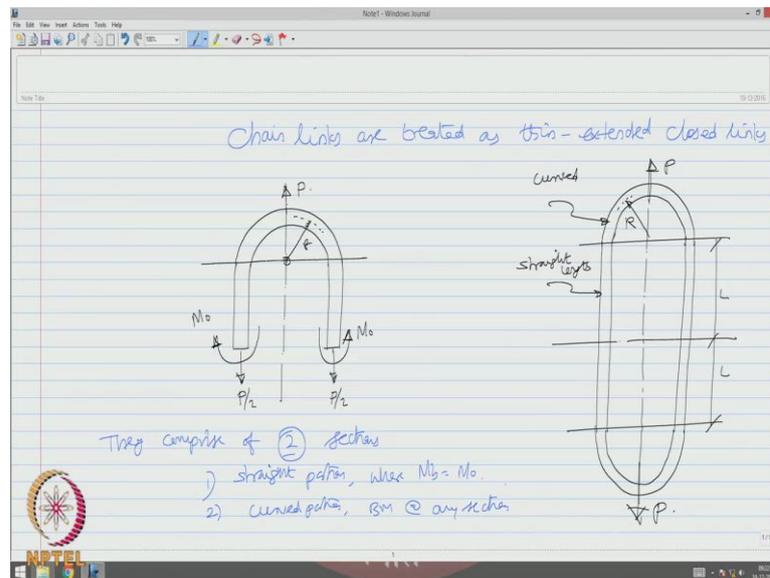


**Offshore structures under special loads including Fire resistance**  
**Prof. Srinivasan Chandrasekaran**  
**Department of Ocean Engineering**  
**Indian Institute of Technology, Madras**

**Module - 02**  
**Advanced Structural Analyses**  
**Lecture - 36**  
**Rings and Chain Links-II**

Friends, we will continue the discussion on the Rings and Chain Links. We did an example problem on closed rings and (Refer Time: 00:28) tensile loads. In this lecture we will derive the equations to find the stresses on chain links and we will derive the equation to determine the bending stresses and also do a numerical example to estimate bending stresses or total stresses in the intrados and extrados of a chain link under the given loads.

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In opening remark we can say that chain links are treated as thin, extended closed links. A typical chain link looks like this, so it is got 2 components; this is the straight component and this is the curved component. Let us call this distance as  $L$  and let us call this radius as  $R$ . Let us say the link is subjected to an axial force; tensile force  $P$  as shown in the figure. Let us take a segment of the ring and the link which I am showing here say this is my  $R$  subjected to loads  $P$  by 2 and moment;  $M_0$  where moment  $M_0$  indicates increase of radius of curvature in the given section. So, as I said they have 2

sections; one the straight portion where  $M_b$  is simply  $M_0$  and for the curved portion bending moment is at any section as shown below.

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$$M_b = M_0 - \frac{P}{2} (R - R \cos \theta)$$
 As per Castigliano's theorem,
 
$$\frac{\partial U}{\partial M_0} = 0$$

$$\Rightarrow \frac{\partial U_{\text{straight}}}{\partial M_0} + \frac{\partial U_{\text{curved part}}}{\partial M_0} = 0$$

$$\left( \frac{\partial U}{\partial M_0} \right)_{\text{total}} = 4 \int_0^L \frac{M_0}{EI} \frac{\partial M_0}{\partial M_0} ds + 4 \int_0^{\pi/2} \frac{M_b}{EI} \left( \frac{\partial M_b}{\partial M_0} \right) ds = 0$$

(Refer Time: 05:13) extend this further close it here, this is  $P$  by  $2$  this is  $M_0$ ; let us say this is  $R$  and this is angle  $\theta$  and this distance is actually  $R$  minus  $R \cos \theta$  subjected to  $P$  by  $2$ . Therefore  $M_b$  is  $M_0$  minus  $P$  by  $2$ ;  $R$  minus  $R \cos \theta$ . So, as per Castigliano's theorem; the strain energy partial derivative should be said to  $0$  which now indicates that strain energy for the straight portion plus strain energy for the curved portion should be said to  $0$ . So, this strain energy total is actually four times of  $0$  to  $l$  m by  $e$  I because  $M$  is  $M_0$ ; in the straight portion and  $d$ ou  $M_0$  by  $d$ ou  $M_0$  which is  $d$  s, that is the straight portion plus curved portion segments  $0$  to  $\pi$  by  $2$ ,  $4$  in number, the moment in this case going to be  $M_b$ ; therefore,  $M_b$  by  $M_0$  and  $d$  s and that to be said to  $0$ ; equation 1.

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$$\Rightarrow 4 \int_0^l \frac{M_0}{EI} ds + 4 \int_0^{\pi/2} \frac{M_b}{EI} \psi ds = 0$$

$$\Rightarrow \frac{4 M_0 l}{EI} + \frac{4R}{EI} \int_0^{\pi/2} \left[ M_0 - \frac{P}{2} (R - R \cos \theta) \right] d\theta = 0$$

$$\Rightarrow \frac{4 M_0 l}{EI} + \frac{4R}{EI} \left[ M_0 \theta - \frac{PR}{2} \theta + \frac{PR}{2} \sin \theta \right]_0^{\pi/2}$$

$$\Rightarrow \frac{4 M_0 l}{EI} + \frac{4R}{EI} \left[ M_0 \frac{\pi}{2} - \frac{PR}{2} \frac{\pi}{2} + \frac{PR}{2} \right] = 0$$

$$\Rightarrow \frac{M_0 l}{EI} + \frac{M_0 \pi R}{2EI} - \frac{PR^2 \pi}{4EI} + \frac{PR^2}{2EI} = 0$$

Which implies that 4 times 0 to l; M 0 by E I; d s plus 4 times pi by 2; M b by E I; d s which indicates 4 R M 0; l by E I plus 4 R by E I 0 to pi by 2 M 0 minus P by 2; R minus R cos theta, d theta which implies 4; M 0 l by E I plus 4 R by E I; M 0 theta minus P R by 2 theta plus P R by 2 sin theta; 0 to pi by 2 which implies 4; M 0 l by E I plus 4 R by E I; M 0 pi by 2 minus P R by 2 pi by 2 plus P R by 2, it should be said to 0 which implies M 0 l by E I because is 4 can get cancelled plus M 0 pi R by 2 E I minus P R square pi by 4 E I plus P R square by 2 E I which can be said to 0.

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$$\Rightarrow M_0 l + \frac{M_0 \pi R}{2} - \frac{PR^2 \pi}{4} + \frac{PR^2}{2} = 0$$

$$\Rightarrow \frac{M_0}{2} (2l + \pi R) = \frac{PR^2}{2} (-1 + \pi/2)$$

$$\Rightarrow M_0 \left( \frac{2l + \pi R}{2} \right) = \frac{PR^2}{2} \left( \frac{\pi - 2}{2} \right) \quad \text{--- (1)}$$

It can be easily shown that,

①  $l=0$ , Eq. (1) reduces to

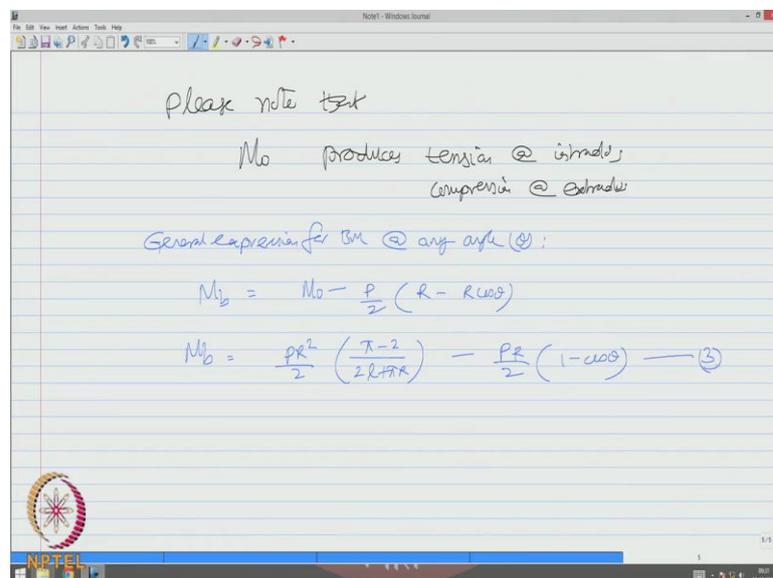
$$\frac{M_0 \pi R}{2} = \frac{PR^2}{2} \left( \frac{\pi - 2}{2} \right)$$

$$= M_0 = \frac{PR}{\pi} \left( \frac{\pi - 2}{2} \right) \quad \text{--- which is same as the fixed ring}$$

Which indicates  $M_0 + PR \sin \theta = \frac{PR^2}{2} (1 + \cos \theta)$ , which implies  $M_0 + PR \sin \theta = \frac{PR^2}{2} (1 + \cos \theta)$  which implies  $M_0 = \frac{PR^2}{2} (1 + \cos \theta) - PR \sin \theta$ , let us do like this  $M_0$  is equal to  $\frac{PR^2}{2} (1 + \cos \theta) - PR \sin \theta$  - equation 2.

Interestingly we can make a comment at this point, it can be easily seen that at  $\theta = 0$  equation 2 reduces to  $M_0 = \frac{PR^2}{2} (1 + 1) - PR \sin 0$  which is  $\frac{PR^2}{2} (2) - 0$  which means  $M_0 = PR^2$ , which is same as the earlier case of a closed ring  $\theta = 0$  means indicated a closed ring we get the same equation as we derived in the last lecture.

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Further please note that  $M_0$  produces tension at intrados and compression at extrados. Now we can write a general expression for bending moment, at any angle  $\theta$  is  $M_b$  which is  $M_0 - \frac{PR}{2} (R - R \cos \theta)$  which gives me  $M_b = \frac{PR^2}{2} (1 + \cos \theta) - PR \sin \theta$ .

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$\theta = 0$ , BM @ B will be given by:  
 $M_b = M_0 = \frac{PR^2}{2} \left( \frac{\pi - 2}{2L + \pi R} \right)$  (a)

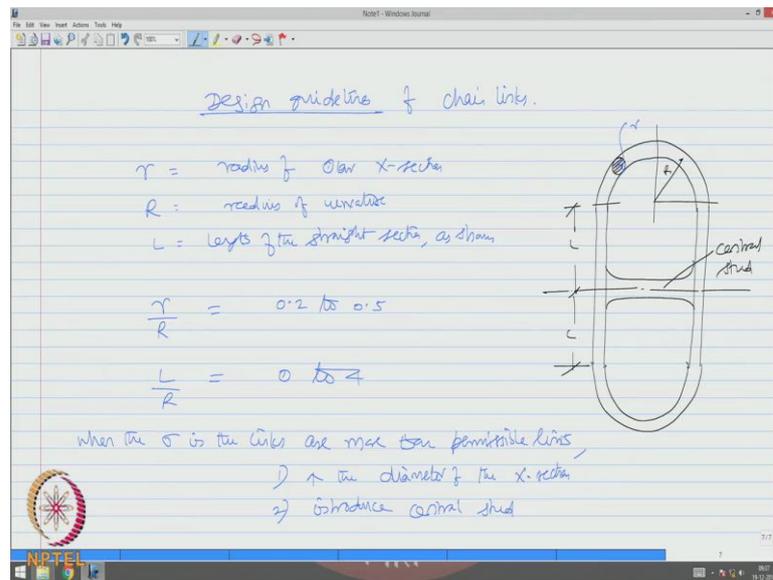
$\theta = \pi/2$ , Max BM @ A-A is given by:  
 $M_b = \frac{PR^2}{2} \left( \frac{\pi - 2}{2L + \pi R} \right) - \frac{PR}{2}$  (b)

Combined normal  $\sigma$ , across the section  
 $\sigma = \pm \frac{M_b}{I_e} \left( \frac{y + e}{y + R} \right) + \frac{P}{2A} \cos \theta$  (tensile is +ve)  
 indicates that direct stress are tensile.

Diagram: A semi-circular arch of radius R and height L. Section A-A is at the crown (top) and section B-B is at the base. The angle theta is measured from the vertical at the base.

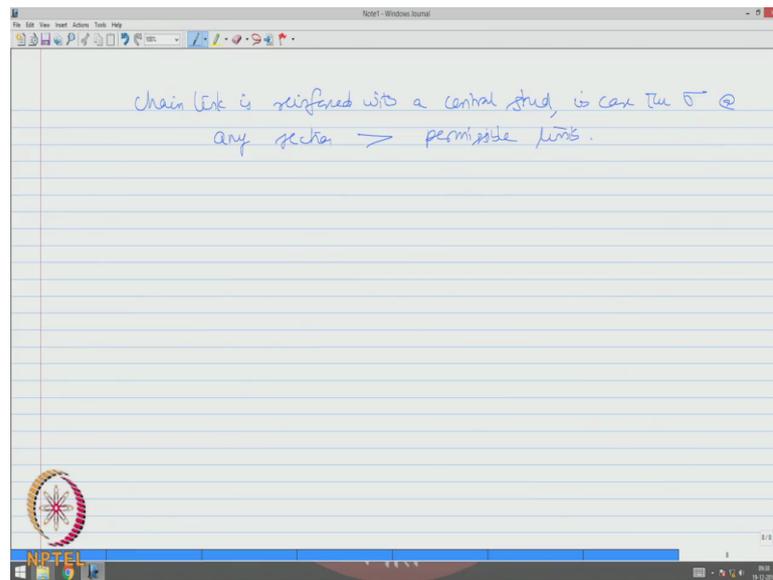
Let us interestingly ask a question, let us say this is my L, this is my section B B, this is my section A A and this my distance R and theta. So at theta equals 0, bending moment at B B will be given by will be  $M_0$  which is simply  $\frac{PR^2}{2} \left( \frac{\pi - 2}{2L + \pi R} \right)$ . At theta equals  $\frac{\pi}{2}$  maximum bending moment will occur at A A and is given by  $M_b = \frac{PR^2}{2} \left( \frac{\pi - 2}{2L + \pi R} \right) - \frac{PR}{2}$ , let us call equation number 4 a and 4 b. If you really want to find the combined normal stresses across this section then you should use the equation  $\frac{M_b}{I_e} \left( \frac{y + e}{y + R} \right) + \frac{P}{2A} \cos \theta$ , this positive indicates that direct stresses are tensile and tensile is positive.

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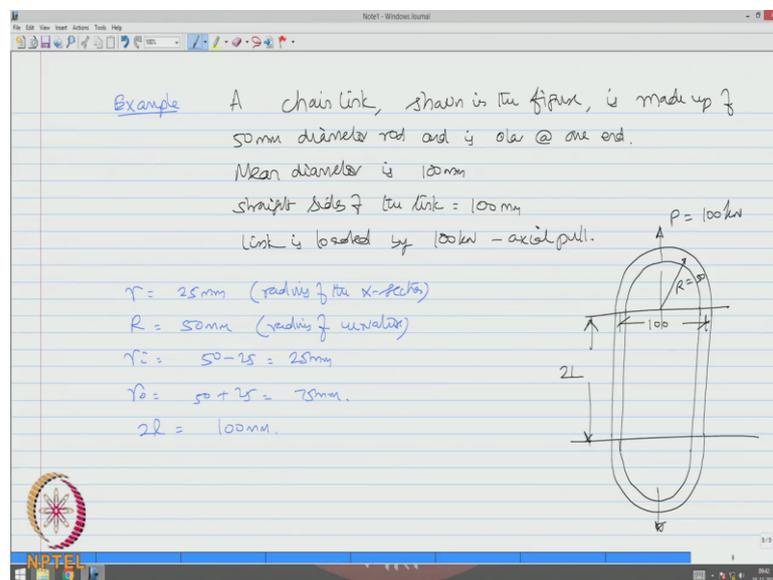
Now, for design guidelines of chain links, let us say this dimension is  $L$  and this is  $R$ , usually this circular cross section. So,  $r$ , be radius of circular cross section, capital  $R$ , be radius of curvature small  $r$  is this,  $L$  length of the straight section as shown. So,  $r$  by  $R$  usually varies from 0.2 to 0.5 for an effective design and  $L$  by  $R$ , usually ranges from 0 to 4, in sense a closed ring to a link. Now interestingly when the stresses in the links are more than permissible, there are 2 ways of correcting it; one increase the diameter of the cross section, alternatively introduce what is called as a central stud. So, central stud is a very interesting component which is introduced as showed figure; now this is called central stud.

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So the ring of the link is reinforced with a central stud, in case the stresses at any section, increases permissible limits.

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Now we will take up one example problem and try to find the stresses. Let us say the problem is like this, a chain link shown in the figure is made up of 50 mm diameter rod and it is circular at one end. Mean diameter is 100 mm, straight sides of the link are 80 mm the link is loaded by 100 kilo Newton axial pull; let us see the figure. So, we know that this length which is actually 2 L or we will say capital L which is 80 millimeters and

this distance is 100 therefore, these distance is going to be 50; this is subjected to (Refer Time: 21:38) pull of 100 kilometers. Let us write down certain data necessary for the numerical R is 25 mm that is radius of the cross section because the diameter is 55 mm, radius of curvature is 55 m; R i is 50 minus 25 which is 25 mm, R o is 50 plus 25 which is 75 mm and 2 L.

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1) To determine 'e' (shift of NA from the centroidal axis for the curved part)

$$e = R - \frac{r^2}{2[R - \sqrt{R^2 - r^2}]}$$

$$= 50 - \frac{(25)^2}{2[50 - \sqrt{50^2 - 25^2}]} = 3.349 \text{ mm}$$

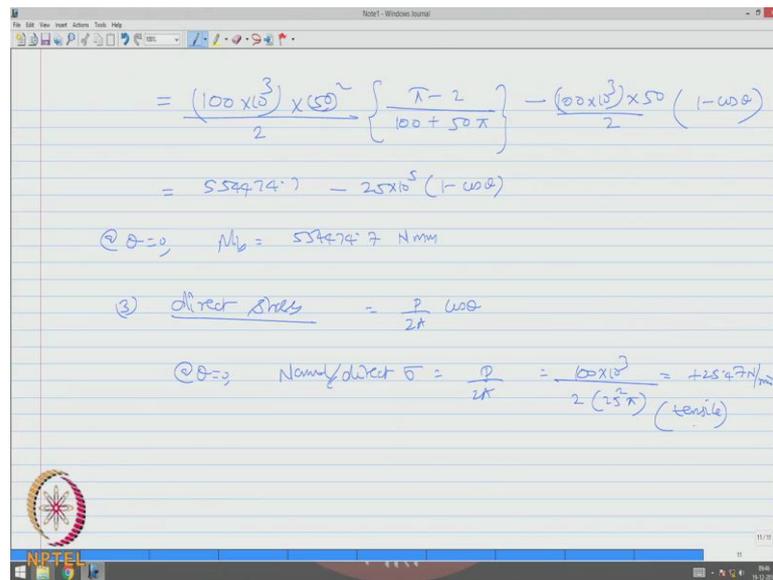
the indicates that NA shifts toward center of curvature

2)  $M_b$  @ any section, @ angle  $\theta$  &  $P, R, L$  by:

$$M_b = \frac{PR^2}{2} \left( \frac{R-r}{2L+\pi R} \right) - \frac{PR}{2} (1 - \cos \theta)$$

Now for the count part we need to find the e value, so to determine e that is shift of neutral axis from the centroidal axis for the curved part. We know the equation e is given by R minus R square by twice of R minus root of R square minus R square. Let us substitute 50 minus 25 square by twice of 50 minus root of 50 square minus 25 square, I will get this value as 3.349 millimeter, positive indicates that neutral axis shifts towards the center of curvature. M b at any section at an angle theta is given by M b is P R square by 2 pi minus 2; 2 L plus pi R minus P R by 2, 1 minus cos theta; we already derived this equation.

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$$= \frac{(100 \times 10^3) \times (50)^2}{2} \left\{ \frac{\pi - 2}{100 + 50\pi} \right\} - \frac{(100 \times 10^3) \times 50}{2} (1 - \cos \theta)$$

$$= 554974.7 - 25 \times 10^5 (1 - \cos \theta)$$

@  $\theta = 0$ ,  $M_b = 554974.7 \text{ Nmm}$

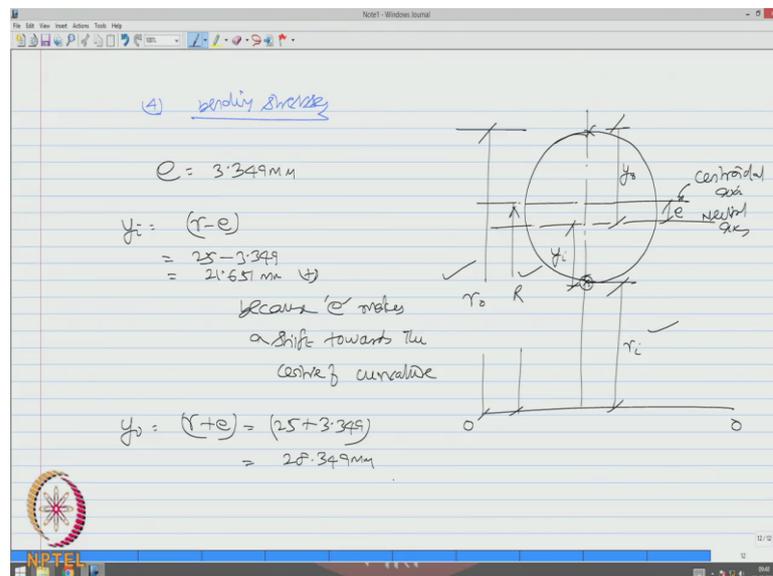
(3) direct stress  $= \frac{P}{2A} \cos \theta$

@  $\theta = 0$  Normal/direct  $\sigma = \frac{P}{2A} = \frac{100 \times 10^3}{2 \times (25\pi)^2} = +25.47 \text{ N/mm}^2$  (tensile)

Let us substitute 100 kilo Newton, so Newton's R is 50, so 50 square divided by 2 pi minus 2 by 2 L is 100 plus 50 pi minus P R by 2. So, 100; 1000 is my P R is 50 by 2 into 1 minus cos theta which is 55, 44, 74, 0.7 minus 25; 10 power 5; 1 minus cos theta.

Now, at theta equals 0 M b is 55, 44, 74, 0.7 Newton millimeter, I want to now find the direct stresses which is P by 2 A cos theta, at theta equals 0 normal stress or direct stress is simply P by 2 A which is 100; 1000 divided by 2 of 25 R square. So, pi R square which is plus 25.47 Newton per mm square plus indicates it is tensile.

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(4) bending stresses

$$e = 3.349 \text{ mm}$$

$$y_i = (r - e)$$

$$= 25 - 3.349$$

$$= 21.651 \text{ mm}$$

because 'e' makes a shift towards the centre of curvature

$$y_o = (r + e) = (25 + 3.349)$$

$$= 28.349 \text{ mm}$$

Now, I want to find the bending stresses, so now cross section is circular is my o o axis let us say this is my c g. So, centroidal axis this is my neutral axis, this distance is e and e is 3.349 millimeter and this distance is R intrados and this distance is R extrados and this distance is R which all are known to us. So, what I want to know is the distance of the extreme inner fiber and the distance of extreme outer fiber from the neutral axis to compute the bending stresses. So, I should say y i is r minus e which is state to be 25 minus 3.349 which is 21.651 millimeter positive because e makes a shift towards the centre of curvature and similarly this is of extreme fiber, extrados will be r plus e which is 25 plus 3.349 which is 28.349 millimeters.

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Handwritten calculations for bending stresses at intrados and extrados:

$$\sigma_i = +25.47 + \frac{557474.7}{(25)^2 \times 3.349} \left( \frac{21.651}{25} \right)$$

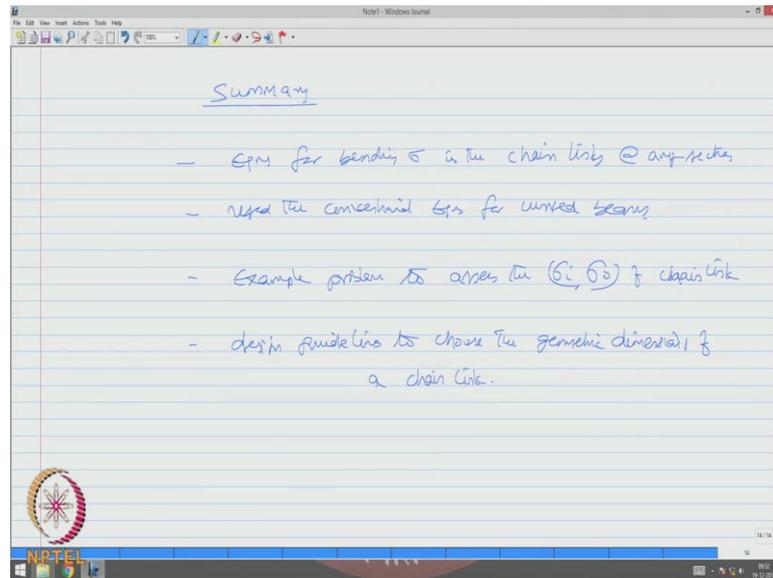
$$= +98.5 \text{ N/mm}^2 \text{ (Tensile)}$$

$$\sigma_o = +25.47 - \frac{557474.7}{(25)^2 \times (3.349)} \left( \frac{28.349}{25} \right)$$

$$= -6.40 \text{ N/mm}^2 \text{ (Compression)}$$

Once again now this I can easily compute the combine stresses at intrados and extrados at section B B which is theta equals 0. So, sigma intrados is plus 25.47 plus M b that is 55, 44, 74, 0.7 by A e which is 25 square pi into e is 3.349 multiplied by distance of extreme fiber inner by R I. Similarly outer will be plus 25.47 minus 55, 44, 74, 0.7 divided by 25 square pi 3.349 multiplied by 28.349 by 75. Please note friends this value will be negative, now this value is plus 98.5 Newton per mm square indicating it is tensile and these values minus 6.40 Newton per mm square indicating it is compressive.

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So, friends we have interestingly estimated the equations or derived equations for bending stress in the chain links. At any section we have used the conventional equations for curved beams which we have derived in the last set of lectures. We have also use an example problem to assess the stresses in intrados and extrados of a chain link with the numeric example, we have also understood certain elementary design guidelines to chose, the geometric dimensions of a chain link and chain links are very common and being used in the mooring lines in offshore structure systems, even for anchoring semicompliment, complaint of fully floating vessels in the sea.

I hope friends these set of lectures what we are discussing in module 2 under the coverage of advanced structural analysis are helpful and address certain special kinds of problems and estimating stresses in cross section, under certain special applications which will be very useful and which is essential in case Offshore Structures under Special Loads.

Thank you very much.