Offshore structures under special loads including Fire resistance

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Module – 2
Advanced Structural Analyses
Lecture – 33
Curved Beam-IV

Friends, we will continue to discussion on curved beams. This is lecture 33, Curved Beams - IV, we will continue discuss the derivation of the geometric parameter small m and the extensive value e of the location of neutral axis from the centroidal axis for a curved beam with large initial curvature.

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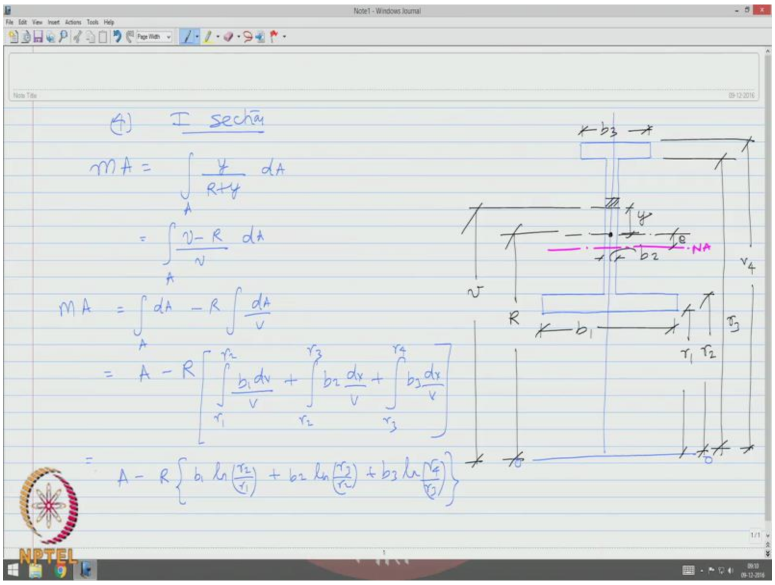

Let us take an I section; let us mark the plane o o; which contains a centre of curvature, let the dimensions be b 1, b 2 and b 3. We do not mark the thickness but we mark different radius from the centre of curvature or from the plane of centre of curvature as r 1, r 2, r 3 and of course this is r 4.

We have the c g located here let us say that is located the distance R from here, we can take a general strip which is y from the centroidal axis. We are about to locate the neutral axis at (Refer Time: 02:51) e from the centroidal axis, we will also define this with a parameter v from here. Now we know that m A is integral y plus R by y, d A for the whole area; which can be y can be expressed as V minus R and R plus y is simply V; integral d A which can be d A minus R; d A by V which can be A minus R times of integral limits R 1 to R 2; the strict area is actually b 1 into d v by v plus for the strip area of width b 2, the limits are going to be R 2 to R 3, b 2 d V by V plus again the last strip which varies from R 3 to R 4; b 3 d V by V. Which we can say this A minus R, b 1 log R 2 by the natural algorithm plus b 2, R 3 by r 2 plus b 3; log R 4 by r 3, so that is going to be m A.

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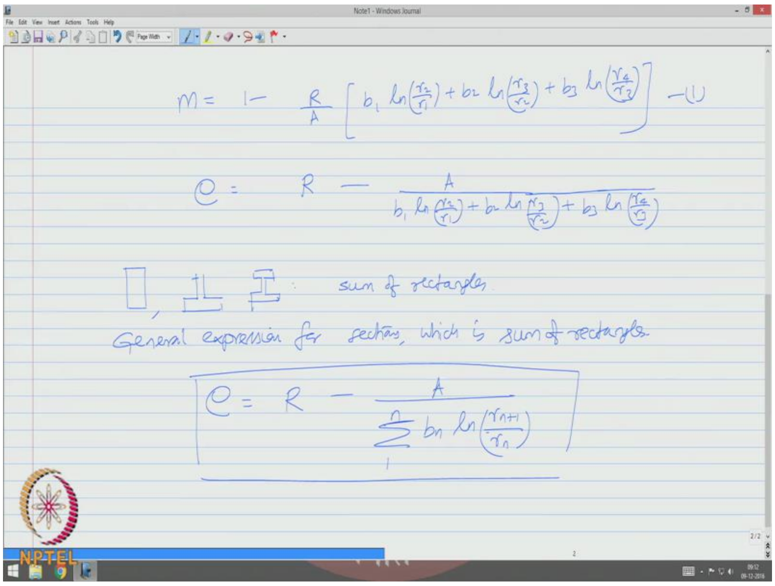

Therefore M is 1 minus R by A of b 1 plus b 2 plus b 3 equation 1; e is then given by R minus A by b 1, r 2 by r 1 plus b 2, r 3 by r 2 plus b 3; log r 4 by r 3. So, now we have got different sections, that is pure rectangular, a T section and I section which are actually sum of rectangles. So, one can now have a very general expression for sections which is sum of rectangles, e in that case will be R minus A by summation of 1 to n; b n natural algorithm r n plus 1 by r n. So that is a general expression we have for e, once I know e can always find M.

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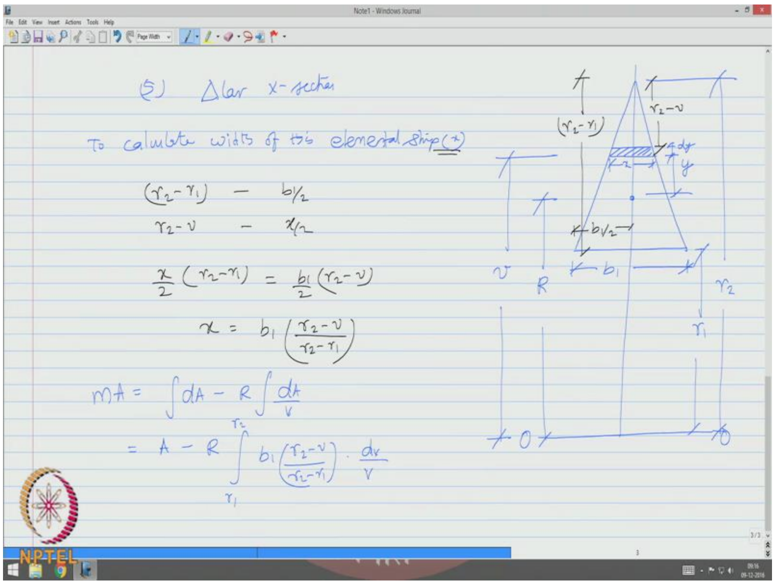

Let us go for your triangle section say this is b 1, this is my o o plane and this is my c g and therefore this distance is R and this is r 1 and this is r 2; as usual we take a strip and let that strip be at a distance V; the radius V from the plane o o, let us say this distance is y and let us say the width of the strip d x. So, it is very interesting now I want to estimate width of this elemental strip that is x. One can use a similar triangle principal you know this value, so actually equal to r 2 minus r 1 and this value. So, actually equal b 1 by 2, so far r 2 minus r 1 we have a variation of b 1 by 2, therefore this variation which is r 2 minus V; will be x by 2.

So, we cross multiplication x by 2; r 2 minus r 1 is b 1 by 2; r 2 minus V, this says x will be b 1, r 2 minus V by r 2 minus r 1, so that is the width of the elemental strip x. Once I know this m A; integral d A minus R; d A by V which is A minus R integral, the limits are this and the elemental area is the width of the strip into thickness is d y, so d 1; r 2 minus V by r 2 minus r 1 into d V by V.

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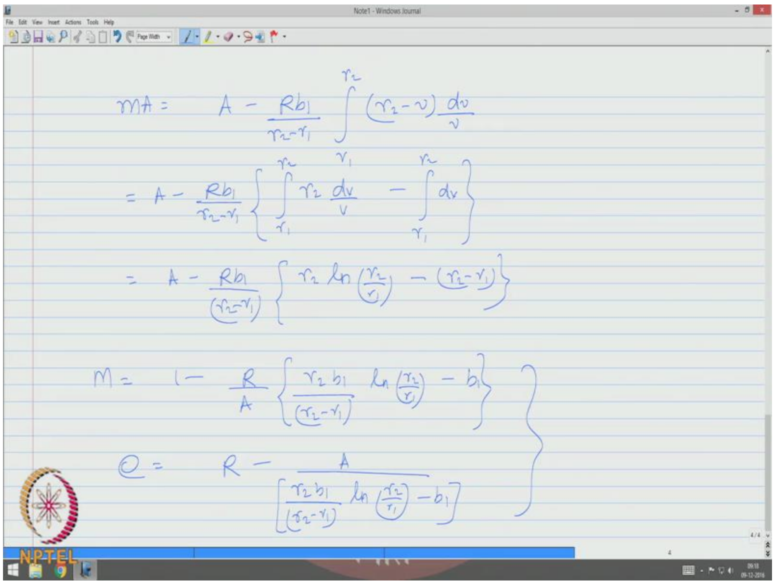

So, m A is A minus R b 1 by r 2 minus r 1; integral limits r 2, r 1, r 2 minus V; d v by v which will be A minus R b 1 by r 2 minus R 1, integral R 2 d v by v minus integral simply d v which will be A minus R b 1 by r 2 minus r 1 of r 2; r 2 by r 1 log minus r 2 by r 1.

Now, M straight away is 1 minus r by A; r 2; b 1 by r 2 minus r 1, r 2 by r 1 minus b 1 and e is R minus A by r 2, b 1 by r 2 minus r 1; r 2 by r 1 minus b 1. So, now, these are the two equations for estimating the location of neutral axis for a triangular section.

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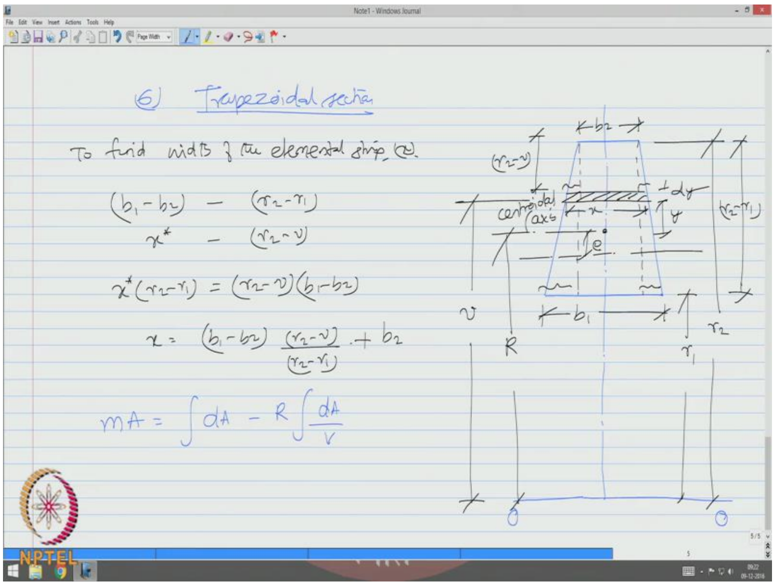

Let us go for trapezoidal section, so look at the o o plane; let us say this is b 1, this is b 2 and this is r 1 and this is r 2. Let us take us strip from the c g which is under radius R which is located at distance y and let the width of the strip be x and let the strip be located at radius V. So, this is my centroidal axis of course, I will look at the neutral axis later here which will be and to distance e from the centroidal axis.

So, now to find width of the elemental strip x, let us say we know; I can divide this as I am showing in the figure. Now this width which is b 1 minus b 2; will have a slope of r 2 minus r 1 because this distance is r 2 minus r 1, therefore the width will be x at distance which is r 2 minus v, so cross multiplying x to r 2, r 1 will be r 2 minus V; b 1 minus b 2, so x is b 1 minus b 2 or 2 minus v r 2 minus r 1. Of course plus b 2 because this only the slope is we are getting only this distance which we can rather call as x star therefore, x will be actually equal to this plus b 2, now once I know this; I can now say my m A is integral d A minus r d A by v which is A minus R integral limits r 2, r 1 which is b 2 plus b 1 minus b 2 by r 2 minus r 1, r 2 minus v of d v by v.

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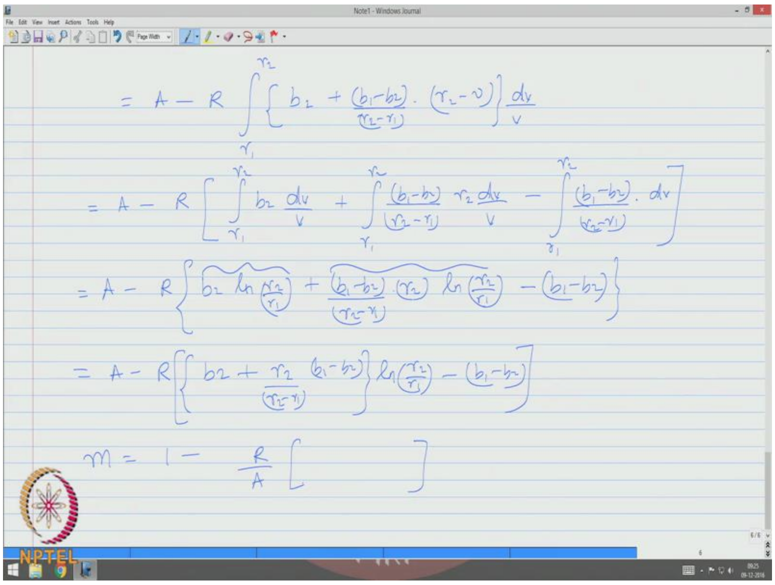

Which can be A minus R; r 1, r 2, b 2; d v by v plus integral limits r 2, r 1, b 1 minus b 2 by r 2 minus r 1; r 2 d v by v minus r 1, r 2; b 1 minus b 2 by r 2 minus r 1 of d v which now is A minus R b 2; log r 2 by r 1 plus b 1 minus b 2 by r 2 minus r 1; r 2; log r 2 by r 1 minus b 1 minus b 2 because when you get the limits and substitute they will be cancelled out, which will be A minus R; b 2 because you know this term and this term can be combined b 2 plus r 2 of b 1 minus b 2 by r 2 minus r 1 of ln r 2 by r 1 minus b 1 minus b 2.

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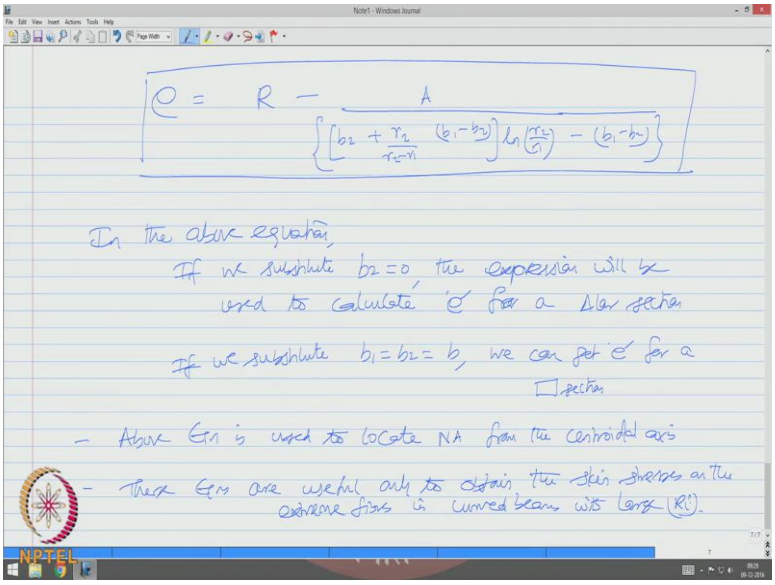

So, I can say m is actually equal to 1 minus R by A of the same parenthesis term and e is r minus A by b 2 plus r 2 by r 2 minus r 1 of b 1 minus b 2 of r 2 by r 1 minus b 1 minus b 2. So, that is going to be my location of neutral axis towards a centre of curvature measured from this centroidal axis. Interestingly, in the above equation if we substitute b 2 equal 0 please see this figure, if b 2 is 0; it becomes a triangle. The expression will be used to calculate e for a triangular section, if we substitute b 1 equals b 2 equals imply b, we can get e for a rectangular section.

So, it is very interesting above equation is used to locate the neutral axis from the centroidal axis. These equations are useful only to obtain the skin stresses on the extreme fibres in curved beams with large initial curvature.

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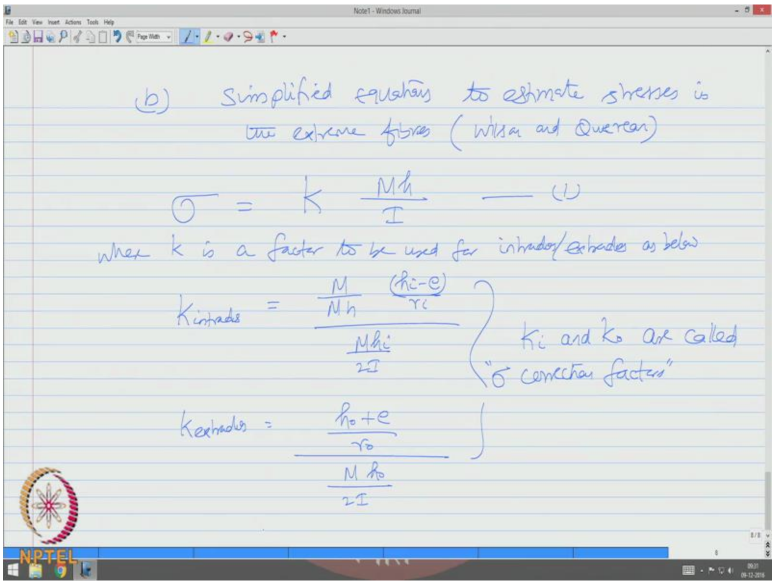

However there are some simplified equations available to estimate the stresses in the extreme fibres. This is given by Wilson and Querean they said stress is given by a factor k M h by I, where k is a factor to be used for intrados and extrados as below for example, k intrados is M by M h; h i minus e by r i divided by M h i by 2 I; k extrados is given by h extrados plus e, you know the positive sin is being is used accordingly by r 0 divided by m h naught by 2 I. So, this k i and k 0 are called correction factors, in fact one can say stress correction factors.

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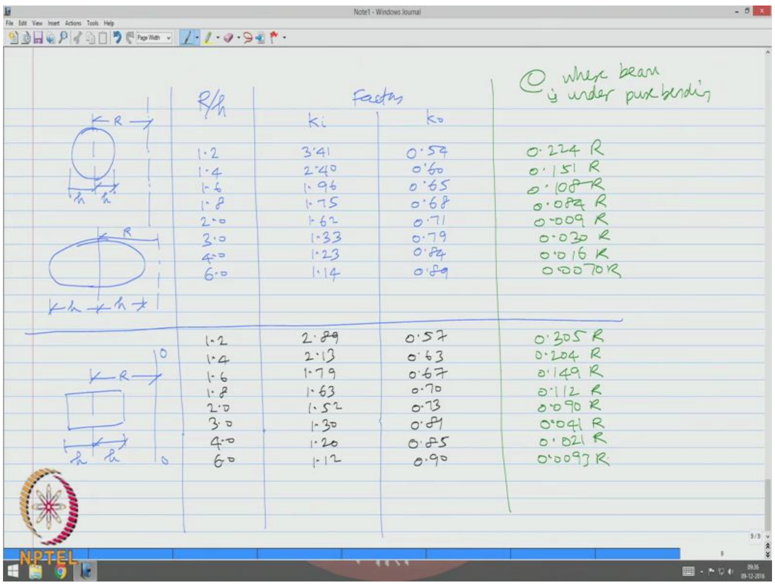

They are given by Wilson for different values of R by h, the factors are given both k i and k extrados are given. For both circular where this is my plane of curvature and this is my r and from the c g they are equidistant, if it is elliptical still one can use a same equation provided you know the dimensions and the depth of the section. For different values 1.2, 1.4, 1.6, 1.8, 2.0, 3.0, 4.0, 6.0; the factors k i and k naught are given 3.41, 2.40, 1.96, 1.75, 1.62, 1.33, 1.23, 1.14 and these values 0.54, 0.6, 0.65, 6 8, 7 1, 7 9, 8 4, 8 9.

For a rectangular section this becomes my plane of radius of curvature and this becomes my R and this becomes my depth of the section h. For 1.2, 1.4 for all ratios of r by h the values are available ask given in the table. Further they have also given the tables extending it for e where the beam is under pure bending. So, this is 0.224 R, 0.151, 0.108, 0.084, 009, 030, 016, 0070, this is 305, 204, 149, 112, 090, 041, 021, 0093.

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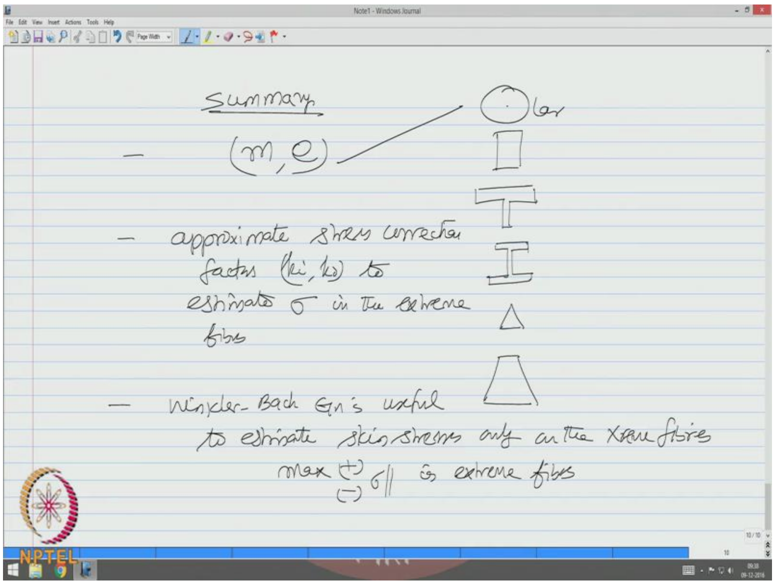

So, friends we have seen how to estimate m and e for different shape of cross sections circular, rectangular, series of rectangles, triangular and trapezoidal. We also found approximate stress correction factors k i and k o; to estimate stresses in the extreme fibres. If we recollect that the Winkler Bach equation is useful to estimate skin stresses only on the extreme fibres because the stress variation along the section is hyperbolic therefore, one cannot do that. So, one can find the maximum compression and tensile stresses in the extreme fibres using the Winkler Bach equation.

So, in the next lecture will take up some numerical examples and solve them and find out stresses at various cross sectional shapes for a given movement, for a curved beam with large initial curvature and so on.

Thank you.