Today's lecture will be continuation of the previous lecture on random process.

(Refer Slide Time: 00:42)



If you recollect what we said in the last lecture we said that the impulse response function or sometimes called as transfer function determines connection between the load and the response, we learned this. Second thing we learnt is it completely defines the properties of the linear system, we also said that this is independent of any given load we learnt it in the last lecture. We will continue this discussion now by extending this further.

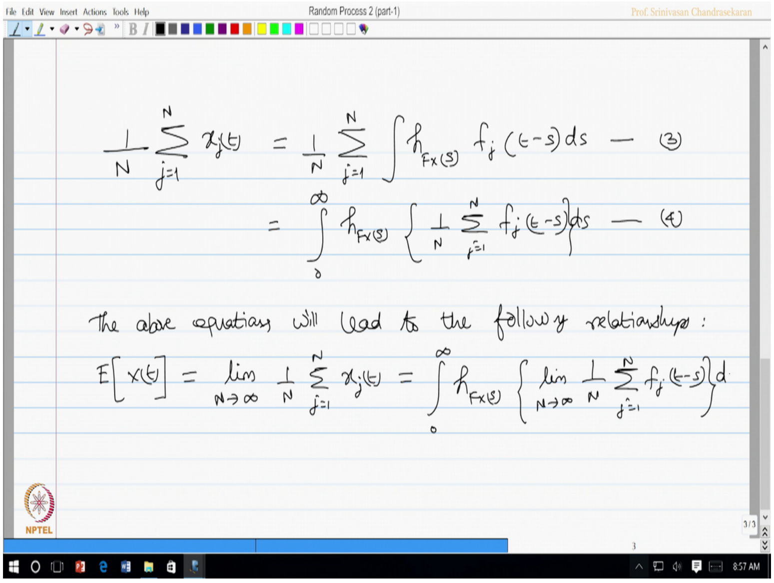
(Refer Slide Time: 02:25)



Let us say I am interested in knowing what is the mean value of the response. Let us assume that f 1 of t, f 2 of t till f N of t is a sequence of realization of f of t and let x 1 of t, x 2 of t till x N of t be the corresponding response realization of x of t.

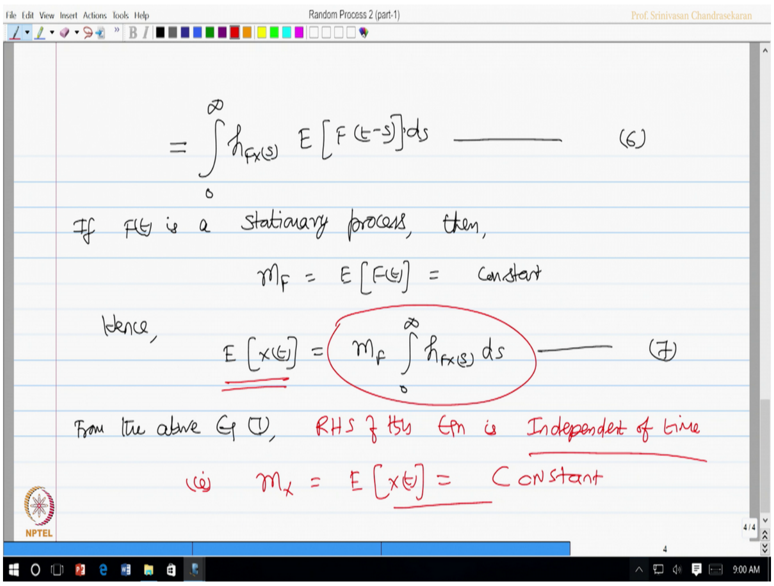
Then following statement is more vital 1 by N of integral j, equals 1 to N x j of t is actually given by 1 by N of summation of j equals 1 to N integral h F x s, f j t minus s ds and call this equation number 3 to continue with the last set of lectures which can be now rewritten as 0 to infinity the transfer function of 1 by N of summation of j equals 1 to N of f j t minus s ds the equation 4.

(Refer Slide Time: 04:05)



Now, the above equations will lead to the following relationships, expected value of x of t which is a set of realizations corresponding to F of t is now expressed as limit N tends to infinity 1 by N of summation of j equals 1 to N x j of t which is now borrowed from the above equation which can be written as 0 to infinity h F x s limit N tends to infinity 1 by N of summation of j equals 1 to N f j t minus s ds. I call this equation number 5. Which can be further simplified as integral 0 to infinity the transfer function expected value of f of t minus s ds. Is it because I am replacing the previous integral in equation 5 with an expected value of f of t minus s ds I call this as the equation 6.

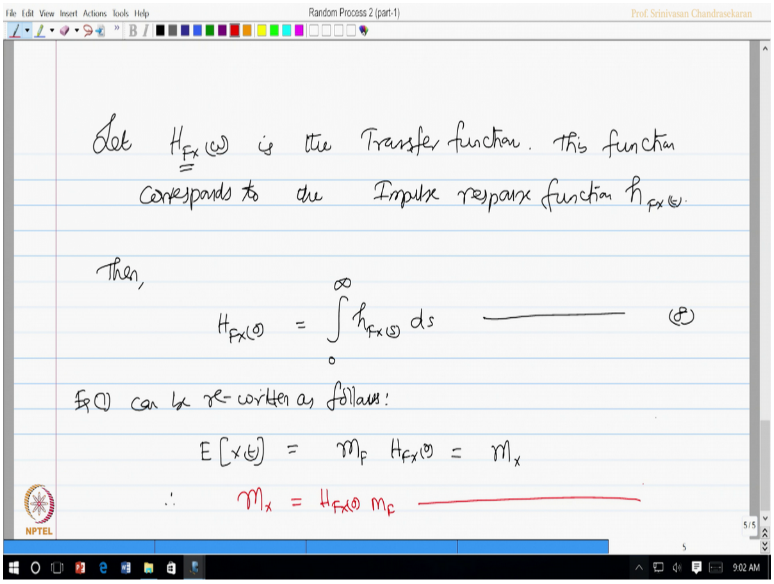
(Refer Slide Time: 07:24)



Now, interestingly F of t is as stationary process which we discussed in the last lecture about stationarity properties, let us assume that the given input loading is a stationary process, then the mean value which is again expected value of f of t is constant I think this statement we already emphasized with an example in the last lecture. Hence expected value of x of t can be now said as mf of integral 0 to infinity the transfer function. I call this as equation number 7.

From the above equation that is equation 7, one can notice that the right hand side of this equation is independent of time that is very interesting characteristic you can see here the right hand side of this equation is independent of time that is m x which is nothing, but the expected value of e of x of t is, expected value of x of t is constant.

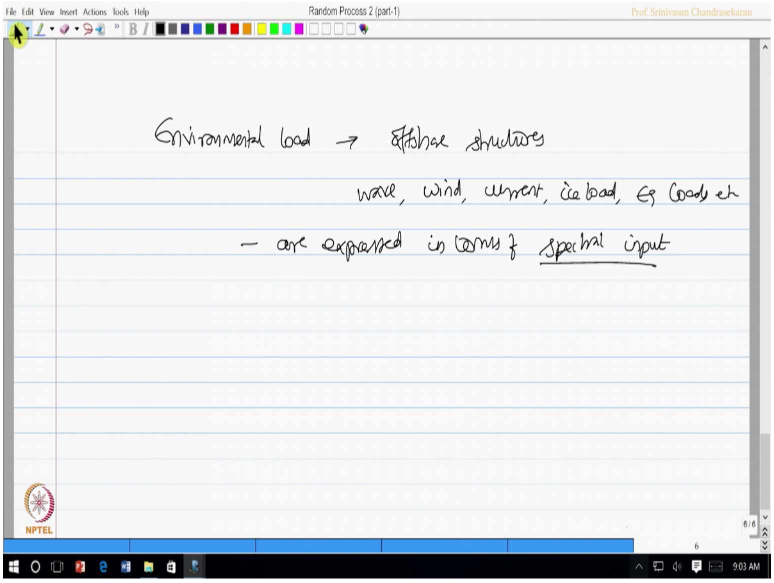
(Refer Slide Time: 09:48)



Having said this let us say my capital f x omega is the transfer function we already know how the notation is understood in stochastic process. This transfer function corresponds to the impulse response function h F x of t then h F x 0 can be expressed as the following integral. I call this as equation number 8.

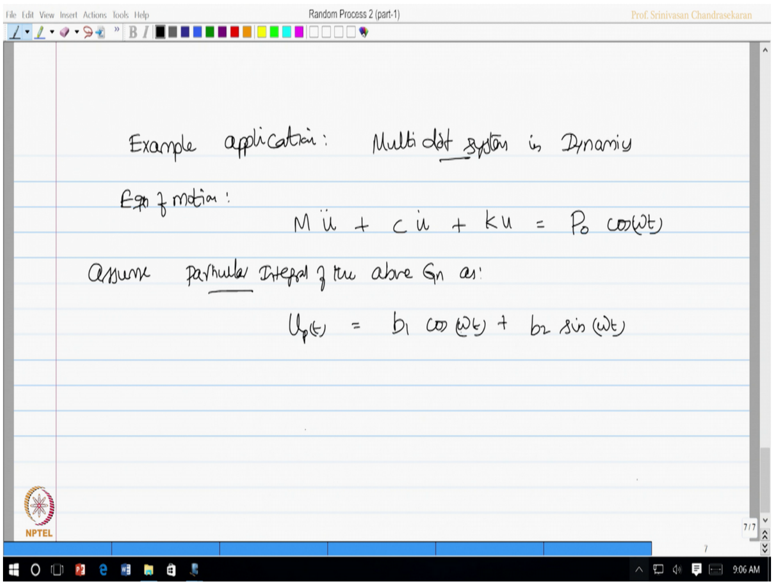
Now equation 7 can be rewritten as follows. The expected value of x of t is m F, H F x 0 which is again same as m x, that is m x is h F x 0 mf let us make a slightly clearer, m f, equation number 9. As in this course we are discussing about the computer methods of structural analysis applied to offshore structures we all know that the environmental loads which act on offshore structures comprises of for example, waves, wind, current, ice loads, earth quake loads etcetera usually are expressed in terms of spectral input.

(Refer Slide Time: 12:11)



As we saw in the earlier modules input loading can be expressed in terms of various spectra for wind, wave etcetera. Let us take a small application example in dynamic analysis and see how this transfer function can be easily connected to the dynamic amplification of factor which you generally derive for dynamic analysis.

(Refer Slide Time: 13:10)

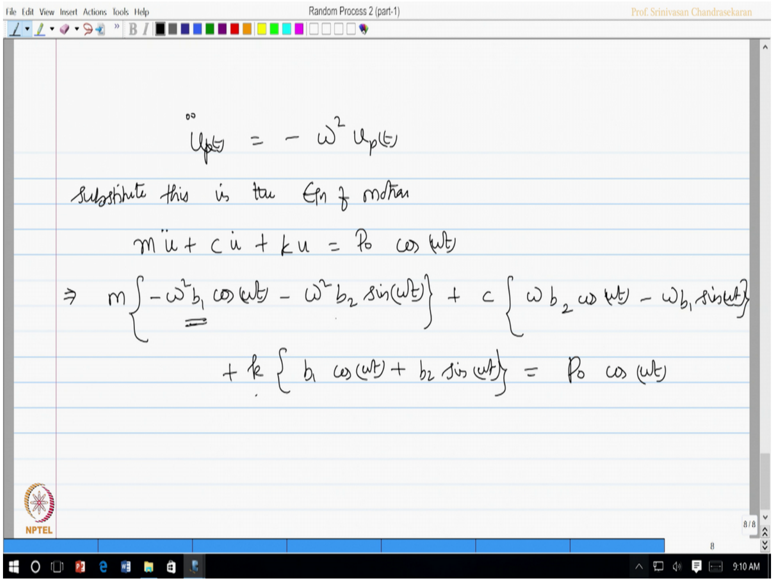


So, let us take an example application which is going to be a multi degree freedom system model in dynamics. We all know equation of motion of the system is given by m u double dot plus c u dot plus k u is p 0 cause omega t. Let us say we have a system representing this. Let us assume particular integral of the above equation as u p of t, p stands for the particular solution. Let it be b 1 cos omega t plus b 2 sin omega t.

We all know the dynamic analysis solutions has got 2 parts - one is the complimentary function other is the particular integral, generally the complimentary function will be dependent on the initial conditions and the free vibration frequency set of the system whereas, particular integral generally gives me a steady state response which will be the function of the forcing frequency omega. So, let us focus only on the particular integral of the above equation let u p t be as written in the above equation. So, let us differentiate this once with respect to time which is very easy minus omega b 1 sin omega t plus omega b 2 cos omega t there is no difficulty and differentiating this. Let us differentiate this once again with respect to time I get acceleration which will be minus omega square b 1 cos omega t minus omega square b 2 sin omega t.

Interestingly I can rewrite this equation as minus omega square b 1 cos omega t plus b 2 sin omega t which is as same as, which is as same as my u p. Therefore, I can now write u p double dot of t as minus omega square u p of t. Now let us substitute this substitute this in the equation of motion. So, equation of motion is given here this is my equation of motion let us substitute this in equation of motion and see what happens. So, m u double dot plus u dot plus k u is P 0 cos omega t. So, I should say now this is equal to m of minus omega square b 1 cos omega t minus omega square b 2 sin omega t.

(Refer Slide Time: 17:07)

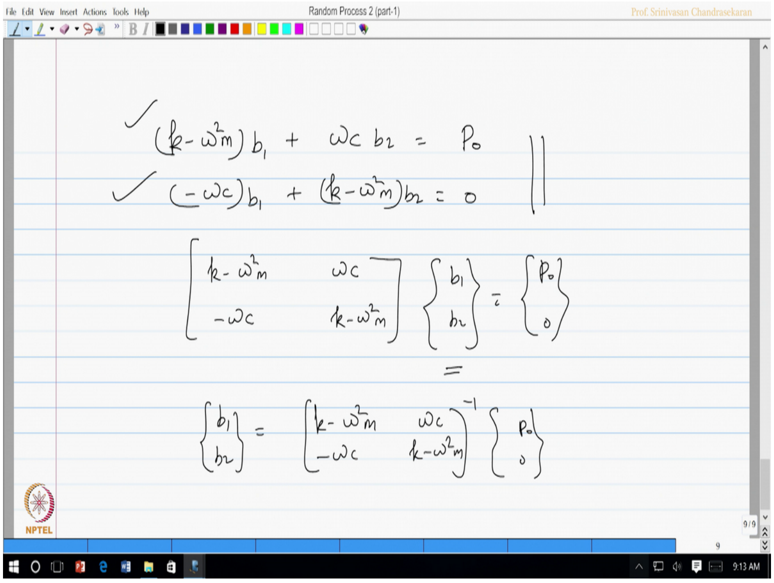


We already saw this just now in the last slide, I am substituting them simply plus c of omega b 2 cos omega t minus omega b 1 sin omega t plus k times of b 1 cos omega t plus b 2 sin omega t which is actually equal to P 0 cos omega t.

All of us know from the first principles of mathematics if you want to really find the solution I should compare the LHS and RHS of this equation in such a trigonometric function we must compare the cosine terms and sin terms get in the equivalence. So, what we are looking for? We are looking for the values of b 1 values of b 1 and b 2, once I know this I can easily find the solution because the particular integral is nothing, but the function of b 1 and b 2.

So, now I am interested in knowing how to estimate b 1 and b 2. So, let us compare the cosine terms between the LHS and RHS let us pick up the cosine terms . So, minus m omega square b 1 that is this term the first term plus c omega b 2 that is the next term here plus k b 1 is P naught I think you can easily do this. Let us compare the sin terms minus m omega square b 2 minus c omega b 1 plus k b 2 is 0.

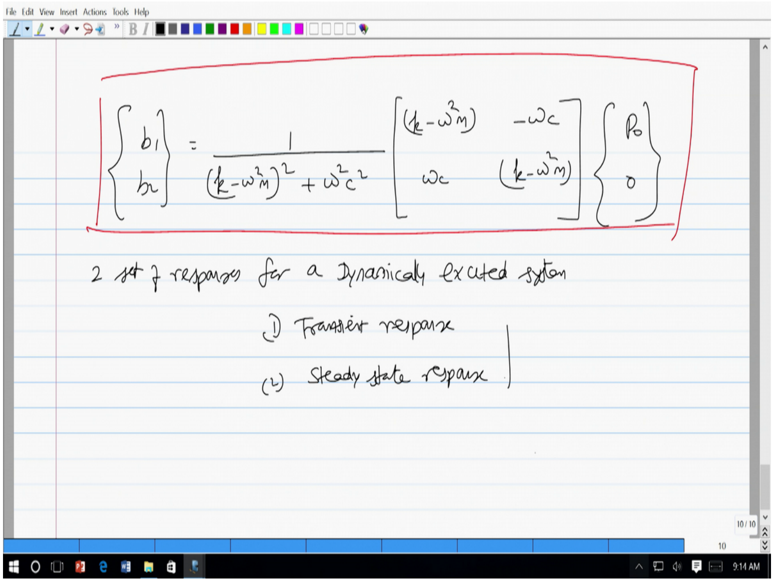
(Refer Slide Time: 19:36)



Once I get this let us proceed the next step writing in a matrix form let us write it like this k minus omega square m of b 1 plus omega c b 2 is p 0 minus omega c of b 1 plus k minus omega square m of b 2 is 0 I get this equation now, in a matrix form k minus omega square m omega c minus omega c k minus omega square m of b 1 and b 2 is P 0 and 0.

You can read this equation the first row with the first column will give you the first equation, the second row with the second column will give you the second equation. Now, my objective is to find the value of the vector b 1 and b 2. So, it is very simple I will invert this matrix. So, b 1 b 2 vector is nothing but inversion of k minus omega square m omega c minus omega c k minus omega square m inverse multiplied by P 0 and 0 let us do the inverse quickly.

(Refer Slide Time: 21:07)



So, 2 by 2 matrix, you can quickly do the inverse. So, b 1 b 2 is actually 1 by k minus omega square m square plus omega square c square of k minus omega square m minus omega c omega c k minus omega square that is going to do the inverse of this matrix multiplied by P 0 0 will give me the b vector .

Let us retain this equation for our reference. Now we also know there are 2 set of responses for a dynamically excited system what are they? You can recollect it easily, one is the transient response the other is the steady state response. And we all agree in general in dynamically excited systems one pays more attention towards steady state response and one does not really bother about the transient response in larger cases because steady state response being a function of forcing frequency will always exist in the response content, but transient response depending upon the initial conditions may decay with prolonging time. But however, it is not always true in offshore structures.

If you look at the special types of responses like springing and ringing responses in offshore platforms there you will also come across the importance of transient response which may be as demanding and vital as the top a steady state response. However, for a more generic case let us assume that the steady state response is more vital for taking it forward for discussions if we agree on that.