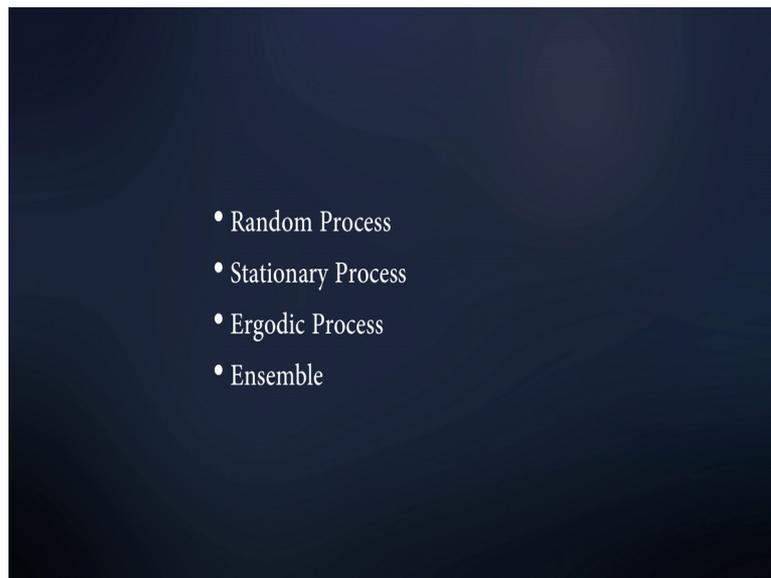


**Computer Methods of Analysis of Offshore Structures**  
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**Module - 03**  
**Lecture - 01**  
**Random Process 1 (Part -1)**

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Module 3 - Stochastic process  
Lecture 1 - Random process

✓ All loads, encountered by offshore structures are random in nature

(1) Wave loads  $\left\{ \begin{array}{l} \text{wave forces} \\ \text{spectrum (PSD)} \end{array} \right.$

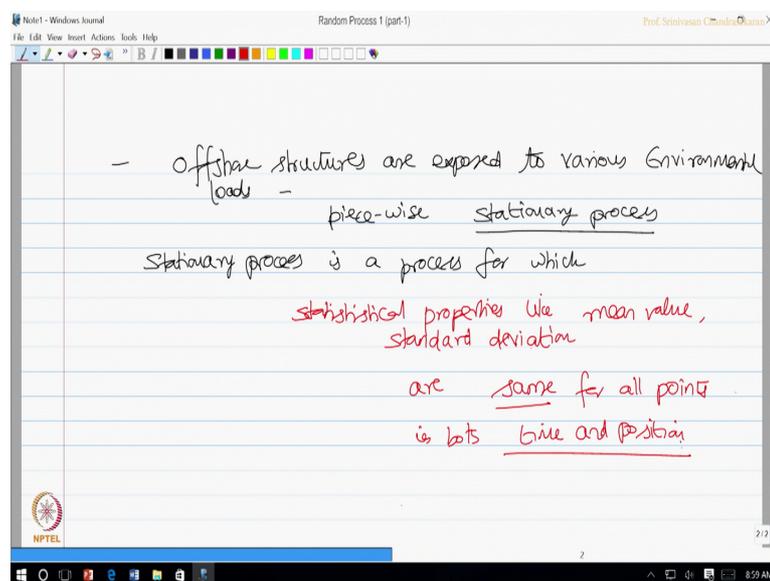
(2) Wind loads  $\left\{ \begin{array}{l} \text{mean component} \\ \text{gust component} \end{array} \right.$

Equivalent wind load, as a static process - Aerodynamic Admittance function

Welcome to the lectures on third module; in this module, we are going to discuss about the stochastic process in lecture 1. In this module, we will talk about random process; friends, let us try to ask a question; how random process is important in computer methods of structural analysis of offshore structures, this question is fundamentally answered with the basic requirement of understanding and saying that all loads encountered by offshore structures are random in nature. There are varieties of loads what we discussed in the last modules; let us quickly sight some examples of these kinds of loads for our understanding: wave loads, we have seen different equations for estimating the wave forces, we have also seen different spectra to estimate wave forces in terms of power spectral density functions.

The second classical load; **that** is apply in offshore structures is wind loads, though wind loads have 2 components which are the mean component and the gust component, we have seen in the last lectures that equivalent wind load as a static process can be obtained using what is called aerodynamic admittance function. Therefore, friends we now understand that all environmental loads acting on offshore structures are random in nature, then how do we actually classify these loads using some simple tools in mathematics and statistics.

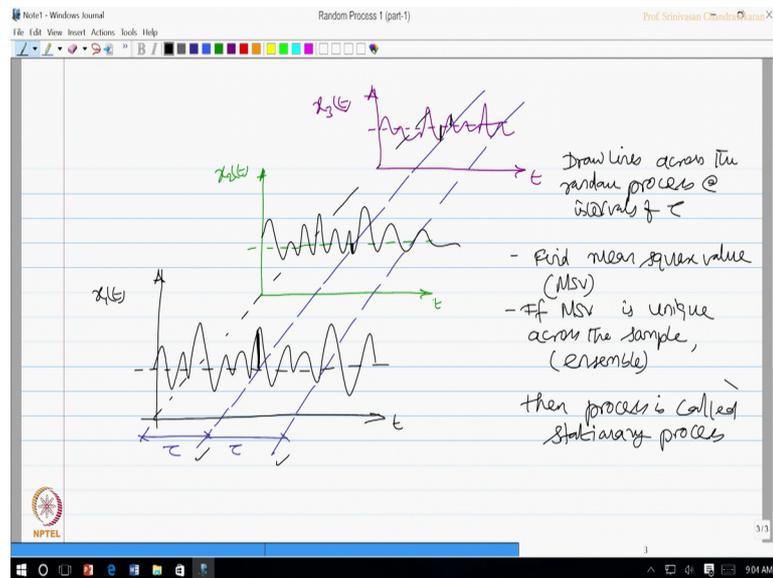
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So, let us make a sentence and statement reinforcing the fact that offshore structures are exposed to various environmental loads which can be modelled as a piecewise stationary

process. Now we need to understand; what is a stationary process? the stationary process is a process for which the statistical properties like for example, mean value, standard deviation are same for all points in both time and position; let us try to understand this slightly in a better manner.

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Let us say I have time history data which is plotted as shown in the screen now let us call this as a time variant process and I call this value, let us say as  $x_1$  of  $t$ . Now to start with, I keep on plotting different processes which is also time variant which will have a different signal; let us say  $x_3$  of  $t$  is another process which has some signal and we all understand that all these processes has their mean value. Now what I do; I take an interval  $\tau$  and draw a line, I draw another line with the same interval now these intervals are equal I do not measure the points of these variables at these intervals.

So, let us explain this process very quickly draw lines across the random process at intervals of  $\tau$ . So, we did that in a 2 lines we drawn now find the mean square value if this MSV is unique across the sample. Now we have a specific name for the sample in stochastic process we call the sample as ensemble; ensemble is actually a qualified sample for doing statistical analysis if the mean square value is unique across the sample then the process is called stationary process extending this discussion to the next level.

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The image shows a digital whiteboard with handwritten text. At the top, it says "Ergodic process - special stationary process". Below this, there are three bullet points: "- let us take about 100 samples", "- find MSV along the time line", and "- This MSV is equal to the unique MSV of the ensemble, then the process is called Ergodic process." At the bottom, there is a definition: "Ergodic process is process representing a single sample is the ensemble which is arbitrarily chosen and has same MSV as that of the unique MSV of the ensemble." The whiteboard has a toolbar at the top and a Windows taskbar at the bottom.

We call the process as ergodic process which is actually a special stationary process.

Let us explain this speciality with an example, let us say; we will consider about 100 samples find the mean square value of all the sample along the time line if you find this, mean square value is exactly equal to the unique mean square value of the ensemble which you worked out in the last slide then the process is called ergodic process. So, by this logic; we can say ergodic process is a process representing a single sample in the ensemble which is arbitrarily chosen and has the same mean square value as that of the unique mean square value of the ensemble.

So, ergodic process is a process which represents a single sample chosen arbitrarily which has the same mean square value as that of the ensemble. So, what we are trying to convert here is any load may be wind load, wave load which are random in nature, we will try to explain and express the time history variation of this process, we take many number of samples try to qualify these samples to be an ensemble for a statistical evaluation for this ensemble at different time intervals of  $\tau$  which are equal in nature, we estimate the mean square value amongst this ensemble pick up any one sample which is arbitrary which has the same mean square value as that of this MSV by this way we are making this process independent of time and position. So, random processes can be conveniently converted to an equivalent simple statistical process when they exhibit stationarity and the ergodicity in the revaluation.

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A stochastic process is Ergodic if the statistical properties ( $\mu, \sigma$ ) can be deduced from a single sufficiently long sample of the process.

It means ERGODIC refers to any sample, chosen from the random process must represent the average statistical properties of that of the entire process.

A stochastic process a stochastic process is ergodic if the statistical properties like mean and standard deviation can be deduced can be derive from a single sufficiently long sample of the process. So, that is very interesting ergodic property will simplify the randomness or random nature in the input loading which is useful for the analysis, it means ergodic refers to any sample chosen from the process I should say random process which must represent the average statistical properties of that of the entire process.

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What is the advantage of Ergodic process?

Regardless of the sample chosen for the analysis, the sample represents the whole process and it doesn't represent only a section of the process.

Next question comes what is the advantage of having the process as ergodic; what is the advantage of ergodic process?

Very interestingly regardless of the sample chosen by you regardless of the chosen by you the sample represents the whole process and it does not represent only a section of the process. So, by this logic, the entire random process what you have chosen take the sample from the process qualify the sample and the sample now represents the whole process and not only the section of the process.

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Define Ergodic process, mathematically.

- constant mean of any process :  $\mu_x = E [x(t)]$  — (1)

auto-covariance is given by

$$r_x(\tau) = E [(x(t) - \mu_x) (x(t+\tau) - \mu_x)]$$

— (2)

which should depend only on the interval ( $\tau$ ), and not on time  $t$

- time independent.

$\mu_x$  and  $r_x(\tau)$  are not time averages - Ensemble averages

The next question comes how do you define ergodicity mathematically we agreed said saying that the process we will have a constant mean the constant mean of any process can be given by.

This equation and the auto covariance is given by the following equation; equation 1, equation 2 which should depend only on the interval tau and not on time t. So, it should be time independent therefore, properties like mu x and auto covariance are not time averages they are ensemble averages. So, that is the fundamental difference we have between a classical random process and an ergodic process, which is the part of the random process.