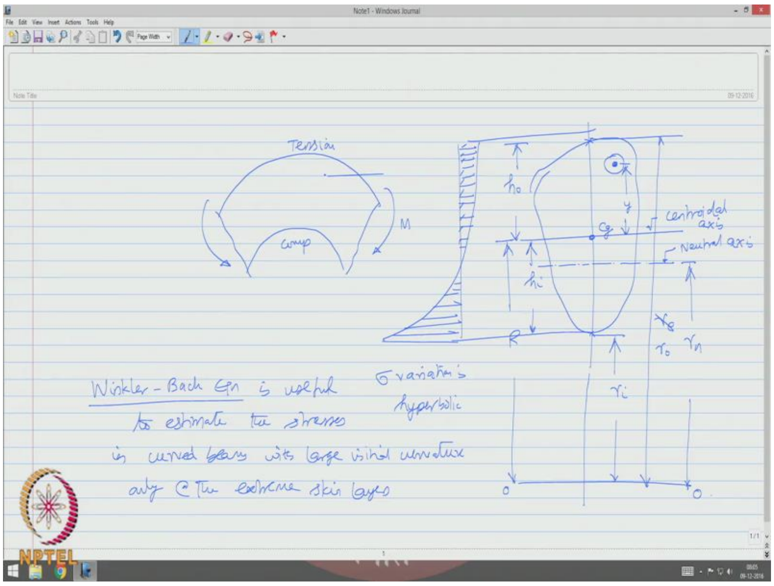
Offshore structures under special loads including Fire resistance
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module – 02
Advanced Structural Analyses
Lecture – 32
Curved Beams – III

Friends, welcome to the 32 lecture. We are continuing with the discussions on curved beams, we have derived equations in the last 2 lectures and curved beams with small initial curvature and large initial curvature. We have derived the expression for Winkler Bach equation, we will continue to the discussions to find the stresses in extreme fibres of curved beams with large initial curvature in this lecture.

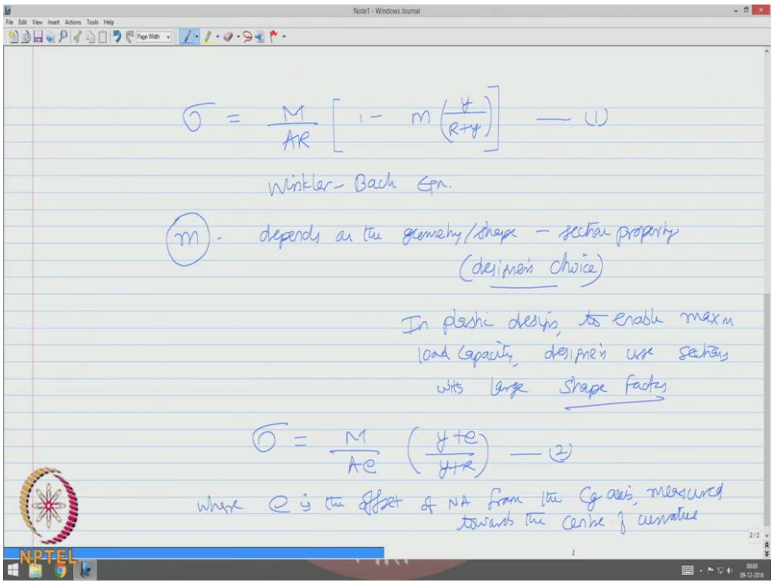
(Refer Slide Time: 00:54)



To recollect back, we have a cross section whose c g is marked; this becomes my axis at which the radius of curvatures located. So the centroidal axis is located at distance R, whereas the beam is subjected to a bending moment of this nature which causes tension in the extrados and compression in the intrados on extreme fibres, the applied moment being M. Therefore, this makes the neutral axis to shift to the bottom that is towards the centre of curvature.

So this is my neutral axis, this is my centroidal axis with curved beams of large initial curvature. We have already said that the centroidal axis, the neutral axis will not coincide and the stress variation along the section is hyperbolic, since they are not equal the neutral axis is not coinciding with the centroidal axis. So, the stress variation is hyperbolic therefore, the Winkler Bach equation is useful to estimate the stresses in curved beams with large initial curvature only at the extreme skin layers. So, please note this equation can give me the stresses only at the extreme points. So, we call this as r intrados and we can call this as r extrados; we can simply say this as r neutral axis, this can also be read as r outer, from the c g the extreme fibre distances are h o and h i, we can take any point; may be here which is at the distance of y from the centroidal axis.

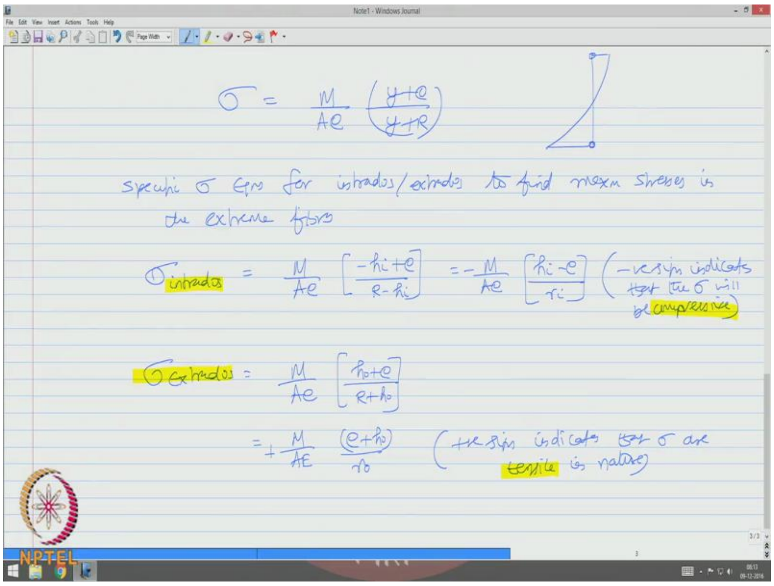
(Refer Slide Time: 05:08)



For this classical terminology we said stress is given by M by AR; 1 minus small m; y by R plus y which is called the famous Winkler Bach equation which we derived in the last lecture. Now m depends on the geometry, shape of the geometrical cross section which is a section property. In fact, I should say it is the property of designers choice, I can quote a parallel example of this. For example, in plastic design to enable maximum load capacity, designers use sections with large shape factors. Similar to this, m is a cross sectional property which is a designers choice to limit the stresses in extreme fibres, in curved beams with large initial curvature, one can choose the cross section accordingly or the value of m to suit the appropriate cross section

There is alternative form of this equation 1, which we also derived in the last lecture. Stress can also be said as m by Ae of y plus e by y plus r, where e is the offset of neutral axis from the centroidal axis measure towards the centre of curvature; that is, this is my e.

(Refer Slide Time: 08:24)

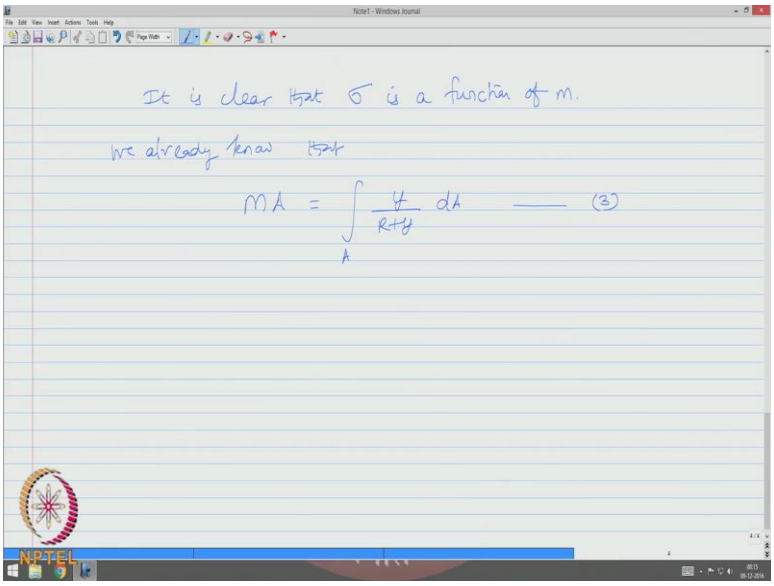


Now, considering equation 2; one can always derive specific stress equations for intrados and extrados to find maximum stresses in the extreme fibres. So look at the cross section here again, the distance of the extreme fibre 1 is what we call as h i, which is intrados; the other one is what we call as h 0 or h o, which is the extrados; basically this distance is essentially this.

We also know this is compression and this is tension, for the given assumption that the neutral axis shifts towards the centre of curvature, tensile stresses or positive. So, with that assumption I want to find intrados stress in the maximum fibre will be given by M by A E, so y in this case going to be minus h i minus h i. So, again rewrite this equation as M by Ae minus of h i minus e by r i. If we look at the figure r minus h i will be actually equal to r i, here negative sin indicates that the stresses will be compressive. Similarly stress for extrados is M by Ae, y in this case going to be h o plus e, R plus h o; which I can say M by Ae; e plus h o by R plus h o can be said as r 0.

So, the positive sin indicates that stresses are tensile in nature, so extrados as got tensile stresses and intrados has got compressive stresses which are evident from this equation of finding out stresses on extreme fibres. Please note this equation is useful only to estimate the stresses on extreme fibres because the stress along the cross section is varying hyperbolically. So, this equation does not give me the stress variation along the depth this can give me the stress only in the extreme fibres of the given cross section.

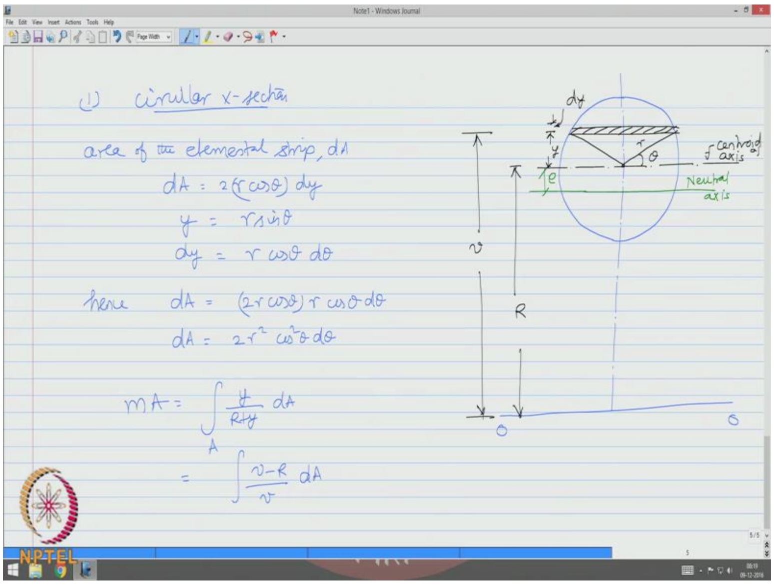
(Refer Slide Time: 12:47)



So now having said this, it is clear that stress is a function of m. So, one need to estimate is value m for various cross sections which are used commonly in curved beams. If we look back the derivation, we already know that M A is actually integral of y by R plus y dA for the entire range.

So using this equation 3, we are now going to derive the cross sectional property M for various geometrical shapes of the cross section.

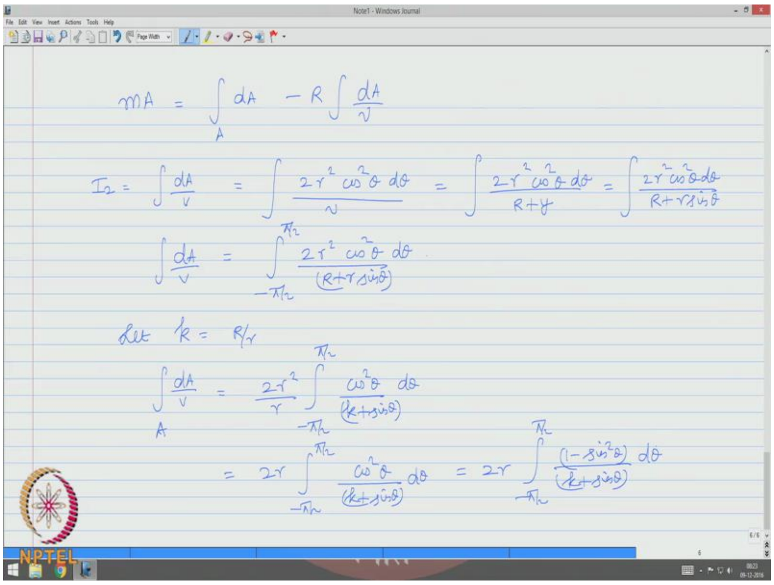
(Refer Slide Time: 13:55)



Let us take a circular cross section, let us say this is my axis of reference where I am going to mark my centre of curvature. Let us say this is my centroidal axis, so that is my centroid and this is going to be my neutral axis. We know that this shift is e and we also know that this distance which is R, let us take a fibre which is at an angle theta; the radius of the circular cross section is small r and let that fibre d at a distance y from the centroidal axis and let this be; d y, let us introduce one more variable which is small v from here.

So, area of the elemental strip dA will be equal to r cos theta twice of that multiplied by d y. We also know y is r sin theta, therefore d y is r cos theta; d theta hence dA will be now 2 r cos theta, r cos theta; d theta which makes it as 2 r square cos square theta; d theta. We now know m A is integral y by R plus y dA for the whole area, I can write this as y can be said as, v minus R and R plus y is v; d A, which is now integral dA minus R, integral dA by v.

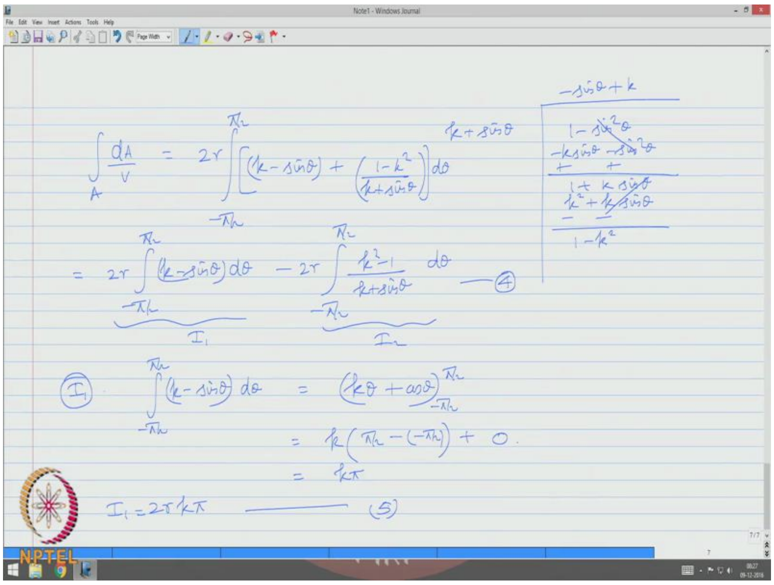
(Refer Slide Time: 18:09)



Let us pick up this integral dA by v, let us do this; dA already we said it is 2 r square cos square theta d theta by v which is 2 r square cos square d theta, v is R plus y which is 2 r square cos per theta, d theta by R plus r sin theta.

Integral dA by v will be now integrated from the limits minus pi by 2; 2 plus pi by 2, 2 r square cos square theta, d theta by R plus r sin theta. Let k be expressed by a ratio R by r, so integral dA by v dA; sorry will be actually equal to 2 r square by r; minus pi by 2 plus pi by 2, cos square theta; d theta by k plus sin theta, which is now 2 r limits cos square theta by k plus and theta d theta which can be expressed as 2 r, 1 minus sin square theta by k plus sin theta; d theta.

(Refer Slide Time: 21:00)

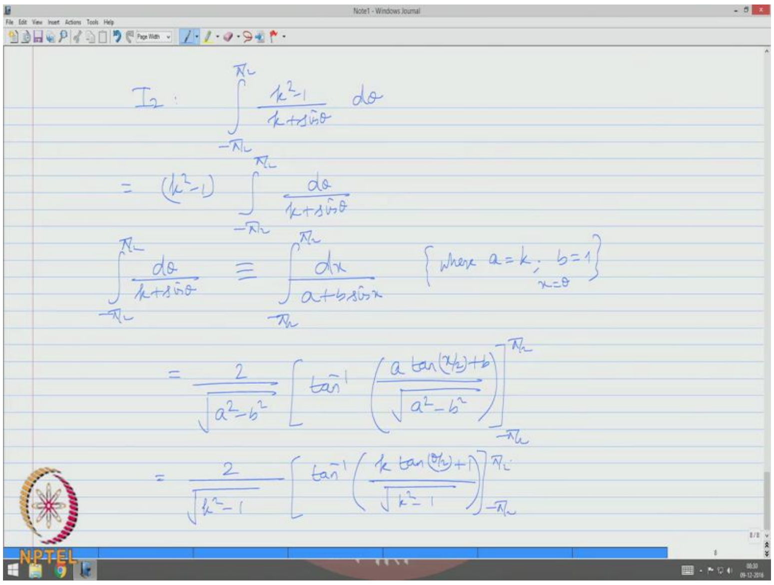


Let us divide this: 1 minus sin square theta, k plus sin theta let us say minus sin theta which makes it as minus k sin theta minus sin square theta which says 1 plus k sin theta.

So, again k so I get k plus k sin theta which gives me 1 minus k, this is k square 1 minus k square. So, therefore I can now express the integral dA by v as 2 r integral minus pi by 2 plus pi by 2, k minus sin theta plus 1 minus k square by k plus sin theta, the whole of d theta, which can be now said as 2 r minus pi by 2 plus pi by 2; k minus sin theta, d theta; let us say minus 2 r minus pi by 2 plus pi by 2, k square minus 1 by k plus sin theta, d theta. Let us take this integral I 1 and this integral I 2 - let us take I 1 first, integral k minus sin theta, d theta minus pi by 2 plus pi by 2, which will be k theta plus cos theta minus pi by 2 plus pi by 2.

So, k pi by 2 minus of minus of pi by 2 plus 0 because cos pi by 2 as my value; so this gives me k; pi. Therefore, 2 r k pi is the value of a first term, I will call this as equation number 4; let us say this is 5.

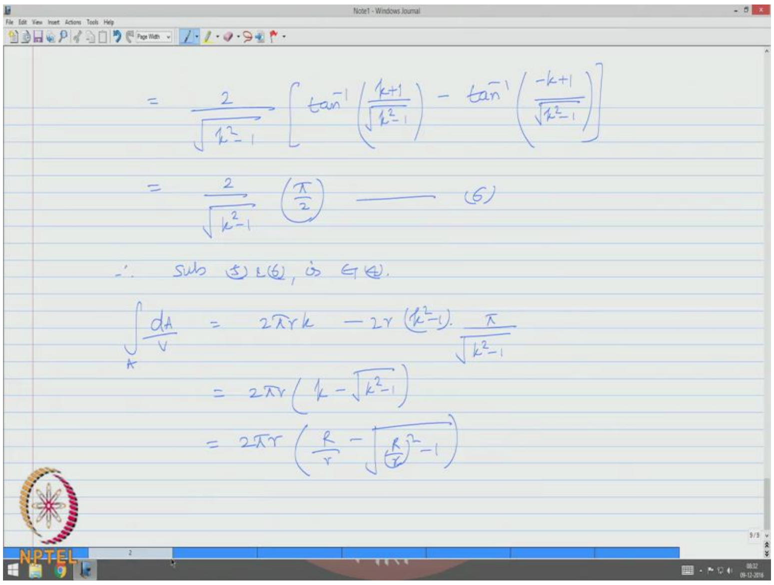
(Refer Slide Time: 24:35)



Let us pick up I 2, I 2 the second integral is actually integration minus pi by 2 plus pi by 2; k square minus 1, k plus sin theta; d theta. In fact, integral actually is minus pi by 2 plus pi by 2, k square minus 1 d theta by k plus sin theta. I can express this integral as similar to d x by a plus b sin x minus pi by 2 plus pi by 2, where a is k and b is unity.

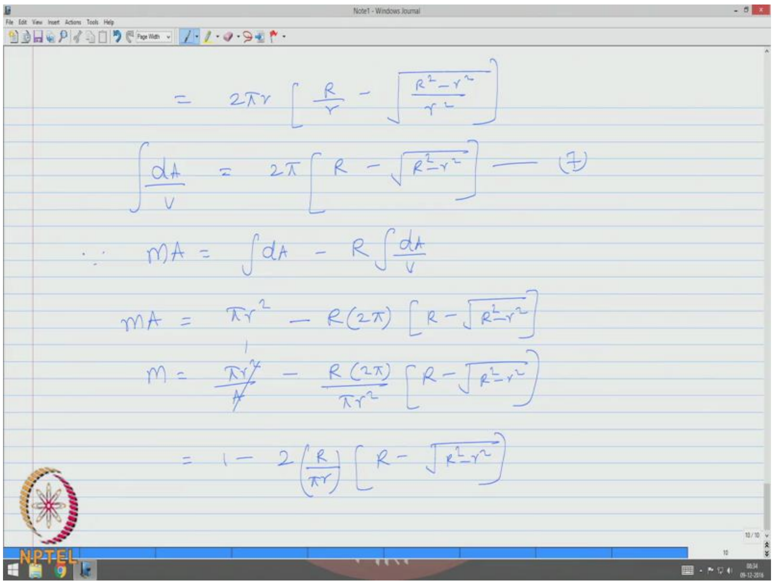
Now this has got to standard relationship, which is given by twice of root a square minus b square, tan inverse; a tan x by 2 plus b by root of a square minus b square limits minus to plus pi by 2 and of course, a x is theta. So, 2 by root of k square minus 1 tan inverse of; k tan theta by 2 plus 1 by root of k square minus 1, applicable to limits minus pi by 2 to plus pi by 2.

(Refer Slide Time: 27:07)



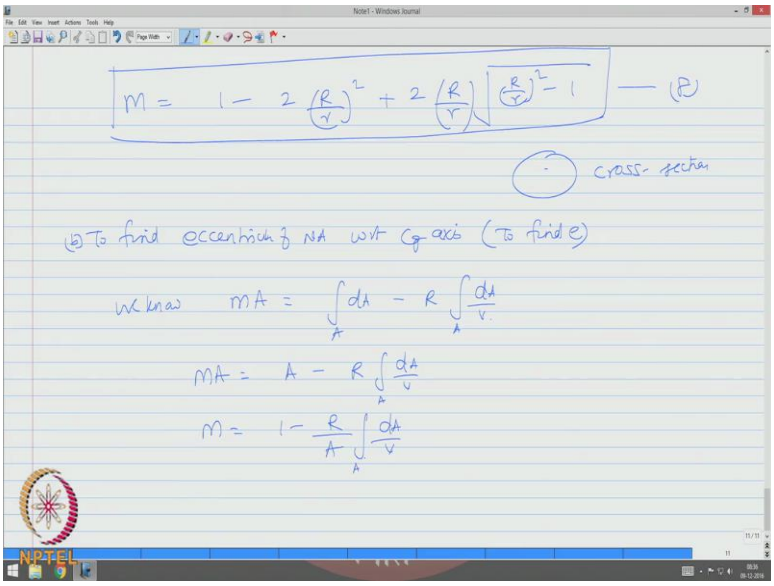
Which can be then simplified 2 by root of k square minus 1; tan inverse of k plus 1 by root of k square minus 1 minus tan inverse of minus k plus 1 by root of k square minus 1 add to substitute in the limits which will then simplify to 2 by root of k square minus 1 of pi 2. Therefore, which I call is equation number 6; substituting 5 and 6 in equation 4, so integral dA by v; will be 2 pi r k minus 2 r, k square minus 1, pi by root of k square minus 1. Which tells me 2 pi r, k minus root of k square minus 1; we know k is r by r let us substitute that R by r minus root of R by r square minus 1, which can be also said as 2 pi r, R by r; root of R square minus r square by r square.

(Refer Slide Time: 29:00)



So dA by v integral is 2 pi R minus root of R square minus r square, so equation number 7. Therefore mA is integral dA minus; R integral dA by v, so mA is pi r square minus R into 2 pi R minus root of R square minus r square. So, therefore m is pi r square by A, minus R into 2 pi by pi r square of R minus R square minus r square, so this becomes 1. So, this is 1 minus; twice of twice of R by pi r; R minus root of R square minus r square plus r square.

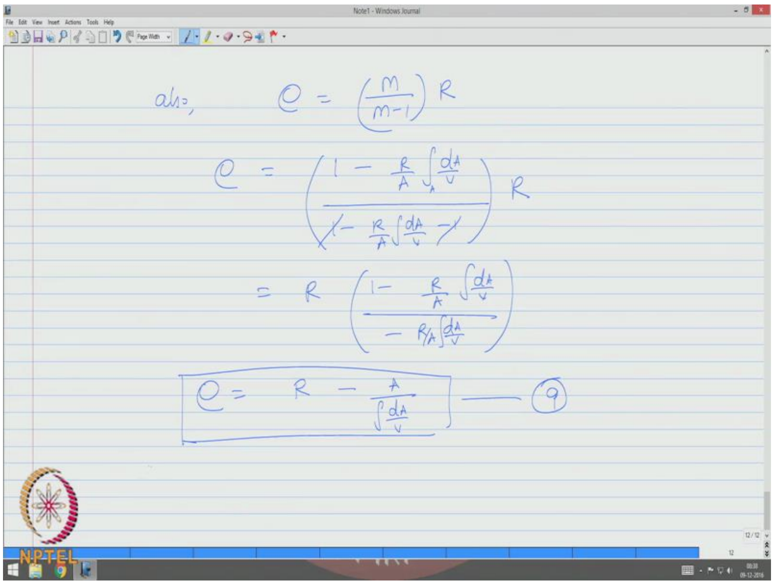
(Refer Slide Time: 30:55)



So, simplifying further m will be 1 minus 2 R by r whole square plus there are three terms here in this equation; first term, second term, and third term to R by r square; so first and second and third term. So, third term will be plus twice of R by r root of R by r the whole square minus 1.

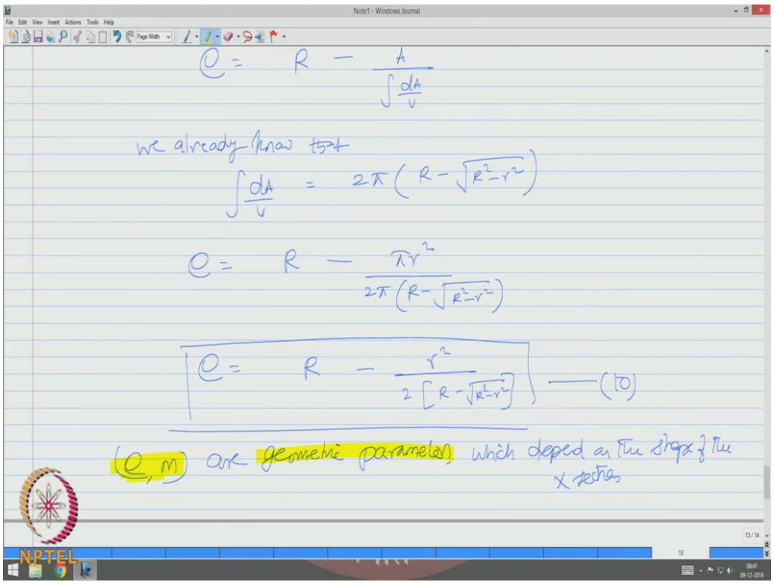
So, that is going to be my cross sectional property parameter m; for a circular cross section; equation number 8. Now I want to find the eccentricity of neutral axis with respect to c g axis that is to find e. We know mA is integral dA minus R integral dA by v that is mA is; A minus r integral dA by v a, so m is 1 minus R by A; dA by v.

(Refer Slide Time: 32:48)



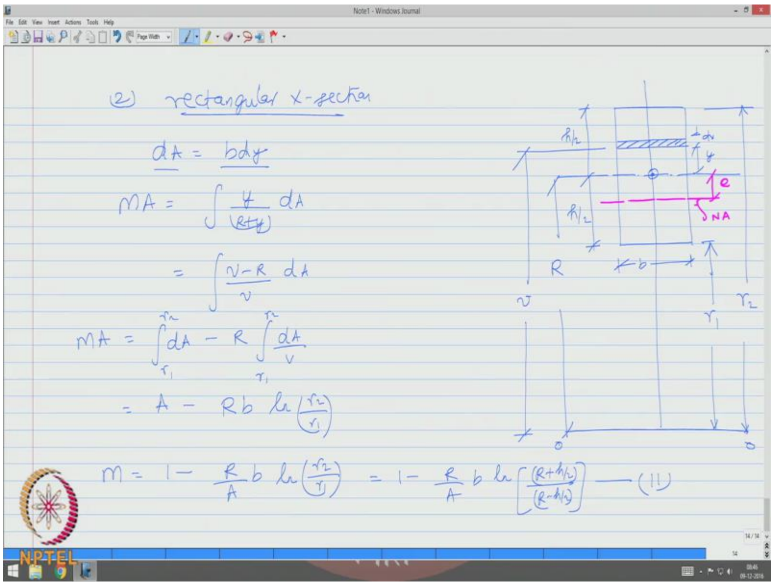
Also e is m by m minus 1 into R, so e is 1 minus R by A integral dA by v divided by 1 minus R by A integral dA by v minus 1 the whole multiply by r. So this two goes away, so R times of 1 minus R by A integral dA by v divided by minus R by A integral dA by v, which I can say as e is R minus A by integral dA by v, so using this relationship which is equation 9; let us find R in our case.

(Refer Slide Time: 34:11)



So, we know e is R minus A by integral dA by v, so we already know that integral dA by v for the circular section is actually 2 pi R minus R square minus r square therefore, e is going to be R minus pi r square 2 pi R minus root of R square minus r square by simplification e will be R minus r square by twice R minus root of R square minus r square, so I get e; as equation number 10. So, both e and m are geometric parameters which depend on shape of the cross section, you can see here the variables are radius of curvature then small r which is the radius of the section and capital R is the radius of curvature. If I know these 2 parameters, I can find e as well as m; that is how we got e and m which are actually geometric parameters for a circular cross section.

(Refer Slide Time: 36:34)

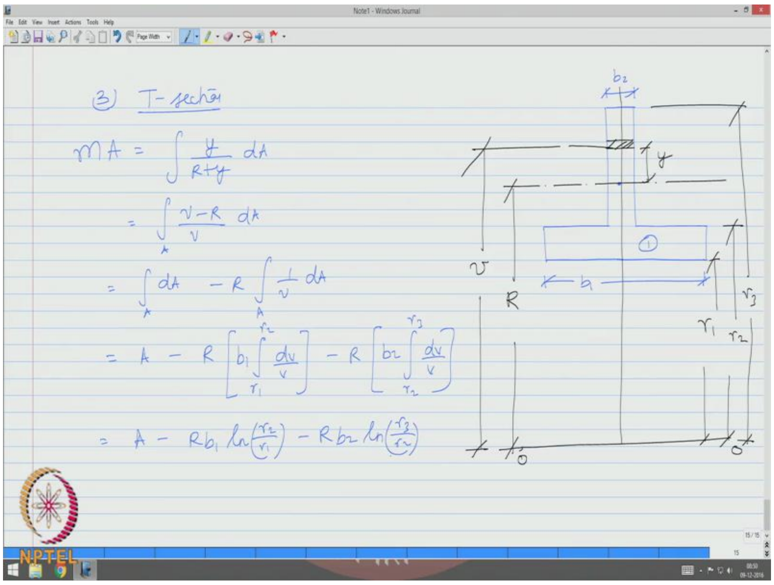


Let us do this for a rectangular cross section, let us take a rectangular cross section where this is my o o; axis of plane, the breadth of the cross section is b, the centroidal axis is at the distance R, we take a strip which is at a distance y from the centroidal axis. The neutral axis of course located at a distance e, this is my neutral axis. Then we say that the intrados, the distance r 1 and the extrados is at a distance r 2 and that the dimensions say h by 2 and h by 2.

Let us say this distance is v from here and this thickness is let us say we know that dA is bdy, mA is integral y by R plus y dA which is v minus r by v of dA, which is integral dA minus r integral dA by v is mA which will be from the limits r 1 to r 2. So, which will give me A minus R b; natural algorithm of r 2 by r 1 because dA is actually b d v.

So, therefore m is 1 minus R by A, b natural algorithm of r 2 by r 1 which can be also said as 1 minus R by A; b natural algorithm of r 2 can be expressed as R plus h by 2 and this is R minus h by 2; equation number 11. So, once I know m we can always find e using the relation which is already derived.

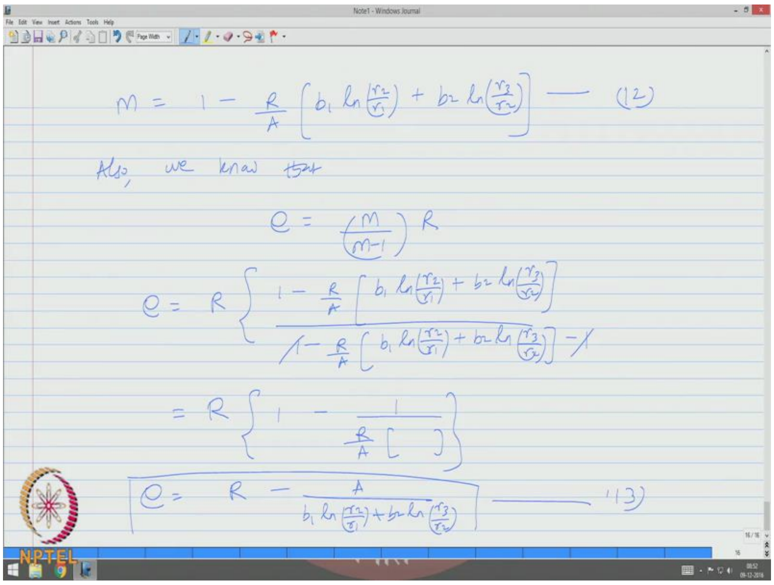
(Refer Slide Time: 41:00)



Let us do it for a; T section let us say this is b 1 and this is b 2; this c g is somewhere here whereas this is my o o plane and this becomes my R and this is my r 1, this is my r 2 and let us mark this as r 3. Let us take a strip at a distance y from the centroidal axis, let us take that value of the strip to be located at v from the plane of centre of curvature.

We know that mA is integral by R plus y dA which can be integral v minus R by v, dA which can integral dA; A minus R integral 1 by v, dA which we say as A minus R times of integral b 1, the limits for b 1 are the piece 1 whereas, from r 1 to r 2 which is d v by v minus R. The second piece b 2 integral again d v by v but the limits varies from r 2 to r 3. So, I can say now as a minus r b 1 natural algorithm of r 2 by r 1 minus r, b 2 natural algorithm of r 3 by r.

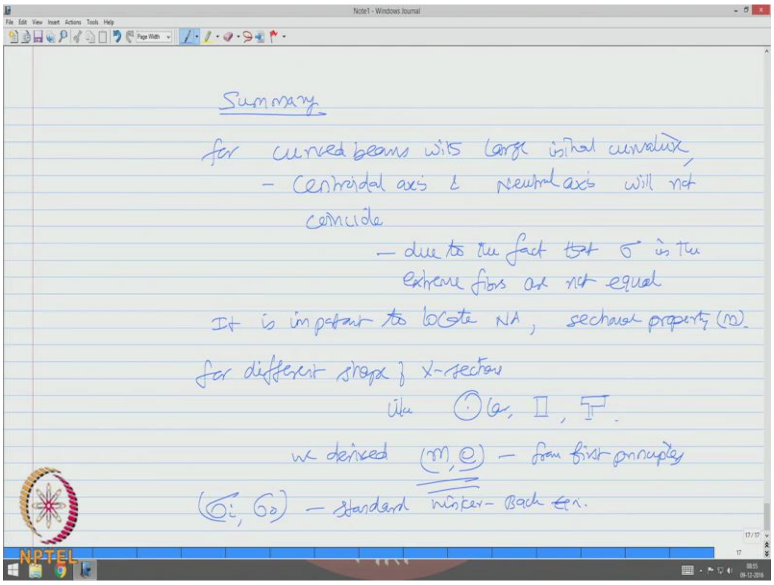
(Refer Slide Time: 44:00)



Therefore m is 1 minus R by A of b 1; log r 2 by r 1 plus b 2 natural algorithm r 3 by r 2; equation number 12, also we know that e is m by m minus 1 into R.

So, let us substitute e is going to be equal to R of 1 minus R by A b 1 log; r 2 by r 1, b 2 log r 3 by r 2 divided by 1 minus R by A; b 1 log r 2 by r 1; b 2 log r 3 by r 2 minus 1 which gets cancelled. So, this can be said as r times of 1 minus 1 by R by A of this expression which can be said as e is equal to r minus A by b 1 log r 2 by r 1 plus b 2 log; r 3 by r 2 - equation number 13.

(Refer Slide Time: 46:20)



So, friends we understood that for curved beams with large initial curvature centroidal axis and neutral axis will not coincide, this is due to the fact that the stress in the extreme fibres are not equal. So, it is important to locate the neutral axis which depends on sectional property m. So we found out for different cross sections, for different shape of cross sections like circular, rectangular, t section we determined or be rather derived m and e from first principles. Once I know these 2 parameters, I can always find the stresses in the extreme fibres using the standard Winkler Bach equation, which we derived. We will also discuss these parameters for other shape of cross sections and also try to tell what could be the approximate variation of these stresses in extreme fibres compared to that of theory of simple bending in the next lecture.

Thank you very much.