Computer Methods of Analysis of Offshore Structures

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Module - 03
Lecture - 01
Random Process 1 (Part - 2)

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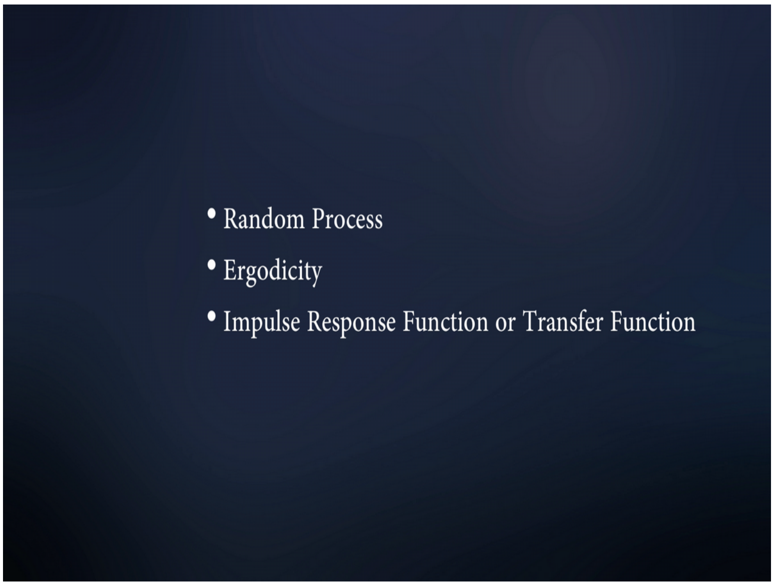

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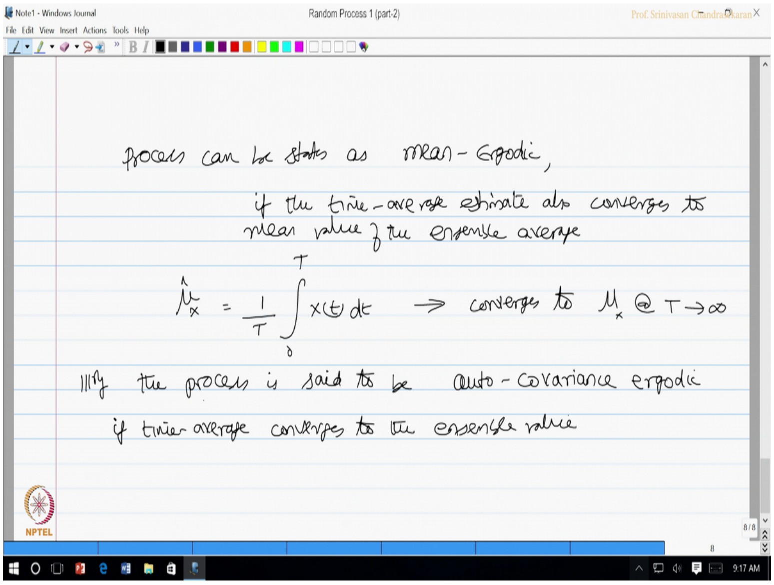

Now further, the process can be stated as mean ergodic if the time average estimate also converges to the mean value of the ensemble average. So, mu hat x is given by the time average over x of t dt which converges to mu x at T tends to infinity. Similarly, the process is said to be auto covariance ergodic if the time average converges to the ensemble average that is auto covariance.

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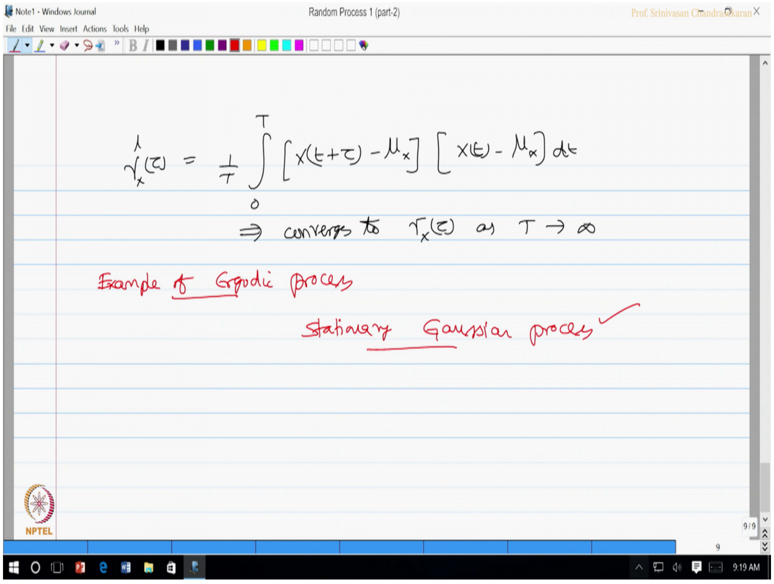

Hat is an time average 0 to t x of t plus tau minus mu x of x of t minus mu x dt which converges to auto covariance of variable tau when T tends to infinity.

Let us quickly ask the question; can we have a classical example of an ergodic process the classical example of a ergodic process is stationary Gaussian process which is classically used as one of the important mathematical model to express various environmental loads in offshore structure, let us take a question of how do you explain or express or confirm ergodicity in case of discrete random process.

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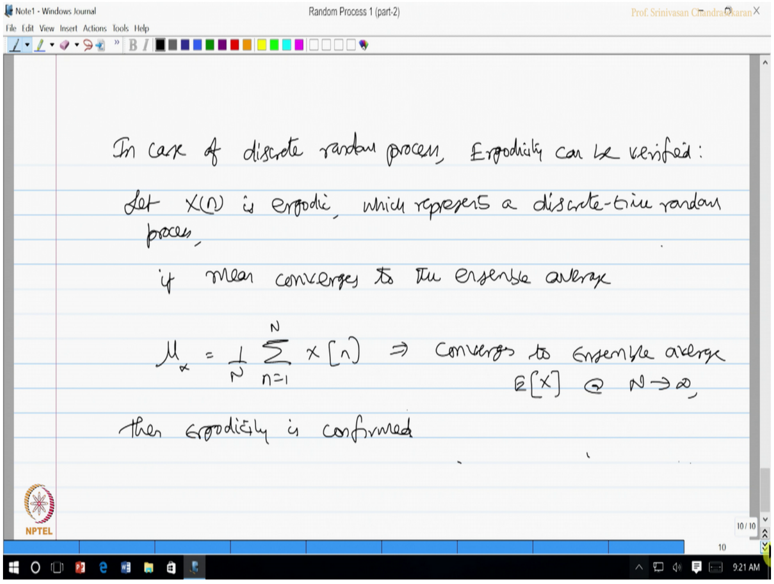

In case of a discrete random process, ergodicity can be verified as follows. Let us say the x of n is ergodic which represents a discrete time random process if the mean converges to the ensemble average that is if the mean converges to the ensemble average of expected value of x at n tends to infinity then the process is ergodicity is confirmed.

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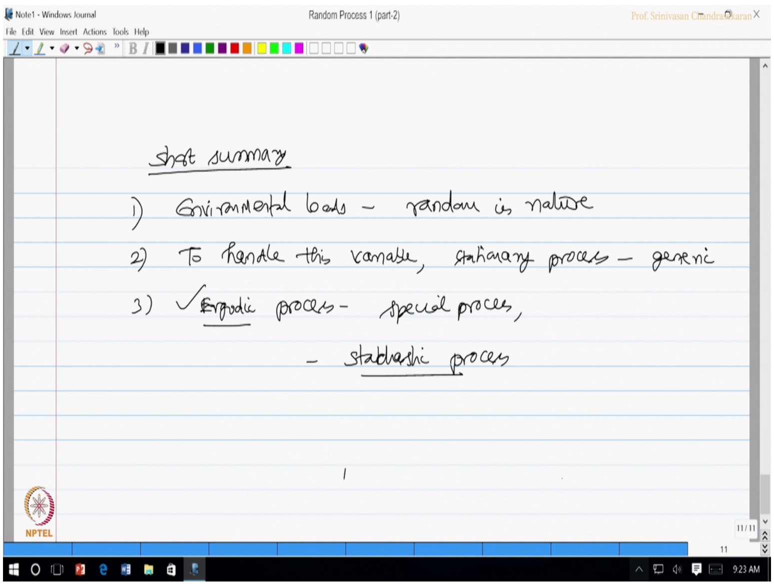

So, friends let us quickly see what we have so far learnt we understood that the environmental loads acting on offshore structures or generally random in nature randomness comes in 2 ways one with time and one with position to handle this mathematically easily we go with what is called searching for a stationary process; stationary process is highly generic still we will have a few complications in expressing the statistical parameters.

Therefore, we go for a specific stationary process called ergodic in nature which is a special process which selects sample from the ensemble arbitrarily which expresses the characteristics of the entire ensemble for the entire period not in the section of the chosen ensemble. So, ergodicity is a easy tool for expressing or converting the randomness in a given system to a mathematically convenient form to express the statistical variations in the stationary process which is very important in terms of stochastic dynamics after establishing a fact that we are able to choose a stationary process.

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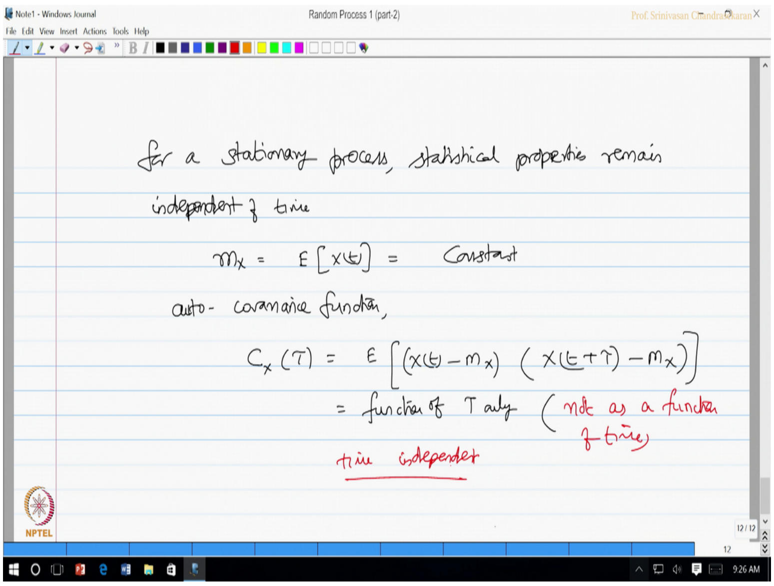

So, for a stationary process, the statistical properties remain independent of time that is the mean value which is expected value on the variable is constant and the auto covariance function c x of tau is expected value of x of t minus mx x of t plus tau minus mx which will remain as function of tau only and not as a function of time. So, it is time independent.

Now friends; what is the advantage of choosing a ergodic stationary process to express the environmental loads in offshore structures for a stationary process.

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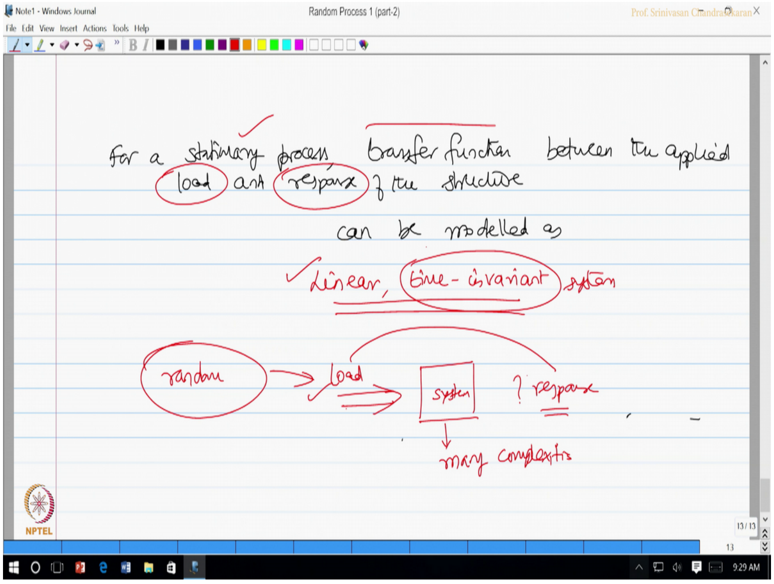

The transfer function between the applied load and response of the structure can be modelled as linear time invariant system

So, that is a very interesting advantage we have in expressing the connectivity between the load and the response. So, after all for a given structural system if an environment load is applied I would like to know; what is the response of the system the applied load is highly varying which is random in nature. Therefore, it is not convenient and easy to determine the response for a system because system also has many complexities I should have a mechanism by which I can express the relationship between this by a simple mean which can give me at least an approximate first level response of the given system for an applied load.

So, if the process is converted or identify or scrutinized and verified to be a stationary process they can explain then a transfer function which will be connecting the load and the response as well as the function will remain linear and time invariant. So, that is a great advantage you have in estimating the response of a given system encountered by environmental loads which are random in nature.

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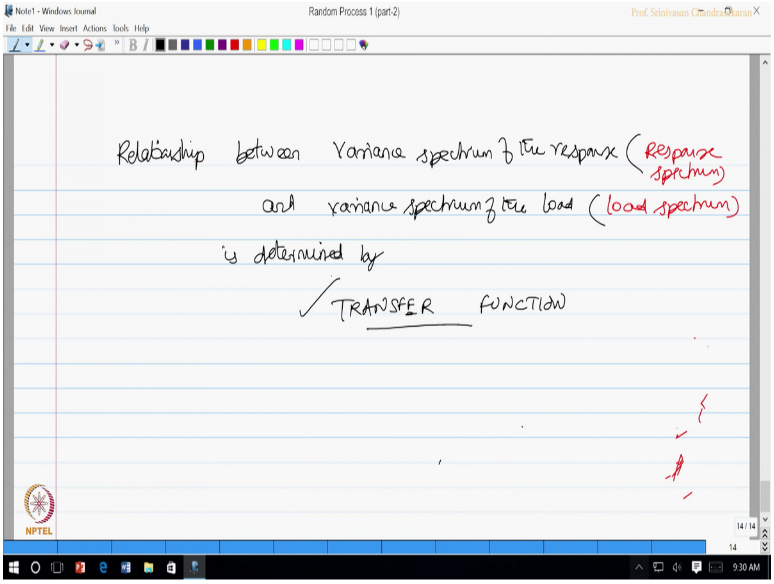

Therefore, the relationship between the variance spectrum of the response which we call as response spectrum and variance spectrum of the load which we call as load spectrum is determined by the transfer function. So, our job now is to determine or derive this transfer function which actually connects the response spectrum to the load spectrum.

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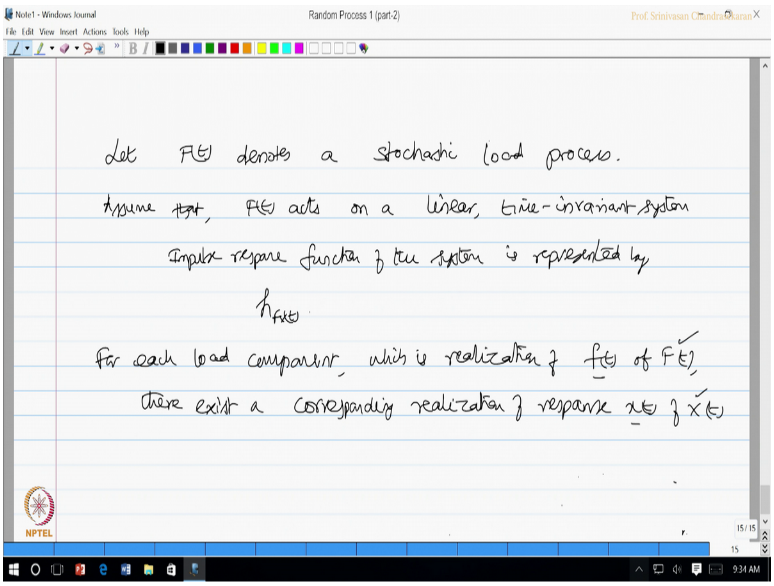

Let F of t denotes a stochastic load process assume that F of t acts on a linear time invariant system. Now the impulse response function of the system is represented by h F x of t for each load component which is realization of F of t of a capital F of t there exist a corresponding realization of response x of t of x of t. So, capital F of t and capital X of t or the complete set of the load and responses small F of t and small x of t are realized values of these complete set of load and response respectively.

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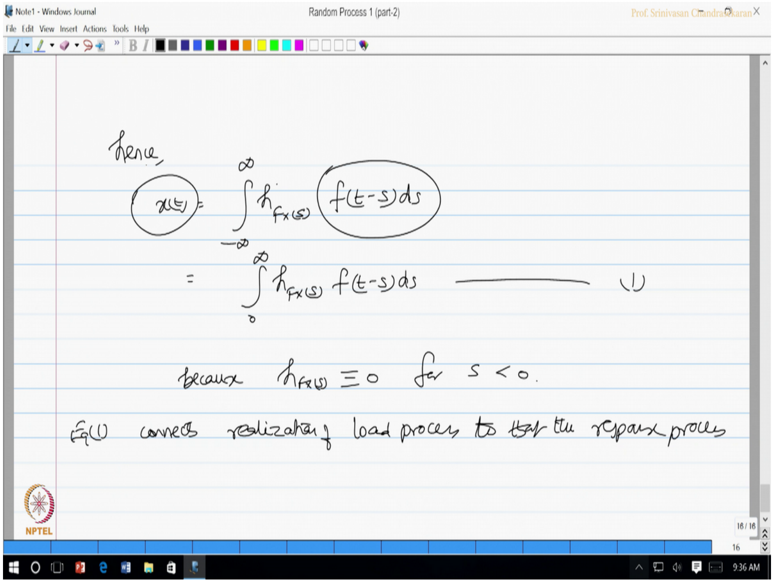

Therefore x of t can be expressed as integral h f x s f of t minus s ds which can be simplified as 0 to infinity h fx s f of t minus s ds because integral can be simplified saying that h f of x is 0 for y s less than 0.

So, no value of this function between minus infinity to 0 therefore, I convert this integral to an integral as given by equation 1. Now equation 1 connects realization of load process to that of the response process you can see very well here this is a realization of the load process a response process.

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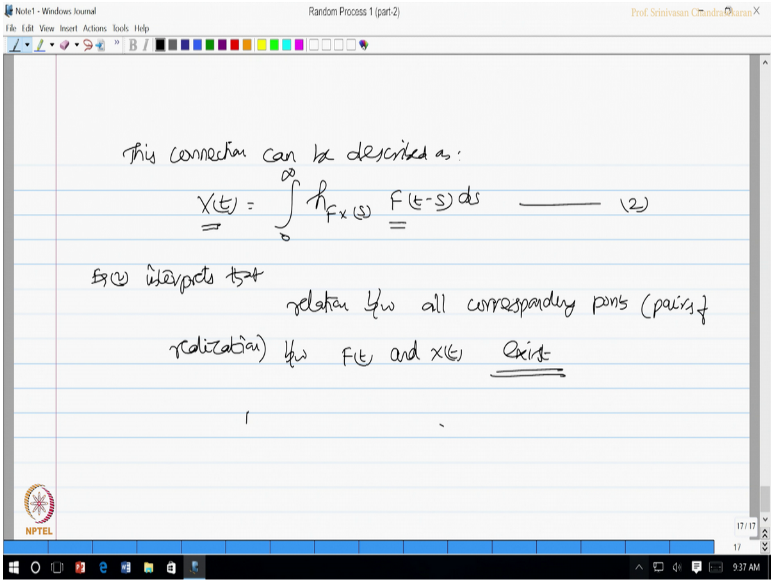

This connection can be further described as X of t is infinity 0 h fx s f of t minus s ds equation 2 interprets that relation between all corresponding points or I should say pairs of realization between F of t and x of t exist. So, that confirms the existence of the relationship between each corresponding pair of the load and the response.

There is a very important observation we can make in this equation.

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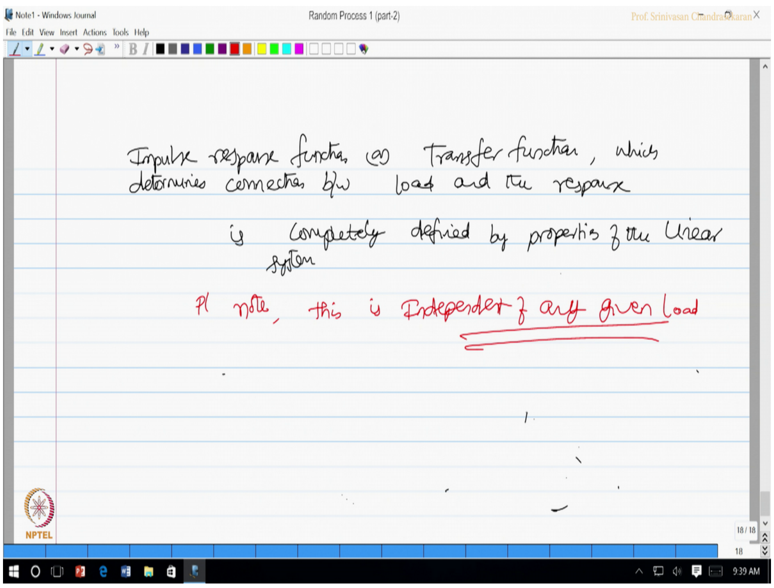

The impulse response function or the transfer function which you saw in equation 2 which connects the load and the corresponding pairs of response which determines connection between the load and the response is completely defined by the properties of the so called linear system. Please note this is independent of any given load.

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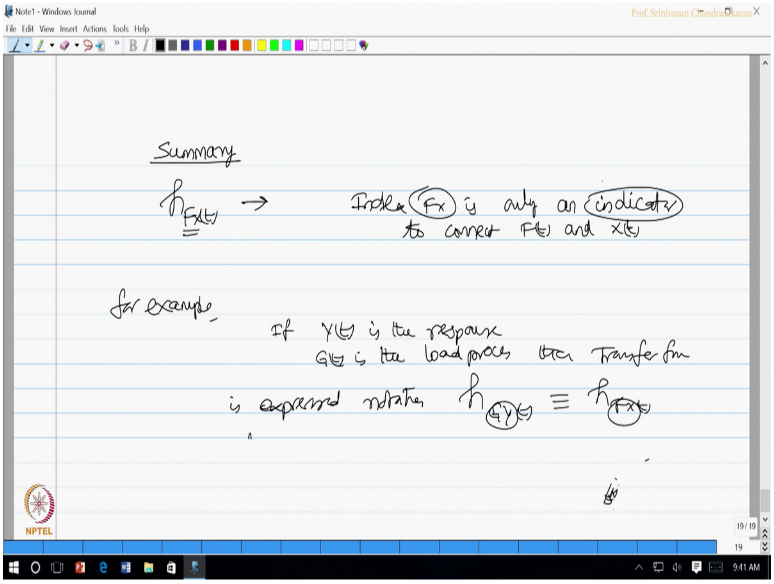

Lastly, we can summarize this saying h F x t is the transfer function the index F x which is indicated here is only an indicator to connect load and the response; for example, if y of t is the response and g of t is a load process. Then the transfer function is expressed with a notation h g y of t which is as identical as h fx of t. So, it is very important that Fx; Fx or gy is an visual indicator which connects the load and the response the load and the response.

So, friends in this lecture we learnt the importance of understanding the random process converting them or identifying a single arbitrary sample which qualifies to remain stationary and ergodic. We also gave you the mathematical definition of a stationary process and an ergodic process. We subsequently expressed how the load being a random process can be connected to the response of a system which is linear and time invariant using what we call as a transfer function. And we also explain how in stochastic process, the transfer function is nomenclated as explained in the last slide.

Thank you very much.