

Computer Methods of Analysis of Offshore Structures
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Module – 02
Lecture – 23
Triceratops – 2 (Part – 2)

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- Triceratops
- Derivation of stiffness matrix
- Structural action of triceratops

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$$K_{66} \theta_6 = 3 \left\{ \frac{(T_0 + \Delta T_6) 2a^2}{L_6} \right\} \theta_6 \quad \text{--- (17)}$$

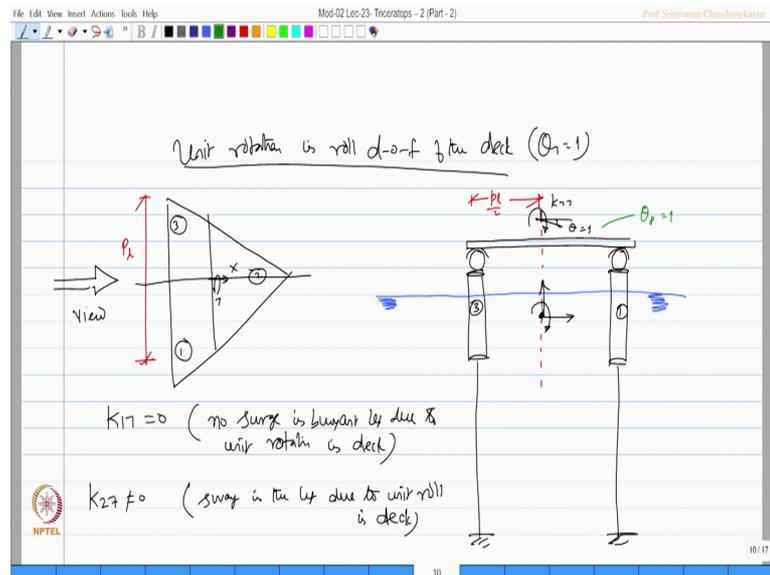
$K_{76} = 0$ (no roll is due due to unit yaw is left)
 $K_{86} = 0$ (no pitch is due due to unit yaw is left)
 $K_{96} = 0$ (no yaw is due due to unit yaw is left)

ball joints do not transfer rotations
 from left to the deck
 (check is partially isolated)

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Let us try to find out what will be the value of K_{66} . Now K_6 is rotational degree of freedom. So, into θ_6 will be equal to T_0 plus ΔT_6 of $2a$ square by l_6 , there are 3 such legs equation number 14. K_{76} will be actually 0 because no roll in deck due to unit yaw in the leg. Similarly K_{86} and K_{96} will also be 0 due to no pitch and no yaw in the deck, due to unit yaw rotation of the leg. The answer is very simple this is because of a simple reason that ball joints do not transfer rotations from the legs to the deck. So, that is deck is partially isolated. Now let us try to go to the unit rotations in deck.

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Let us say we want to now give unit rotation in roll degree of freedom of the deck that is θ_6 . So, if I draw the figure say this is my deck position, supported by the ball joints the buoyant legs which are tethered to the sea bed. If I have the deck plan which is triangular which has got 2 legs on one side, and one on other side and if this is my c g if this my x axis this is the roll which I am looking at to look at this I must rotate it about x axis. So, let us view this from here this is a view direction if I call this as a leg 1 and leg 3 and leg 2; obviously, this will be leg 1 and this will be leg 3, and I will have to from this direction you see if this is my P_1 this will be P_1 by 2 this will be P_1 by 2 correct this is the centre.

So, let us mark the water line, c g will be somewhere let us say here. So, at the point on the deck we are giving unit rotation and I will be able to get K_{77} right. So, we will be able to get 3 corresponding degrees, which we will mark as we derive them. Now let us

understand very clearly that K_{17} will be 0, that is no surge in buoyant leg due to unit rotation in deck please understand this because rotation is not transferred. Let us try to find out K_{27} will not be equal to 0. So, K_{27} will be sway in the legs unit roll in deck.

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The slide shows two equations written in black ink on a white background with blue horizontal lines. The first equation is $K_{27} = \frac{(T_0 + \Delta T_7)}{\frac{P_1}{2} \tan \theta_7}$ labeled (15). The second equation is $K_{37} = \frac{(T_0 + \Delta T_7)}{\frac{P_1}{2} \tan \theta_7}$.

Which will be equal to $T_0 + \Delta T_7$ that is a new tension divided by P_1 by 2 $\tan \theta_7$ that is equation number 15. K_{37} will be $T_0 + \Delta T_7$ by P_1 by 2 $\tan \theta_7$.

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The slide shows five equations written in black ink on a white background with blue horizontal lines. The first equation is $K_{27} = \frac{T_0 + \Delta T_7}{\sqrt{S_{wB}^2 + 1}}$ labeled (15). The second equation is $K_{37} = \frac{(T_0 + \Delta T_7)}{\frac{P_1}{2} \tan \theta_7}$ labeled (16). The third equation is $SwB = \frac{P_1}{2} (1 - \cos \theta_7)$ labeled (17). The fourth equation is $Q_7 = P_1 \sin \theta_7$ labeled (18a). The fifth equation is $\theta_7 = \tan^{-1} \left[\frac{(P_1 \tan \theta_7)}{SwB} \right]$ labeled (18b).

K_{27} is $T_0 + \Delta T_7$ by square root of S_{wB} square plus 1 square, where S_{wB} is actually P_1 by 2 $(1 - \cos \theta_7)$.

So, the eccentricity will be given by $h \sin \theta$ or let us call this as θ . So, θ is actually $\tan^{-1} \left(\frac{P_1}{2} \tan \theta \right)$. It is very important to note that $S w B$ given by the equation 17 depends on K_{47} .

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$S w B$ depends on K_{47}
 $K_{47} \theta = K_{27} S w B$ ——— (19)
 $K_{57} = 0$ (no pitch in leg due to unit roll in deck)
 $K_{67} = 0$ (no yaw in leg due to unit roll in deck)
 $K_{77} \theta + (T_0 + \Delta T_7) \frac{P_1}{2} - (T_0 + \Delta T_7) P_1 + F_b e_7 = 0$
 $K_{77} = \frac{P_1 \Delta T_7 + F_b e_7}{\theta}$ ——— (20)
 $K_{57} = 0$ (no pitch in deck, due to unit roll in deck)
 $K_{67} = 0$ (no yaw in deck, due to unit roll in deck)

So, K_{47} into θ is actually K_{27} into $S w B$ into h equation number 19. K_{57} will be 0 that is no pitch in leg due to unit roll in deck, K_{67} will also be 0 because no yaw in legs due to unit roll in deck. K_{77} can be computed by this equation which is equilibrium in the roll degree of freedom ΔT_7 of P_1 by 2 minus T_0 plus ΔT_7 of P_1 by 2 where we can say letter T_7 dash that is the tension in the father leg P_1 by 2 plus buoyancy into eccentricity this should be equal to 0 that is the equilibrium equation.

From this one can estimate K_{77} as $P_1 \Delta T_7 + F_b e_7$ by θ . K_{87} will be 0 that is no pitch in deck due to unit roll in deck, K_{97} will also be 0 that is no yaw in deck due to unit roll in deck. So, we have worked out the seventh column of the stiffness matrix.

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Unit pitch rotation to the deck ($\theta_p = 1$)

$$K_{1P} = \frac{3T_0 + 2\Delta T_1 - \Delta T_2}{\sqrt{S_{u0}^2 + 1}} \quad (21)$$

$$S_{u0} = \frac{2P_b}{3} (1 - \cos\theta_p) \quad (22)$$

$K_{2P} = 0$ (No sway in the leg due to unit pitch in the deck)

$K_{3P} \neq 0$

$$K_{3P} = \frac{3T_0 + 2\Delta T_1 - \Delta T_2}{z_1} \quad (23)$$

The diagrams illustrate a tricopter deck with two buoyant legs. The top diagram shows a side view with a unit pitch rotation $\theta_p = 1$ and a horizontal force P_b applied at a height $P_b/3$. The bottom diagram shows a plan view of the deck with two legs, each of length z_1 , and a horizontal force $P_b/3$ applied at a distance $z_1/3$ from the center. Stiffness coefficients K_{1P} , K_{2P} , and K_{3P} are indicated at the deck level.

Let us give unit displacement or unit pitch rotation to the deck, that is let us give theta 8 as unity. So, the figure is similar to what we have here. So, we want to give unit rotation now. So, let us do that. So, we will be getting portion of the deck, which is connected by the legs these are the buoyant legs, which are connected to the sea bed by tethers let us mark the water line ok.

Let us draw the plan on the top. So, this is my x axis, I am now giving unit pitch that is theta 8 is unity. So, I must look at the system from this view. So, if this is like 1, this is like 3 and this is leg 2 this will be leg 1 and 3 this will be leg 2 and the c g will be here somewhere here. So, this distance is $P_b/3$ because this is P_b and this is two-third P_b let us now mark the degrees of freedom corresponding to the deck and to this these are the points where I am going to mark the degrees of freedom. So, this will be K_{18} and of course, this is K_{88} sorry this is K_{58} and this is K_{38} and we are giving unit rotation theta 8 therefore, this will be K_{88} on the deck. So, K_{18} will be $3T_0 + 2\Delta T_1 - \Delta T_2$ but let us keep this as 1 and this as 2 that is the delta T is in different legs are going to be different because these legs are closer these legs are farther.

So, I had taken 2 different notations here to arrive at them divided by $S_{uB}^2 + 1$ square the root. So, let this equation be equation number 21, where S_{uB} is given by $2P_b/3 (1 - \cos\theta_8)$. Now K_{28} will be 0 that is no sway in the leg due to unit pitch in the deck rotate will not transferred. K_{38} will not be equal to 0. So, K_{38} is given by 3

T_0 plus $2 \Delta T_1$ minus ΔT_2 because one will be slacking and one will be on increase in tension divided by Z_1 equation 23.

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$$Z_1 = \frac{P_b}{3} \tan \theta_8$$

$$Z_2 = \frac{2P_b}{3} \tan \theta_8$$

$$\Delta T_1 = \left(\frac{AE}{L} \right) Z_1$$

$$\Delta T_2 = \left(\frac{AE}{L} \right) Z_2$$

$$e_8 = \bar{h} \sin \theta_{58}$$

$$\theta_{58} = \tan^{-1} \left(\frac{Sub}{Z_2} \right)$$

Where Z_1 is P_b by $3 \tan \theta_8$ and Z_2 is two-third P_b half $\tan \theta_8$, ΔT_1 the nearer leg will have AE by L of Z_1 and ΔT_2 the farther leg will have AE by L of Z_2 . So, now, eccentricity will be $e_8 = \bar{h} \sin \theta_{58}$ and θ_{58} is given by \tan inverse of Sub by Z_2 .

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$$k_{sp} = 0 \text{ (no roll in the leg due to unis pin in the deck)}$$

$$k_{sf} \neq 0.$$

$$k_{sf} = \frac{1}{\theta_{sf}} (k_{sp} \sin \bar{h}) \quad \text{--- (25)}$$

$$\tan \theta_{sf} = \frac{Z_1}{Sub} \quad \text{--- (26)}$$

$$k_{sp} \parallel = 0 \text{ no transfer of rotation from deck to the leg}$$

$$k_{sf} \theta_{sf} = (T_0 + \Delta T_1) \frac{2P_b}{3} - (T_0 + \Delta T_2) \frac{2P_b}{3} + F_0 e_8 \quad \text{--- (27)}$$

So, let us say equation number 24. Now K_{48} will be 0 that is no roll in the leg due to unit pitch in the deck. K_{58} will not be equal to 0 because there is a differential heave K_{58} will be equal to $K_{38} \sin \theta_{58}$ equation number 25, where θ_{58} is given by Z_1 by \sin equation 26. Now K_{68} and K_{78} will be 0 because no transfer of rotation from deck to the leg K_{88} into θ_{88} will be given by T_0 plus ΔT_{81} of 2 P b.

By 3 the closer legs minus T_0 plus ΔT_{82} of 2 P b by 3 plus f_b into e_8 where e_8 is given by this equation.

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$K_{49} = 0$ (no yaw in deck due to unit pitch of the deck)
 unit rotation in yaw deg to the deck

$$\Delta T_9 = \frac{GJ_{deck}}{L_{deck}} (\theta_9 - 1) \quad \text{--- (28)}$$
 $K_{19} = 0$ || no surge/sway in the leg
 $K_{29} = 0$ || due to unit yaw rotation of the deck

$$K_{39} = \frac{3 T_0 \left(\frac{L}{L_0} - 1 \right) + \Delta T_9 \left(\frac{L}{L_0} \right)}{\Delta T_9 / (L_0 - 1)} \quad \text{--- (29)}$$

The diagram shows a trapezoidal structure with vertices labeled 1, 2, 3, 4. A horizontal dashed line represents the original position. A solid line represents the rotated position. A point 'c' is marked on the horizontal line. Vectors k_{19} and k_{29} are shown. Angles θ_9 and θ_{19} are indicated. A red line is drawn from point 'c' to the top vertex 1.

So, K_{98} will be 0 no yaw in the deck due to unit pitch of the deck. Lastly let us give unit rotation in yaw degree of freedom to the deck, let us say this is my original position leg 1, leg 3 and leg 2 we get a new position because let us mark the c_g and give unit rotation about this point, let us say unit rotation about this point, let us say its rotated like this or let us slightly increase this line and let us mark it like this ok.

So, I get at the c_g we have rotation θ_9 as unity, I get K_{99} and I will also get K_{59} . So, change in tension ΔT_9 is given by GJ of the deck by t of the deck into 1_9 minus 1 equation number 28; where K_{19} will be 0 K_{29} is also 0 because no surge and sway in the legs due to unit yaw rotation of the deck, because ball joint is not trans to the rotation K_{39} will be T_0 change in length minus original length of 3 legs min plus 3 ΔT_{91} by 1_9 of ΔT_{919} minus 1 ok.

Let us look at the summary; friends, in the 2 lecture this lecture and the previous lecture we understood the structural action of triceratop under unit displacements and rotations. We derived the stiffness matrix of the triceratop we have stiffness matrix now which is 9 by 9 which we expected to derive from the beginning from first principles. So, the computer method of deriving stiffness matrix by knowing the coefficients k_{ij} will be helpful to actually write a program to derive this matrices on first principles.

Thank you very much.