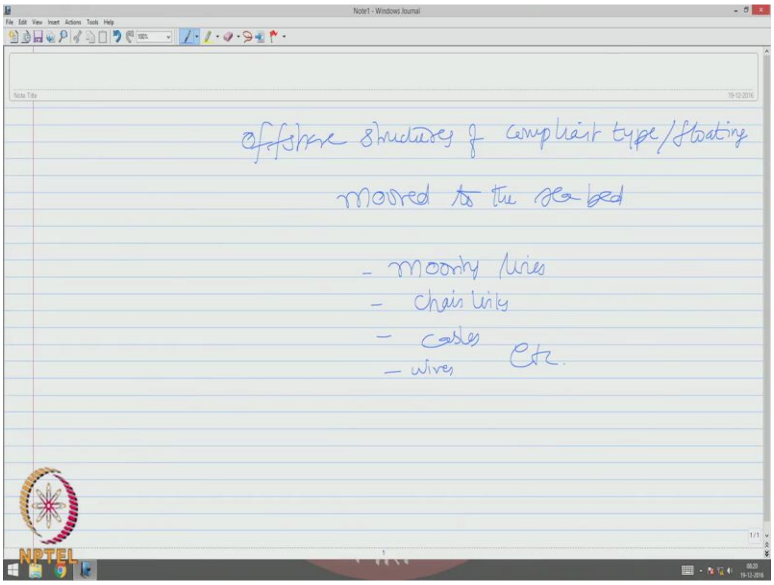
Offshore structures under special loads including Fire resistance
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Advanced Structural Analyses
Lecture - 35
Rings and Chain Links

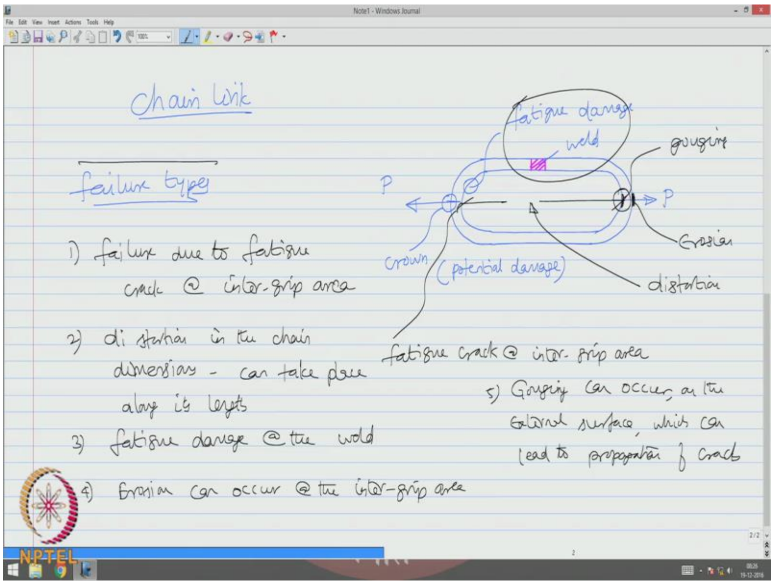
Friends, welcome to the 35th lecture title Rings and Chain Links, which is an extended study of curved beams what we discussed in the last set of lectures. This is under module 2; Advanced Structural Analyses of the course, offshore structures under special loads including fire resistance design.

(Refer Slide Time: 00:41)



We do agree that, so offshore structures of complaint type are generally moored to the sea bed, in fact complaint type and floating type are generally moored to the sea bed using mooring lines, chain links, cables wires etcetera. So, it is interesting for us to understand what would be the nature of stresses which are acting on these chain links and rings so that we can design them in a proper manner.

(Refer Slide Time: 01:47)

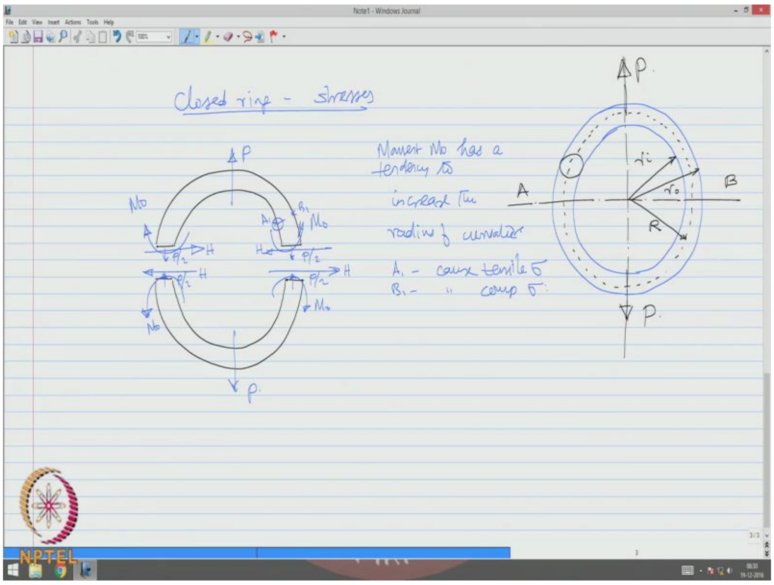


Let us take a typical chain link to understand the failure mode of this. Let us say there can be an issue related to the failure in the weld; let us say this is a typical weld. The intrados part can have fatigue damage, the crown part can also have a potential damage when the chain link is subjected to axial pull or push essentially it will be pull.

There are different failure types which can be envisaged in these kinds of chain links. If I say different failure modes, there can be possibly a fatigue crack at inner or inter grip area, so there can be failure due to fatigue crack occurring at the inter grip area. The second can be within this length, there can be a distortion in the chain dimensions which can take place along its length.

Thirdly there can be a fatigue damage at the weld, fourthly at the intrados part and the extrados part here erosion can take place; can occur at the inter-grip area and in this sector there can be gouging, gouging can occur on the external surface which can lead to propagation of cracks. So, there are different failure types by which a chain link can fail and the chain link fails; obviously, we understand that the station keeping of the floating vessels cannot be ascertained.

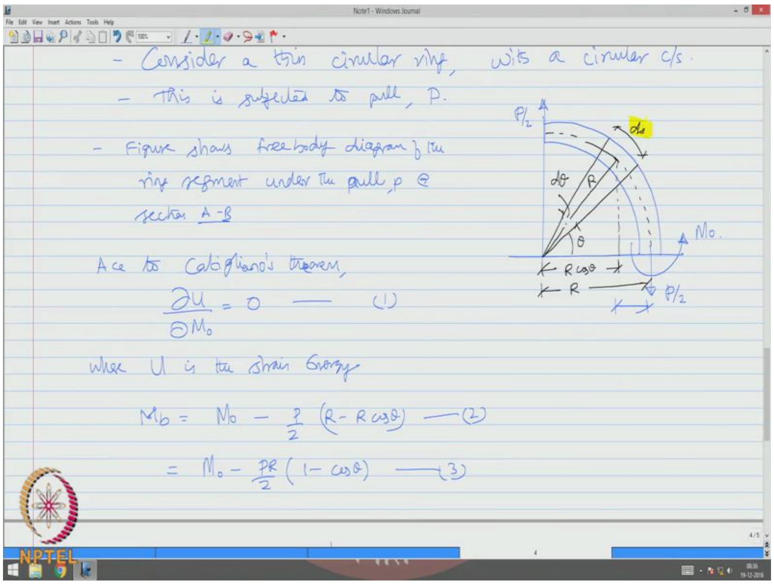
(Refer Slide Time: 06:38)



For our study now, let us consider closed ring and work out stresses in a closed ring. A typical closed ring is drawn in the figure; let us say it is subjected to forces of axial pull P. Let us take an horizontal axis of symmetry A B and of course, mark the centroidal axis of the cross section and mark these dimensions as r i, r o and this becomes R of the centroid, which usually called as capital R.

Once the ring is subjected to the force P then let us try to draw the split part of the ring, let us mark the forces to which they are subjected. Let us say this is my force P which will be acting on the system, now this will be resisted and opposed by P by 2, in addition there will be H acting and they are again in equilibrium. In addition there will be also a moment which will be acting as shown which are again in equilibrium at the marked section. Please note the moment M o has a tendency to increase the radius of curvature, it means at any point here mark this as; let us say A 1; at A 1; it will cause tensile stresses and at the corresponding B 1, it may cause compressive stresses.

(Refer Slide Time: 10:58)

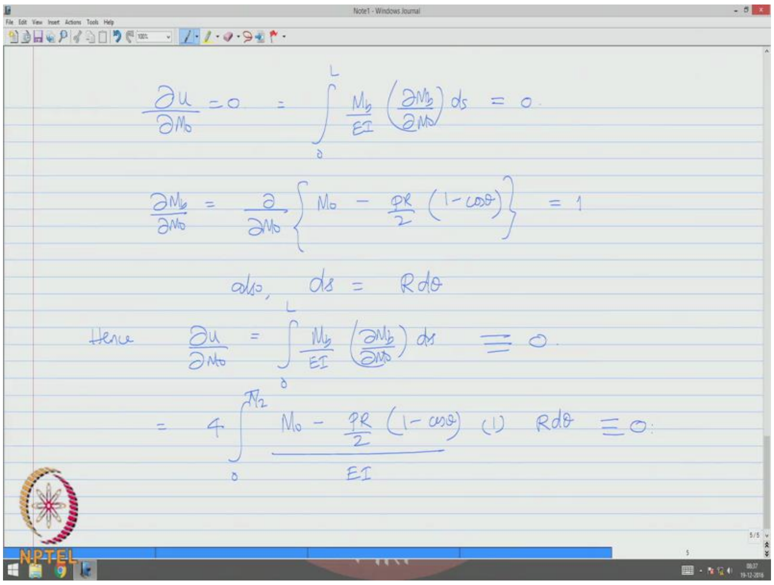


Having said this, let us consider a thin circular ring with a circular cross section; this is subjected to pull P. Let us try to draw the free body diagram acting on the section A B, let us say this is subjected to M o; there is a force as we just now saw P by 2 acting here, to maintain the equilibrium P by 2 is acting here.

Let us take a strip whose length is d s, at radius R, at an angle strip theta and included angle is d theta. We know that this distance will be R cos theta and we also do agree that this distance is of course, R. The figure shows free body diagram of the ring segment under the pull P at section A B; A B refers to this one; this section A B. According to Castigliano's theorem, the equation with respect to this is 0; where U is the strain energy.

The bending moment M b; at the consider section will be now equal to M 0 minus P by 2; R minus R cos theta. Actually I am looking for this distance and this going to be clockwise in nature caused by P by 0 and M 0 is anticlockwise in nature therefore, the resultant moment M b will be given by equation 2. One can always say this further simplified as minus P R by 2; 1 minus cos theta.

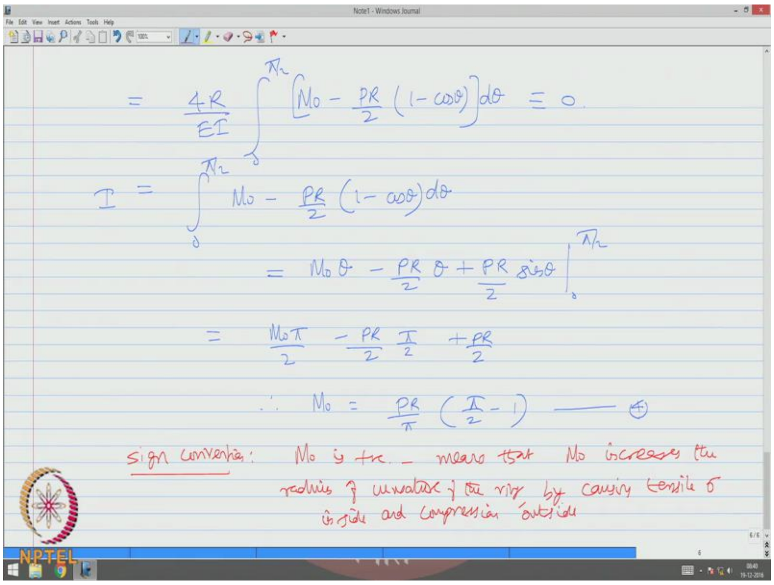
(Refer Slide Time: 15:16)



Now, let us say partially differentiate this with respect to the moment and make it to 0, that is the application of Castigliano's theorem. So, now this is for the length L; M b by E I; dou M b; dou naught, the differential length of the strip d s and said it to 0. Let us say dou M b; by dou M naught is actually dou by dou M naught of M b is given by M naught minus P R by 2; 1 minus cos theta which is very easy to understand is going to be unity and also the element length d s can be also expressed as R d theta, you can see here the element length d s can be expressed as R d theta. Hence dou u by dou M naught is integral 0 to l M b by E I; dou M b by dou M naught d s and that should be said to 0.

There are four segments, so let us say 4; varying from 0 to pi by 2, there is a validity of theta; M b is actually M naught minus P R by 2; 1 minus cos theta, let us say divided by E I because I want E I here and dou M b; dou naught is unity and d s is R d theta said this to 0.

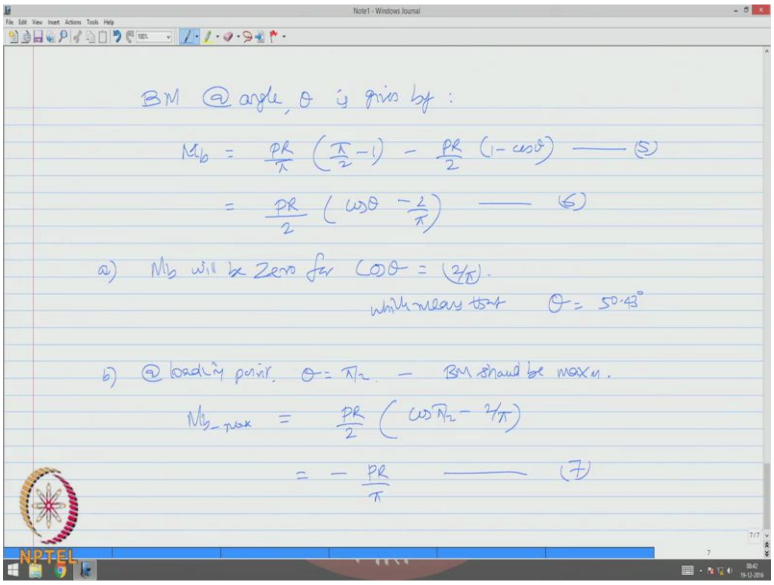
(Refer Slide Time: 17:27)



Simplifying, it is going to be 4 R by E I; 0 to pi by 2; M 0 minus P R by 2; 1 minus cos theta d and this should be said to 0.

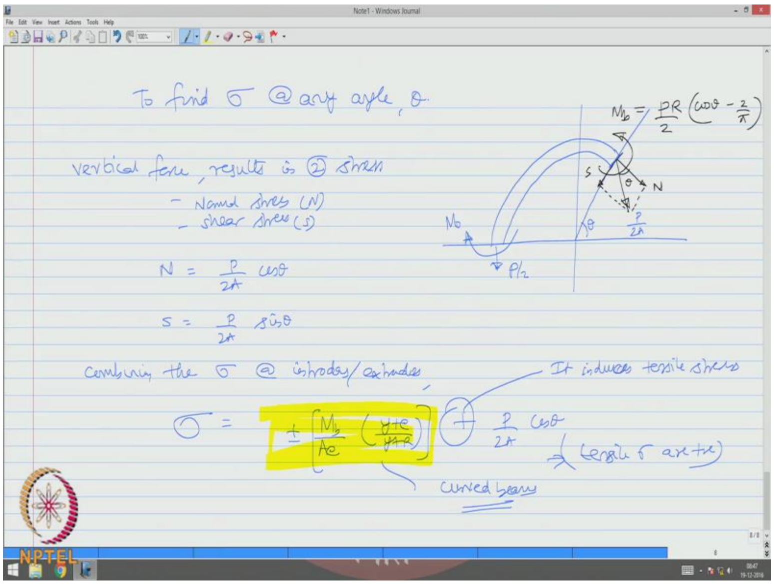
Further integration of 0 to pi by 2; M naught minus P R by 2; 1 minus cos theta, d theta will be equal to M naught theta minus P R by 2 theta plus P R by 2 sin theta limits 0 to pi by 2 which will give me M 0 pi by 2 minus P R by 2 pi by 2 plus P R by 2, which gives me M 0 is P R by pi by 2 minus 1, I call this equation number 4. It is very important to see the sign convention here, the sign convention in above statement is M 0 is considered to be positive which means that M 0 increases the radius of curvature of the ring by causing tensile stresses inside and compressions outside, that is very important.

(Refer Slide Time: 19:51)



Now we would like to know bending moment at any angle theta, which is given by M b is equal to P R by pi; pi by 2 minus 1, minus P R by 2; 1 minus cos theta, R is equal to P R by 2 cos theta minus 2 by pi. Now it is interesting to note that M b will be 0 for cos theta is equal to 2 by pi which means that theta is about 50.43 degrees. Now at the loading point, we know the theta is pi by 2; here bending moment should be the maximum. So, I should say M b maximum is P R by 2 cos pi by 2 minus 2 pi; which will be equal to minus P R by pi - equation number 7.

(Refer Slide Time: 21:38)

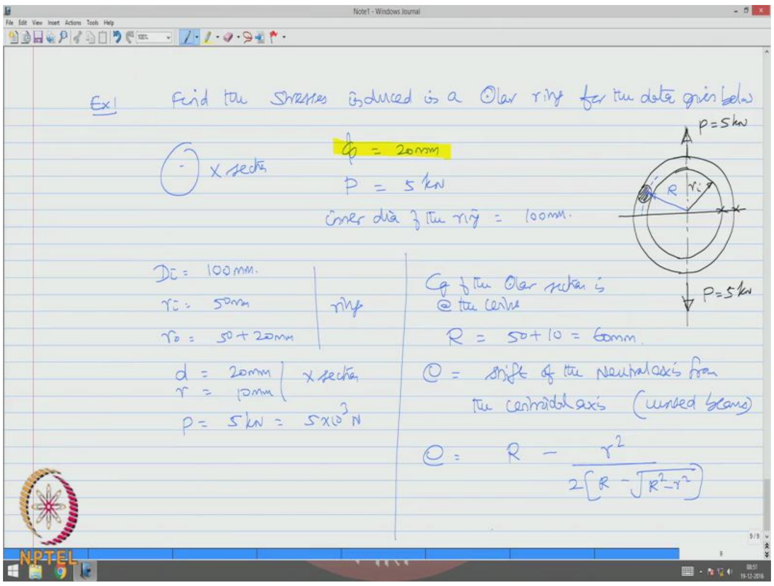


Now, if we really want to find the stress at any angle theta; then let us mark this figure. So, we understand here is going to be P by 2 for equilibrium and we also know this is the sign convention what we are using for M 0. Let us consider at any angle theta, there will be stresses as shown here, one will be the normal stress to a surface, one will be the tangent stress to a surface and these two will set in a resultant.

As we know very well now this angle, so this angle this theta it will be now subjected to M b and M b is actually equal to P R by 2 cos theta minus 2 by pi. The vertical force results in 2 stresses, which is normal stress and the shear stress. Where the normal stress is given by P by 2 A cos theta because this going to be P by 2 A and the shear will be P by 2 A sin theta.

We also know that combining the stresses at intrados and extrados, we now say stress is simply given by M b by A e of y plus e by y plus R, plus P by 2 A cos theta, this is positive because it induces tensile stresses. So, in this equation we know tensile stresses or positive that is very important and friends you should recollect that this equation, what we written here is actually equation derived for curved beams. So, we will do a numerical to understand how this can be applied directly for a given problem.

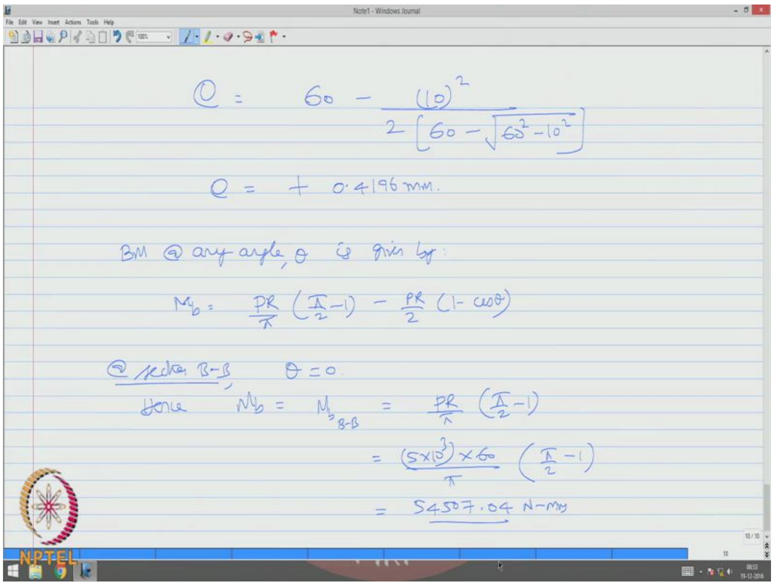
(Refer Slide Time: 25:32)



Let us say, we will have a numerical example. Find the stresses induced in a circular ring for the data given below. It is going to circular cross section, the diameter is 20 subjected to load P which is 5 kilometer and inner dia of the ring is 100 mm. So let us draw a ring, it has a circular cross section; the ring is subjected to your vertical load or a pull which is 5 kilometer. Let us take a cross section and try to find the stresses at this point, we know that this is r i and D i is 100 millimeter. Therefore, r i is 50 millimeter, r outer is going to be 50 plus 20 millimeter because that is the diameter of the cross section. Therefore, the diameter of the ring small d is 20 millimeter and therefore, the radius is 10 millimeter; this is for the cross section; this is for the ring.

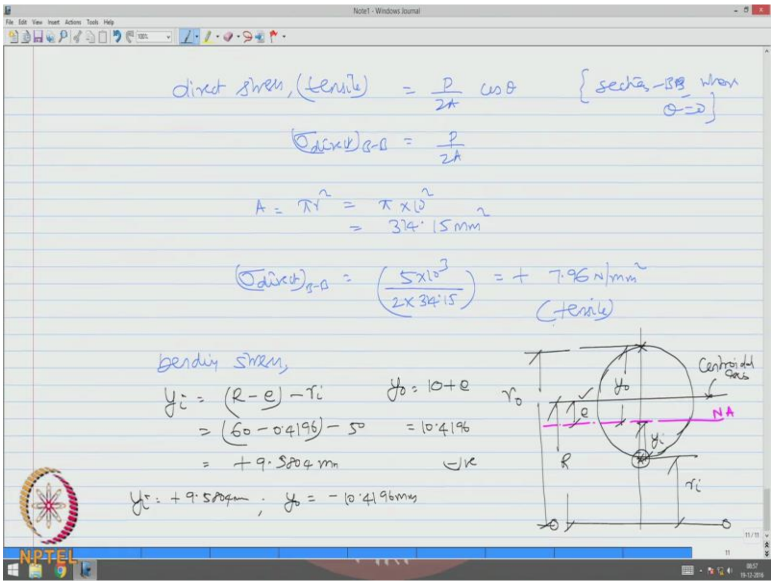
Subjected to a load which is 5 kilo Newton, so many Newton's, we all know that the c g of the circular section is at the centre therefore, R will be now 50 plus 10; which is 60 millimeter. So, I can mark R which is 60 millimeter. We already know from the equation, we calculate e; e is the (Refer Time: 28:45) of the neutral axis from the centroidal axis, which we have explained very clearly in the lectures on curved beams and e is given by R minus r square, twice of R minus root of R square minus r square. Please refer this equation for the curved beam discussion with the circular cross section.

(Refer Slide Time: 29:32)



Let us substitute in this case which is 60 minus R square, twice 60 minus square root of 60 square minus 10 square, which gives me e as positive 0.419 millimeters. So, we also know bending moment at any angle theta is given by M b is equal to P R by pi; pi by 2 minus 1 minus P R by 2 1 minus cos theta. So, let us say at section B B; we understand that the theta is 0, hence M b is what we say M b at B B is P R by pi; pi by 2 minus 1 because the second term goes away, let us substitute P; 5000 Newton, R 60 pi by 2 minus 1 which gives me the moment as 54507.04 Newton millimeter.

(Refer Slide Time: 31:26)



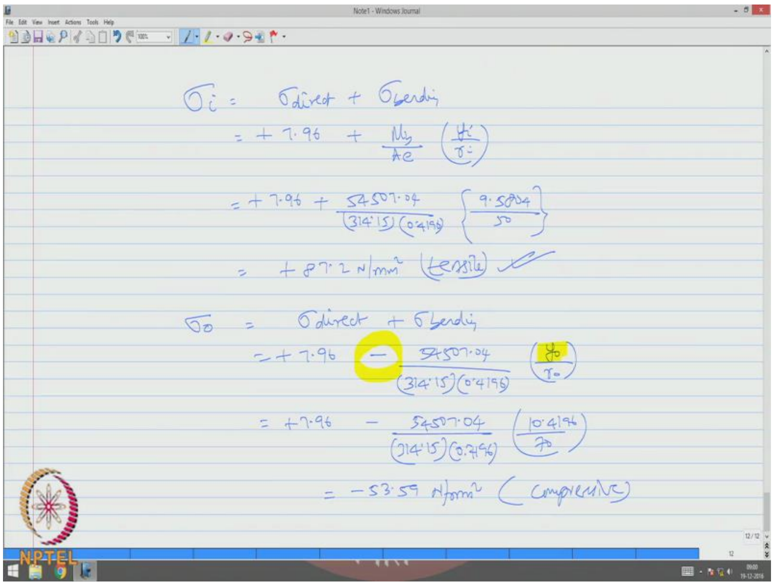
Since the cross section is subjected to a direct force, there will be also a direct stress which will be tensile; which will be given by P by 2 A cos theta. So, now we are looking for section B B where theta is 0, hence the direct stress at section B B will be simply P by 2 A; area in our case is pi r square which is pi 10 square that is the area of my cross section which is 314.15 mm square, therefore the direct stress at section B B will be 5; 10 power 3 by 2; 314.15, which gives me plus 7.96 Newton per mm square plus indicate it is tensile.

So that is the first part of it, I also want to find the bending stress. To find the bending stress, we need to do a small exercise; let us say this is my circle, let us say this is my o o plane and we know that this is my centroidal axis, this is my neutral axis and we do agree that there is a shift of this which is e, which we know, which we computed and we know that this distance is r i and this distance is r o.

I want to now know the distance of the inner most fiber and extern most fiber, so I call this as y I; inner and this as y outer. So obviously y inner will be; we know that this value is R, so R minus e minus r I, let us substitute all of them; 60 minus 0.4196 minus 50, so this is plus 9.5804 millimeter.

Similarly, y outer will be in similar terms I want to find this distance is going to be; one can say simply the diameter plus e which is 10.4196, this is negative because this heading towards the compressive stress therefore, y i is plus 9.5804 and y o is minus 10.4196 millimeters, once I have the distance of extreme fiber from the neutral axis, I can now compute the bending stress using this equation which we already derive in the curved beams section.

(Refer Slide Time: 35:38)

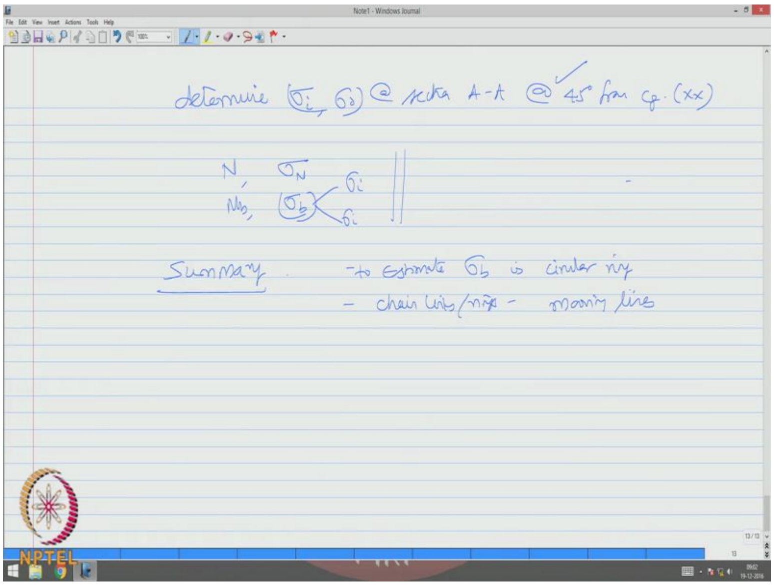


So, stress at intrados is stress direct plus stress bending, stress direct is tensile 7.96 plus stress bending at M b be is M b; A e; y i by r i which will be plus 7.96 plus 54507.04 divided by 314.15 and e is 0.4196 and this ratio is y i distance of inner fiber 04, divided by r i is 50. So, I get this value as plus 87.2 Newton per mm square plus indicate it is tensile as expected that the moment is increase in the radius of curvature, inner portion is in tensile which we are already getting verified on the results.

Similarly, stress at the outrados is again sigma direct plus let us say sigma bending, sigma direct is again tensile all the time. Whereas, this will be minus 54507.04 divided by 314.15 into 0.4196 this is going to be y outer by r outer. Now y outer is negative in our case therefore, plus 7.96 minus 54507.04 by 314.15 into 0.4196 multiplied by 10.4196 by 70. So, the minus sign of the y 0 is actually taken care of here, which gives me minus 53.59 Newton per mm square and that is going to be compressive.

I will give a small exercise, go back to this figure; we have found out the stresses at the section B B, we can always find a similar section which is at some angle let us say A A; at 45 degree.

(Refer Slide Time: 38:38)



So, the exercise determines the stress at intrados and outrados at section A-A, which is at 45 degrees from the centroidal axis xx. So, you know the equations for M b, you know the equations for n, one can obtain the normal stress and find sigma normal and one can find the bending stress, bending moment and then find bending stress; both at intrados and extrados bending. Then combine them to find a stresses as we can check at 45 degrees, I think you can be able to straight away apply the equation derived and substitute them properly to get the answers.

So friends in this lecture, we discussed the method to estimate bending stress in a circular ring, rings and chain links are common in mooring lines. So in mooring line analysis under special loads, you need to understand how to estimate stresses using the curved beam derivations what we discussed in the soft lithography lectures. One example problem explains you how to estimate the stresses in the intrados and extrados of any section drawn across the circular ring to estimate the stresses at this point both direct and bending stresses at intrados and extrados. We have also explained how to use the proper sign conventions for M 0 and the stresses as you apply in the equations.

Thank you very much.