

**Computer Methods of Analysis of Offshore Structures**  
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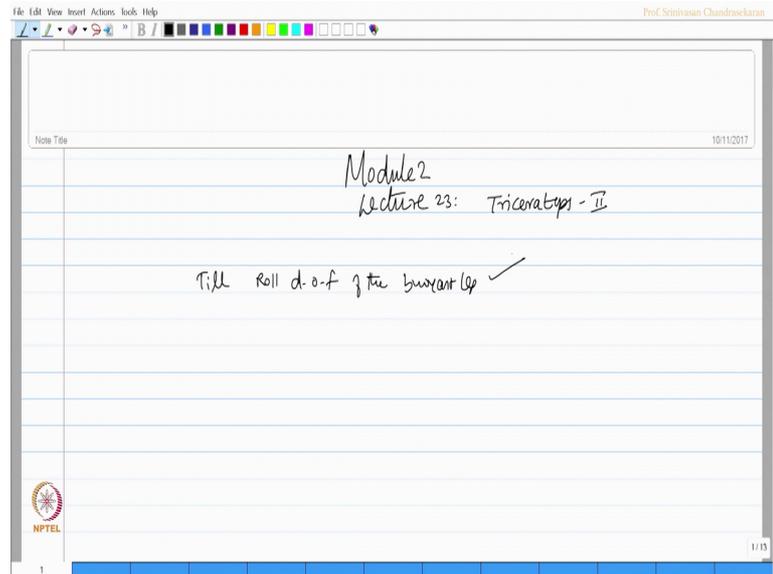
**Module – 02**  
**Lecture – 23**  
**Triceratops – 2 (Part –1)**

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- Triceratops
- Derivation of stiffness matrix
- Structural action of triceratops

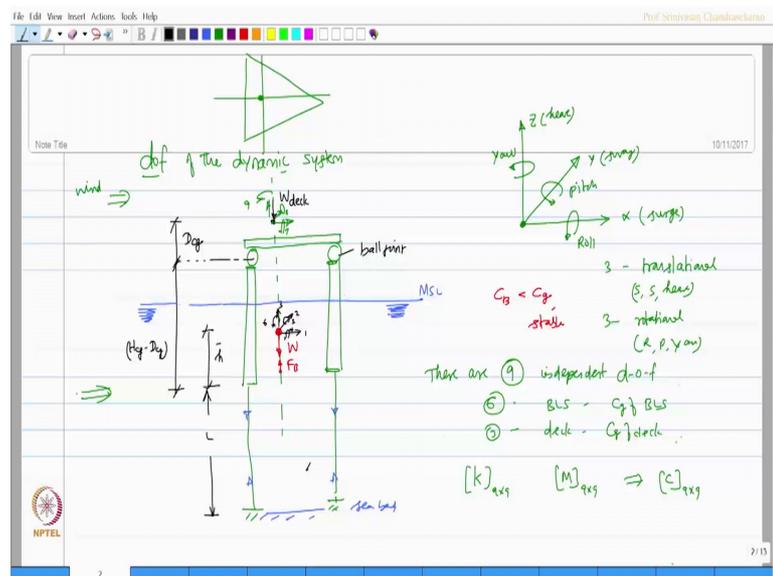
Friends, let us continue to derive the stiffness coefficients of triceratops. We discussed the last lecture the stiffness coefficients till role degree of freedom of the buoyant leg.

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Now we will start discussing the stiffness coefficients from the pitch degree of freedom of the buoyant leg. So, just for recollection you know, triceratops has 9 degrees of freedom there are 6 degrees of freedom of the buoyant leg as you see here.

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And 3 degrees of freedom for the deck numbered sequentially as 1 to 6 for the buoyant legs and 7 8 9 are independent rotations of the deck which are not connected and transferred from that of the buoyant leg, that happens because of the presence of ball joint.

So, the dimensional variations are given in the figure as shown in the screen now, the degrees of freedom are anyway marked. So, we are getting 3 degrees of freedom for the deck and 6 independent degrees of freedom for the buoyant leg which makes it 9, and we now understand that my stiffness and mass matrices will be of size 9 by 9.

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Let unit rotation be applied in pitch dir of the buoyant leg

$$K_{15} = \frac{2(T_0 + \Delta T_s)}{S_w + L^2} \quad (1)$$

$$S_w = (H_{cg} - d) \sin \theta_s \quad (2)$$

$K_{25} = 0$  (no sway response in the buoyant leg due to unit pitch rotation of the buoyant leg)

$$\Delta T_s = \frac{A\epsilon}{\lambda} \left( \frac{2}{3} p_b \omega_s \right) \theta_s - 2 \left( \frac{A\epsilon}{\lambda} \right) \left( \frac{p_b}{3} \omega_s \right) \theta_s \quad (3)$$

Let us continue to discuss the derivation of stiffness coefficients from the first principles from the pitch degree of freedom. Let us say this is my deck I have ball joint, which is now connected to the buoyant legs, which are anchored to the sea bed using high initial tension tethers similar to the tougher tension like platform, being a triangular geometry Cg will be shifted towards the left side, let us draw this line as my Cg line.

So, let us mark the degrees of freedom, this is my water line this is my centre of buoyancy and centre of gravity, let us say this my Cg point, where my degrees of freedom are marked as K 15 and K 3 5 and K 55; centre of buoyancy is located slightly in offset because of a simple reason you are giving pitch is a rotational degree therefore, there may be a possibility of a small shift in the centre of buoyancy. So, the shift between this 2 is eccentricity, which you call as e 5 and change in tension because of the shift will be indicated as T 0 plus delta T 5.

So, from this distance this place or this leg p b by 3 and two-third where of course, this distance is p b in the plant dimension the deck will have an independent rotation component which will be K 85. So, now, referring to this figure, let unit rotation be

applied in pitch degree of freedom of the buoyant leg. Say  $K_{15}$  is given by  $T_0 + \Delta T_5$  there are 3 such legs divided by sum of root of squares of  $SuD$  square plus 1 square where I will call this equation as let us say start with the new number here 1, where  $SuD$  is  $H_{cg} - \bar{h} \sin \theta_5$ .  $K_{25}$  will be 0 because no sway response in the buoyant leg, due to unit pitch rotation of the buoyant leg.  $\Delta T_5$  is given by  $2 \frac{p_b \cos \theta_5}{3}$  of  $\theta_5$  of  $AE$  by 1 because we are looking for an axial stiffness minus  $2 AE$  by you can see here there are 2 legs, which are closer to one-third. So, I should say  $2 AE$  by  $1 \frac{p_b}{3 \cos \theta_5}$  of  $\theta_5$ .

So, equation 3 will give me the change in tether tension caused because of unit rotation in the pitch degree of freedom.

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The slide contains the following handwritten equations:

$$K_{35 z_b} = 2 (T_0 + \Delta T_5) \cos \theta_5 + (T_0 + \Delta T_5) \cos \theta_5 - 3 T_0 \quad (4)$$

$$K_{55 \theta_5} = F_b e_5 + (T_0 + \Delta T_5) 2 (s_1 - e_5) - (T_0 + \Delta T_5) (s_2 + e_5) - W_{deck} SuD \quad (5)$$

Where  $e_5 = \bar{h} \sin \theta_5$

$$s_1 = \frac{p_b}{3} + e_5 \quad (6)$$

$$s_2 = p_b - s_1$$

So,  $K_{35 z_b}$  will be given by  $2 T_0 + \Delta T_5 \cos \theta_5$ , 2 such legs plus  $T_0 + \Delta T_5$  of one additional leg separately, because in this figure 2 legs are oriented on the same line whereas one leg is separate. So, I am writing it separately here minus  $3 T_0$  that is the initial tension the cable, that is equation number 4.  $K_{55 \theta_5}$  will be now equal to  $F_b e_5 + T_0 + \Delta T_5$  of twice of  $s_1 - e_5$  minus  $T_0 + \Delta T_5$ , separate leg which is  $s_2 + e_5$  minus  $w_{deck}$  of  $SuD$ . Where  $e_5$  is actually  $\bar{h} \sin \theta_5$  and  $s_1$  is actually  $\frac{p_b}{3} + e_5$ , and  $s_2$  is  $p_b - s_1$  equation number 6.  $K_{45}$  will be 0.

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$K_{65} = 0$  (rotation in roll dof due to unit pitch rotation is zero of the buoyant leg)  
 $K_{65} = 0$  (No yaw motion in the buoyant leg due to unit pitch rotation in the buoyant leg)  
 $K_{75} = 0$  (no motion about sway axis of the deck due to unit pitch rotation of the buoyant leg)  
 $K_{85} = \checkmark$  (influence on rotation about surge axis due to unit pitch rotation of the buoyant leg)  
 Variasi dengan inputs differential heave  
 this is transferred from the leg to the deck  
 That causes  $K_{85}$

That is rotation in roll degree of freedom due to unit pitch rotation is 0, because any way roll and pitch are not directly coupled of the buoyant leg these are all for the buoyant leg.  $K_{65}$  is 0 because no yaw motion in the buoyant leg due to unit pitch rotation in the buoyant leg.  $K_{75}$  is also 0 because no motion about sway axis of the deck because we are talking about degree of freedom 7, due to unit pitch rotation of the buoyant leg.  $K_{85}$  will be having some value because this will have influence on rotation about surge axis due to unit pitch rotation of the buoyant leg.

One may ask me a question how rotation from the buoyant leg is transferred to the deck, it is not transferred actually; when look at this figure when the deck is rotated because of submergence happening between the legs there is no transfer of this rotation from the leg to the deck, but deck alone will have some influence of variable tension which will make the deck to rotate. Please understand variable tension imposes heave difference. So, variable tension imposes differential heave and this is transferred from the leg to the deck and that causes  $K_{85}$ .

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The screenshot shows a presentation slide with the following content:

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$$K_{\theta 5} = k_{35} z_b \text{ SuD} + [k_{15} \text{ SuD} (H_{cg} - \bar{h})] \quad \text{--- (7)}$$

$$z_t = (H - \bar{h}) \left( \frac{\text{SuD}}{\tan \theta_5} \right) \quad \text{--- (8)}$$

$$z_b, \bar{h} = \left( \frac{e_5}{\tan \theta_5} \right) \quad \text{--- (9)}$$

$$\theta_{\theta 5} = \tan^{-1} \left( \frac{z_t}{(p_b/3)} \right) \quad \text{--- (10)}$$

$k_{95} = 0$  (no yaw motion in the deck due to unit pitch in the legs)

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So,  $K_{\theta 5}$  into  $\theta_5$  will be essentially arising from  $K_{\theta 5} z_b \text{ SuD}$  plus  $K_{15} \text{ SuD} H_{cg} - \bar{h}$  calls the equation number 7. So, from this equation you can say  $z_t$  top will be actually  $H - \bar{h}$  of  $\text{SuD}$  by  $\tan \theta_5$  and  $z_b$  bottom is  $\bar{h} - e_5$  by  $\tan \theta_5$  which is equation 9 and  $\theta_{\theta 5}$  is actually  $\tan^{-1}$  of  $z_t$  by  $p_b/3$  which is one of the planned dimensions of the platform, further  $K_{95}$  is 0 that is no yaw motion in the deck due to unit pitch in the legs ok.

So, by this logic I have worked out  $K_{15}$ ,  $K_{25}$ ,  $K_{35}$ ,  $K_{45}$ ,  $K_{55}$ ,  $K_{65}$ ,  $K_{75}$ ,  $K_{85}$  and  $K_{95}$ . So, I have got all the 9 coefficients 6 for the legs and 3 for the deck for unit rotation in the pitch degree of freedom. So, we derive now the fifth column of the stiffness matrix.

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Do the same for unit rotation in yaw dof of the buoyant leg ( $\theta_6 = 1$ )

$K_{16} = 0$  (no surge force due to unit yaw rotation of the buoyant leg)

$K_{26} = 0$  (no sway force in the buoyant leg due to unit yaw rotation of the leg)

Unit  $\theta_6$  to the left from the geometry,

$$a^2 = (p_1)^2 + (p_2)^2$$

$$l_6 = \sqrt{l^2 + 2(a\theta_6)^2} \quad \text{--- (11)}$$

So, let us now do for do the same for do the same for unit rotation in yaw degree of freedom of the buoyant leg that is theta 6 is given to be unit that is I want to do this. Let us draw the figure this is my initial configuration of the platform which having a triangular deck

There are 3 legs 1 2 and 3, I name them as 1 2 and 3 let us now draw the new position by rotating it. Let us say a rotate by some angle at the Cg. We know this is p 1 and this is p b equation will be now available for deriving the stiffness matrix coefficients, let us say this is my new position of the first leg let us call this leg as 3 and this leg as 2. So, the new legs 1 3 and 2. So, now, they will have new tension which is T 0 plus delta T 6, T 0 plus delta T 6 and T 0 plus delta T 6. Now K 1 6 let us say now what are the degrees of freedom we are going to mark we are imposing theta 6 unit rotation.

So, we will get K 66 and further things. So, let us say K 1 6 is 0, that is no surge force due to unit yaw rotation of the buoyant leg K 2 6 is also 0. So, we should say no sway force in the buoyant leg due to unit yaw rotation of the leg. So, by giving unit rotation that is let us impose unit theta 6 to the legs, now from the geometry we can say that a square and b square can be new variables, which can be p 1 by 2 the whole square plus p b by 3 the whole square and l 6 the new length of the tether because of this twist will be root of sum of squares of l square plus 2 a theta 6 the whole square I call equation number 11.

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Change in initial pre-tension in each leg is given by:

$$\Delta T_6 = \frac{AE}{l} (l_6 - l) \quad \text{--- (12)}$$

Force in heave dof

$$K_{36} (l_6 - l) = 3 [T_0 + \Delta T_6 - T_0] \quad \text{--- 13}$$

$K_{46} = 0$  (no roll due to unit yaw rotation of the leg)

$K_{56} = 0$  (no pitch due to unit yaw rotation of the leg)

So, change in initial pre tension in each leg is given by  $\Delta T_6$  which is  $l_6$  minus  $l$  of  $AE$  by 1 equation number 12, let us now summarize the force in heave degree of freedom. So,  $K_{36}$  of  $l_6$  minus  $l$  will be actually equal to or will be balanced by  $T_0$  plus  $\Delta T_6$  minus  $T_0$  of the top 3 legs equation 13.  $K_{46}$  will be 0 that is no roll due to unit yaw rotation of the leg,  $K_{56}$  will also be 0 that is no pitch due to unit yaw rotation of the leg.