Offshore structures under special loads including Fire resistance
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Module – 02
Advanced Structural Analyses
Lecture - 31
Curved Beams – II

Welcome to the 31st lecture which we will continue to discuss on curved beams, under module 2, title Advanced Structural Analyses, under the NPTEL course, offshore structures under special loads including Fire resistance. In the last lecture, we discussed about the derivations for obtaining stresses; bending stress in A curved beam with small initial curvature, we also worked out equations for calculating the deflection from the first principles and we discussed certain idealizations and assumptions made in deriving the standard equation for curved beams with small initial curvature.

In this lecture, we will extend the discussion for curved beams with large initial curvature and see how this can be now estimated for the given problem.

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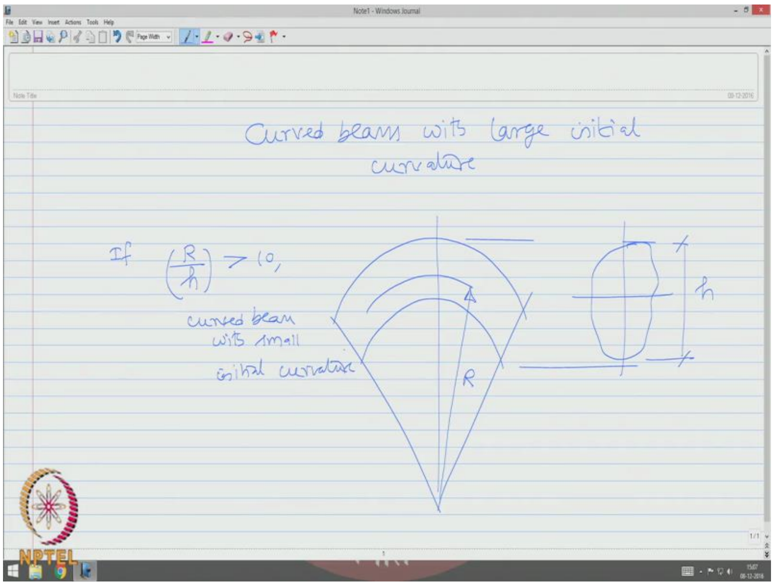

Now we are talking about curved beams with large initial curvature, let me remind you, if this is my curved beam, which has radius of curvature as R and the cross section as depth, let us say h, if ratio of initial radius of curvature to the cross sectional thickness exceeds 10 then we call this as curved beam with small initial curvature.

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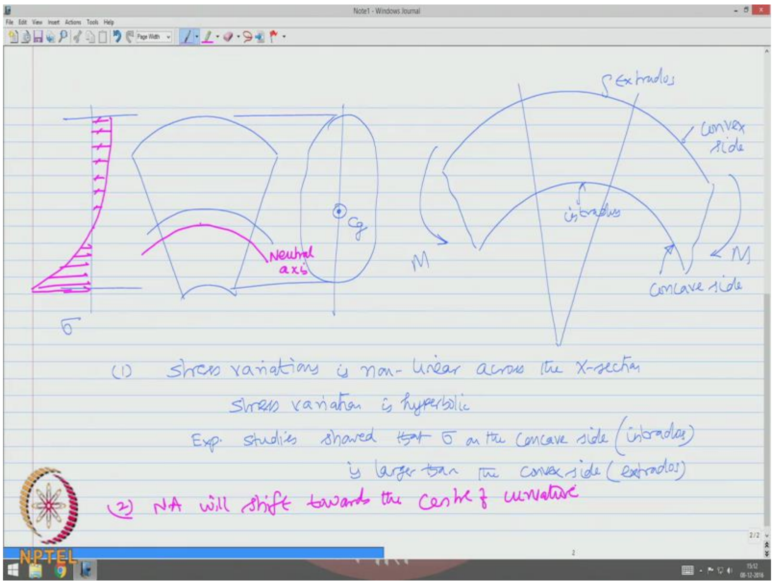

If you look at curved beam with large initial curvature subjected to moment M, we call this surface as concave side and this surface is convex side. If you take A segment and try to draw only the segment with some cross sectional dimension and shape where this becomes the c g it is seen from the literature that the stress variations is non-linear across the cross section.

Typical stress variation is hyperbolic, here it is interesting to note that the stress variation is hyperbolic and the experimental studies show that stress on the concave side that is intrados, you can also call this as intrados and this as extrados is larger than the convex side, therefore, it is interesting that the neutral axis will shift towards the centre of curvature.

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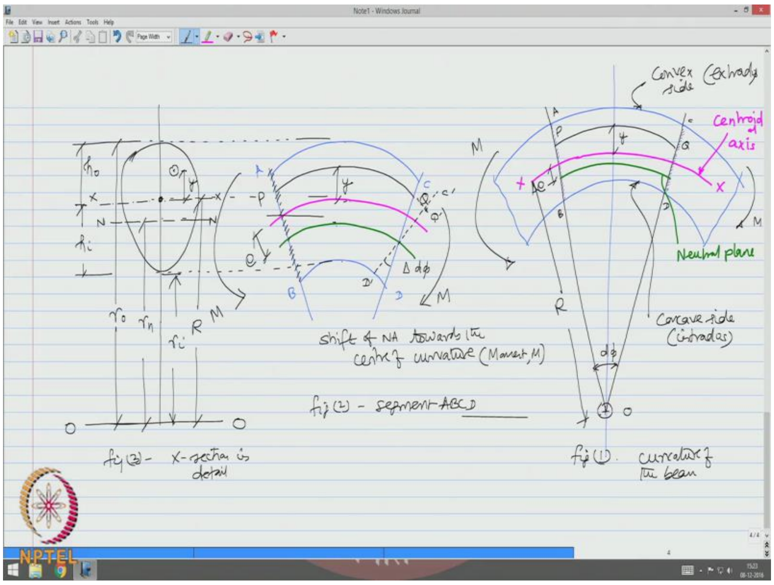

It is also interesting to note that neutral axis will not pass through the Centroid of the cross section because stress, the concave side is not equal to stress in the convex side. So, let us now understand how to estimate these stresses an intrados and extrados for a curved beam with large initial curvature.

We will take a curved beam, will take a layer P Q which substance rangle D phi at the point o, let us say this fiber is P Q which intersects the extrados and intrados at points A B and C D. So, 2 sections, 2 planes, A B and C D are now considered, let us say the centroidal axis is marked as x x, as we just now said the neutral layer will shift towards the centre of curvature which will be now at an extendicity e from the centroidal axis and the fiber is measured the distance y from the centroidal axis.

Let us say the beam is now subjected to an applied moment M and the beam has got initial radius of curvature R as shown in the figure, parallelly let us take up the segment A B C D which is A B C and D, which has the neutral layer and the centroidal layer and the fiber P Q, we know that we have understood this distance as e and this distance is measured as y, let us say on application of moment M let us keep the plane A B undisturbed and let us say C D has shifted. So, it has become now C dash and D dash and that tilt is delta d phi. So, we call this point as Q dash and D is moved to D dash. So, I can say now, there is a shift of neutral axis towards the centre of curvature on application of moment M.

Let us try to draw the cross section of some standard shape, let us say the cross section is symbolically indicated this way and C g is marked here, let us say the fiber is marked somewhere here and this distance is y and this axis is the x x axis which indicated here and this axis is the neutral axis inclined as N N and we call this distance as h i, i stands for intrados and this distance is h o, o stands for outer or extrados. From the center of curvature what we call as line o o, this distance is r i and this distance is r o and this distance is r n, n stands for neutral axis and of course, we know that this distance is capital R. So, let us say figure one showing the curvature of the beam, figure 2 shows segment A B C D and figure 3 shows the cross section more in detail and we call this surface as concave side and this surface as convex side which can be extrados and this is intrados.

With these 3 reference figures, let us try to derive the equation of finding the stress at outer and inner fibers.

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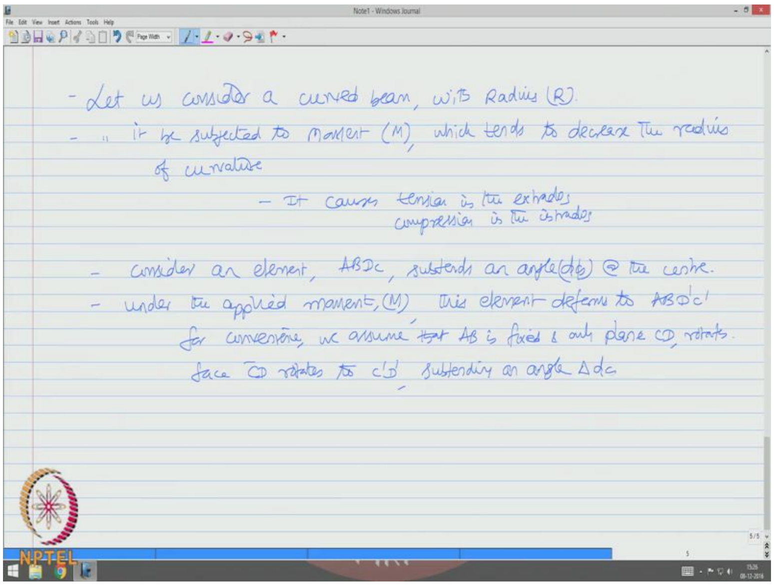

Let us say, consider a curved beam with radius R, let it be subjected to moment M which tends to decrease the radius of curvature that is it causes tension in the extrados and compression in the intrados. So, then let us consider an element A B D C, an element A B D C, which subtends an angle D phi, you can see the angle D phi at the centre. Under the applied moment M, this element deforms to A B D dash C dash, A B A B D dash C dash, it is for convenience we assume that A B is fixed and only plane C D rotates. So, the face C D rotates to C dash D dash C dash D dash subtending an angle delta D phi, this angle is delta D phi.

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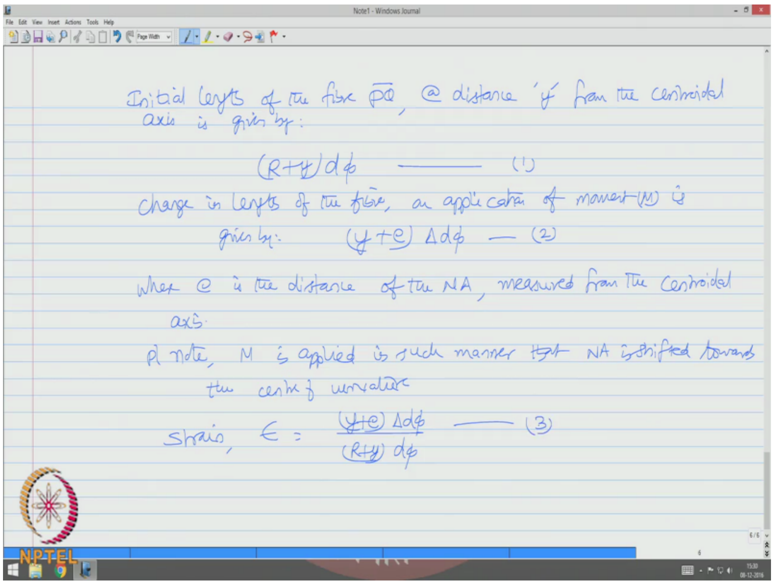

Then initial length of the fiber P Q which is at a distance y from the centroidal axis is given by R plus y D phi equation 1, change in length of the fiber on application of moment M is given by y plus e delta D phi. So, you know it is y plus e, y plus e delta D phi equation 2, where e is the distance of the neutral axis measured from the centroidal axis. Please note M is applied in such a manner that neutral axis is shifted towards the center of curvature. Now strain epsilon is change in length by original length, it is called equation 3.

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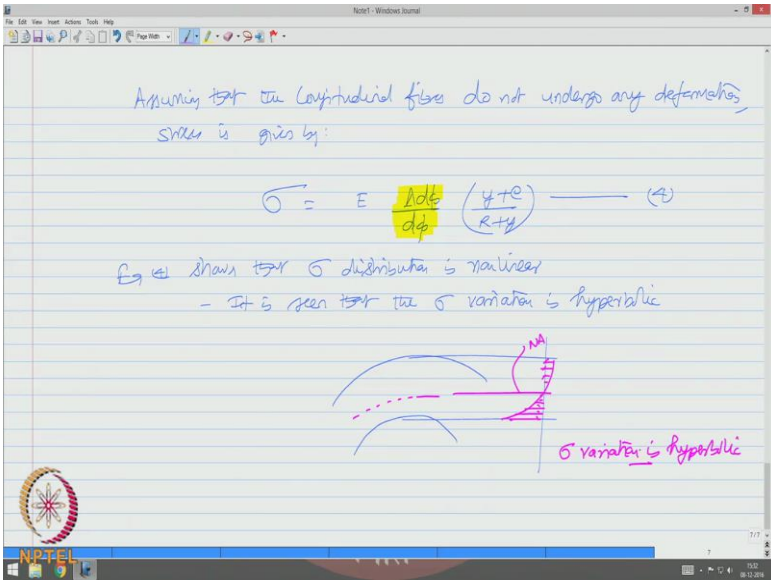

We assume that the longitudinal fibers do not undergo any deformations; stress is given by e delta D pi by D phi of y plus e by R plus y equation number 4.

Equation 4 shows that the stress distribution is non-linear because you got the components of delta D phi D phi, it is seen that the stress variation is hyperbolic. So, if you look at the segment A B C D and try to plot the stress variation, it shows that this is a neutral axis. So, the stress variation is hyperbolic.

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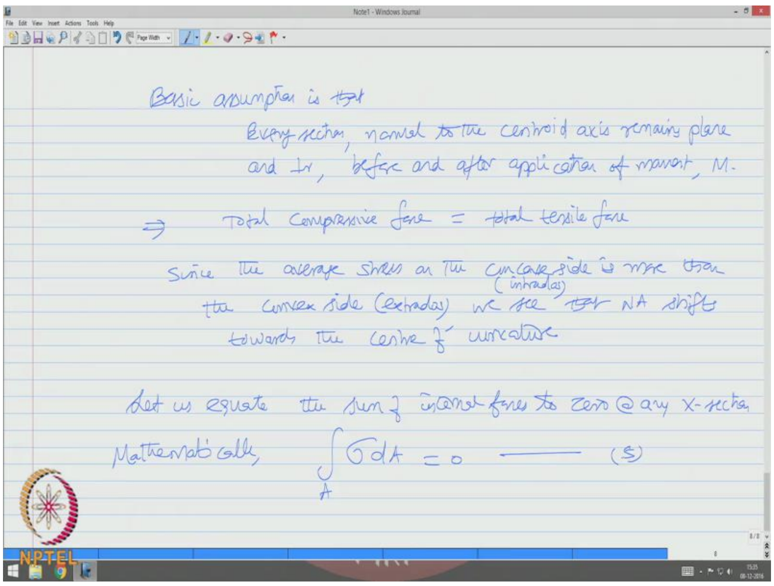

There is a basic assumption here; every section normal to the Centroid axis remains plane and perpendicular before and after application of moment M which implies that total compressive force should be equal to the total tensile force. Since the average stress on the concave side is more than the convex side, concave side intrados and convex side is the extrados we see that neutral axis shifts towards the centre of curvature.

Now, let us equate the sum of internal forces to 0 at any cross section mathematically integral stress D A, A should be 0, we already have the equation for stress from equation 4.

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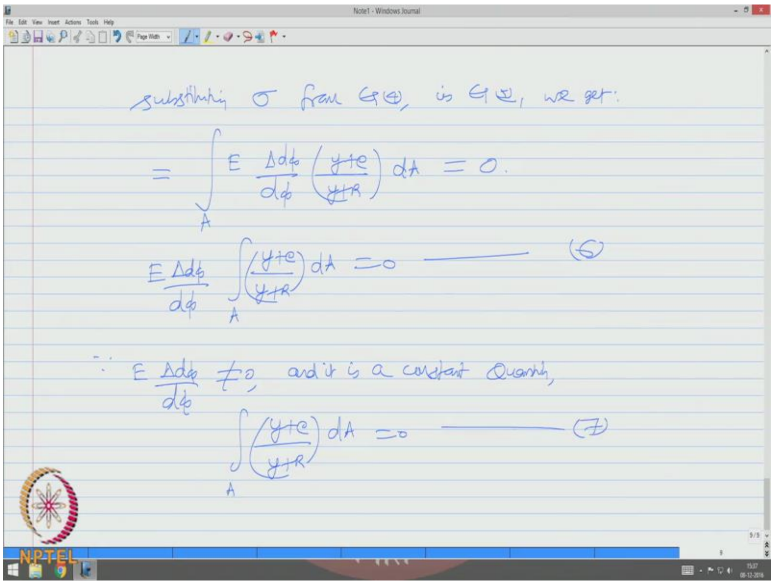

Let us substitute stress from equation 4 in equation 5, we get E delta D phi by D phi y plus e by y plus R dA or A should be 0 that is E delta D phi by D phi integral A y plus e by y plus R dA should be 0. Since E delta D phi by D phi cannot be 0 and it is constant quantities equate y plus e by y plus R dA as 0.

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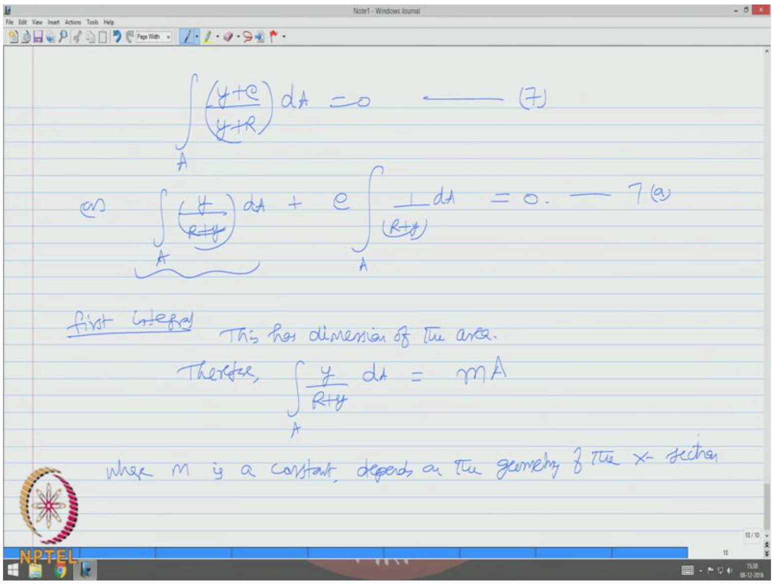

Let us look at this equation 7, integral y plus e y plus R dA is 0, I split this equation - integral y by R plus y dA of A plus e 1 by R plus y dA is 0, let us call the equation 7 a, this is equation 7. Let us look at the first integral, this has dimension of the area therefore, integral y by R plus y dA for A can be sum multiplier of A where M is a constant which depends on the geometry of the cross section.

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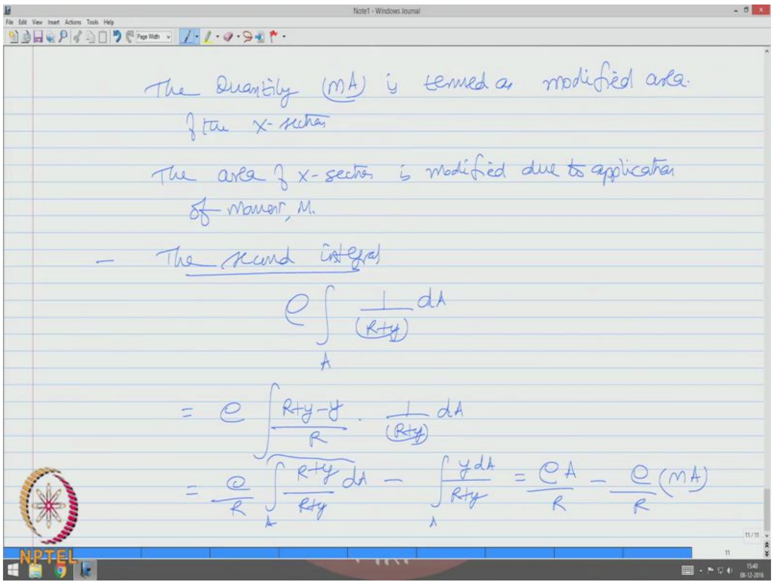

The quantity mA is termed as modified area of the cross section that is the original area of the cross section is modified due to application of moment M.

Let us look at the second integral, the second integral is e 1 by R plus y dA which can be simplified as R plus y minus y by R 1 by R plus y dA which can be said as e by R integral R plus y by R plus y dA minus integral y dA by R plus y which is now e A by R because this integral will give you the area and this already we have said, it is e by R of mA because this is having an area dimension. So, let us look at the equation 7 a, we have evolved both integrals.

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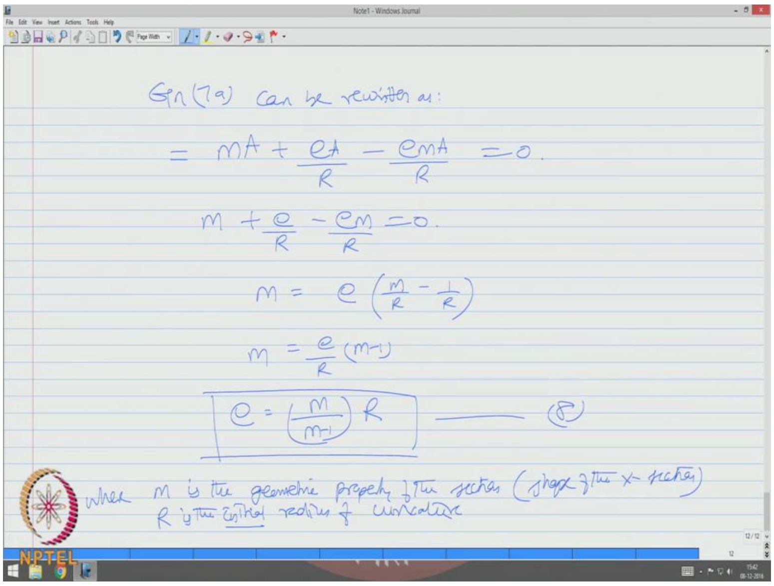

Now equation 7 a can be re written as mA plus e A by R minus e mA by R which is said to 0 is it not? That is said to 0 A, you can see here said to 0; that is M plus e by R minus e M by R is 0 M will be equal to e M by R minus 1 by R that is e by R M minus 1 or e is equal to M by M minus 1 of R that is equation 8 where M is the geometric property of the section depends upon the shape of the cross section and of course, R is the initial radius of curvature, if I know these 2 can always find the shift of neutral axis from the centroidal axis towards the centre of curvature.

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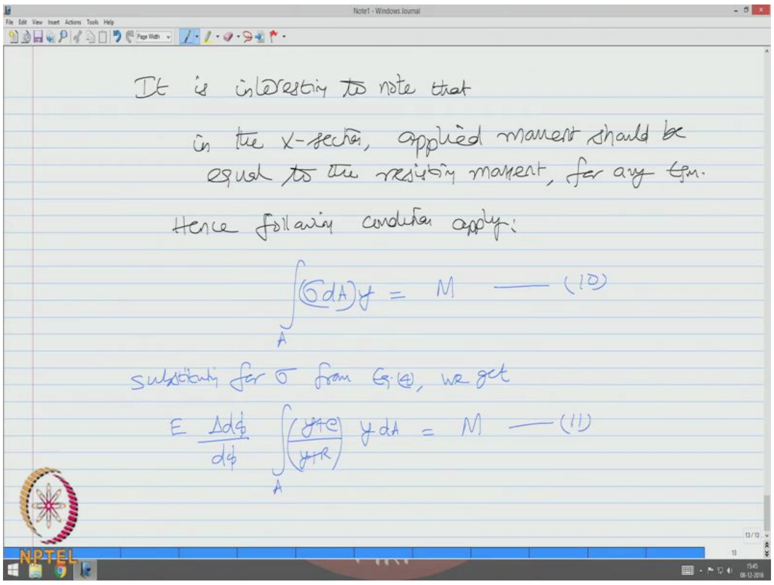

It is very interesting to note that in the cross section applied moment should be equal to the resisting moment for any equilibrium, hence following condition apply integral sigma dA into y for the area A should be actually equal to M equation number 10; this equation actually 8, this equation 9, so 10. Substituting for stress from equation 4, we get E delta d phi by d phi integral y plus e y plus R y d A, A is M.

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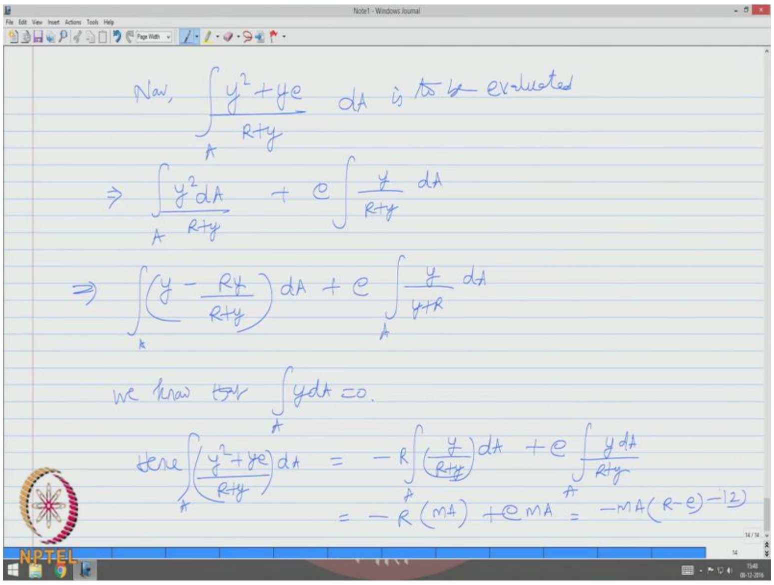

That is y square plus y e by R plus y integral A dA is to be evaluated is it not, you can see this equation y square plus y e by R plus y is to be evaluated, which is simplified as y square dA by R plus y A plus e y by R plus y dA which implies, I can simplify this first integral as y minus R y by R plus y of d A, you multiply this becomes R y minus R y cancels y square, so that is what you get here; plus e integral y y plus R dA for the entire area.

We already know that y dA is 0, hence y square plus y e by R plus y dA for integral will be now equal to minus integral R y by R plus y dA plus e y dA plus R plus y for the entire area which is minus R mA plus e mA which is otherwise minus mA R minus e which I call equation number 12.

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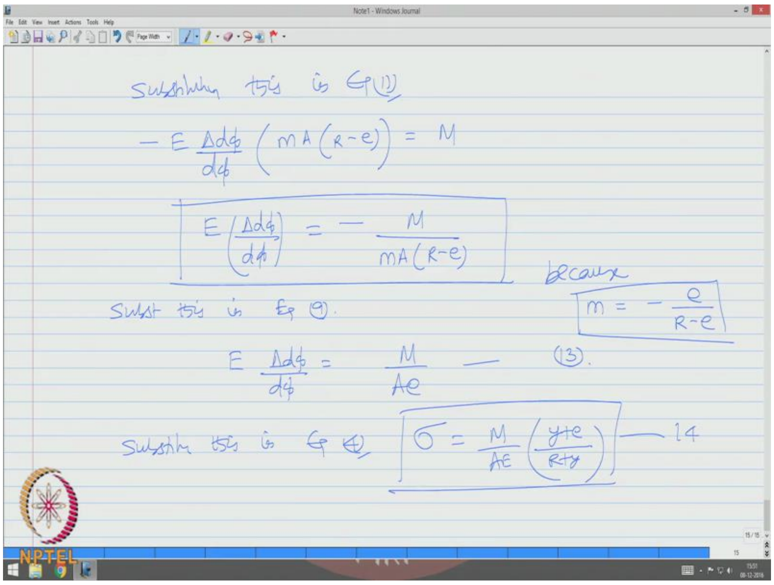

Substitute this in equation 11, minus e delta d phi by d phi of mA R minus e will be M because equation 11 is related to this; this equation which says that e delta d phi by d phi will be equal minus M by mA R minus e, now substitute this in equation 9, equation 9 is here. So, we have some value e expressed in terms of M and R in equation 9, we also have e to be expressed in terms of M and R here. So, let us say e delta d phi by d phi is actually M by A e because e is actually M by M minus 1 into R call this equation number 13, because M is equals minus e by R minus e substitute for this you will get equation 13.

Now, substituting this in equation 4, we get stress as M by A E y plus e by R plus y.

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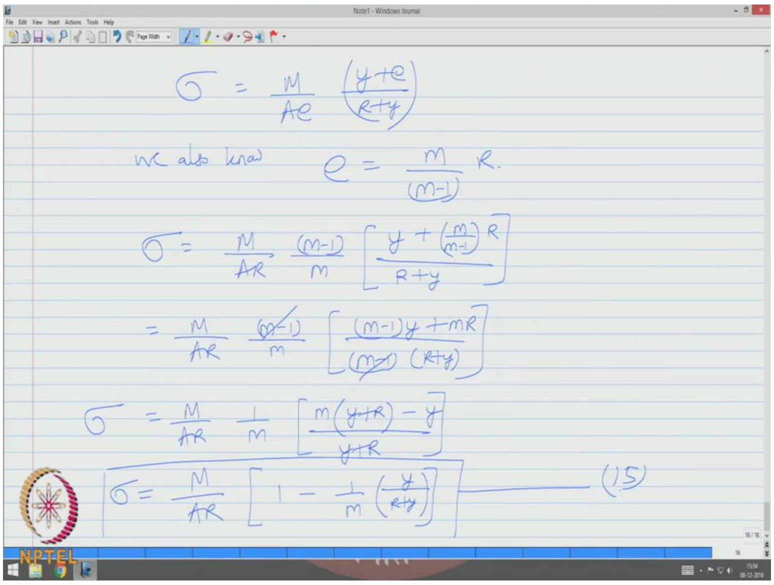

Let us say stress is M by A e y plus e by R plus y, we also know e is M by M minus 1 into R therefore, stress is M by A R, M minus 1 by M of y plus M by M minus 1 of R divided by R plus y which tells me M by A R, M minus 1 by M, M minus 1 y plus M R by M minus 1 R plus y - it says M by A R because these 2 will go away M of y plus R minus y by y plus R which is the stress or in simplified forms M by A R. If you multiply the first product with this you get 1 minus 1 by M of y by R plus y - I call this equation number 15.

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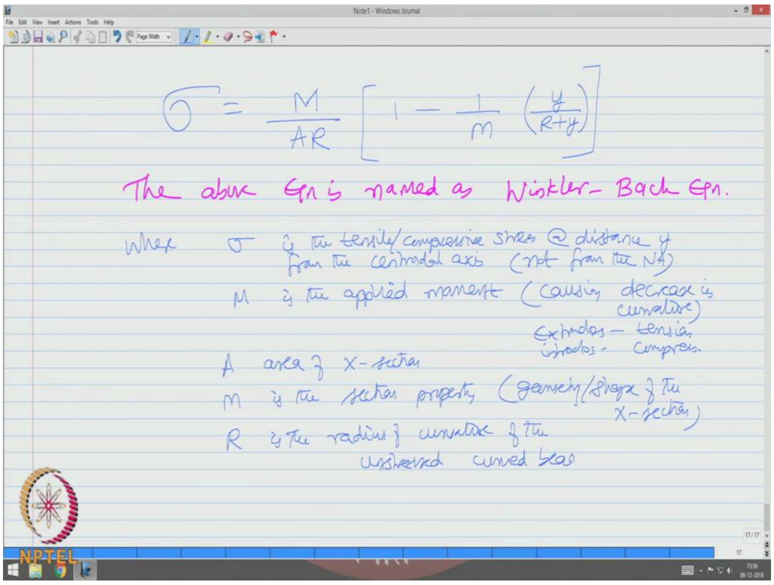

Stress is applied moment by A R 1 minus 1 by M y R plus y, I can get the stress, I know the value of M and R where this equation called is named as Winkler back equation, in this equation where sigma is the tensile or compressive stress at distance y from the centroidal axis it is not from the neutral axis please note. M is the applied moment causing decrease in curvature that is extrados in tension and intrados in compression. A is the area of cross section, M is the sectional property depends on geometry, shape of the cross section and R is the radius of curvature of the unstressed curved beam.

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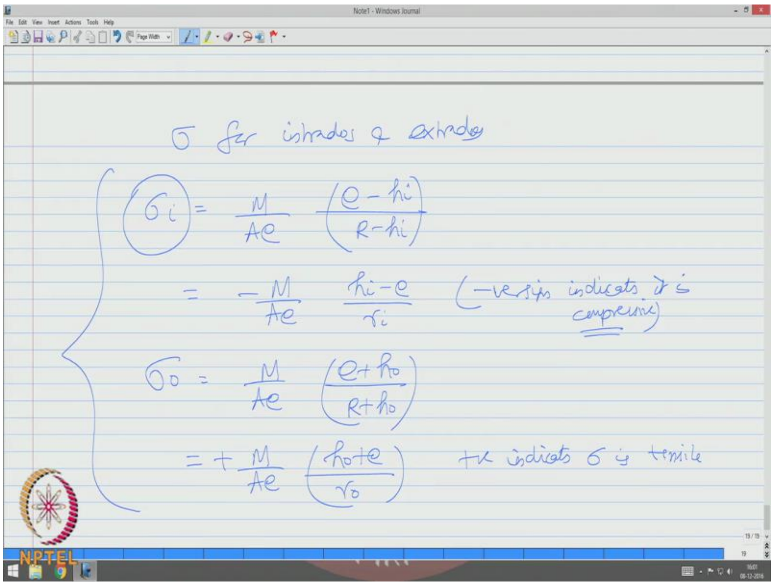

This equation has A very interesting sign convention, y is negative when measured towards the concave side, this is my curved beam, this is my concave side, this is my convex side, y is measured positive, the other way it is measured positive when measured towards the convex side where this is the centroidal axis not the neutral axis; neutral axis somewhere here be shifted, this is the neutral axis. So, with this sign convention we say negative stress indicates compressive stress and positive stress indicates tensile stress.

Let us workout these stresses for intrados and extrados stress, for intrados and extrados. If we look at the cross section R plus h o and minus h I will give you the limits of the section. So, I should say R plus h o can be R o R minus h I can be R I, use this relation in calculating stresses at the intrados and extrados. So, stress intrados is simply A e e minus h I by R minus h I which will be minus M by A e h i minus e by R i. So, negative sign indicates it is compressive, we also said intrados the stress will be compressive. For the extrados, it is M by A e e plus h o by R plus h o which will be plus M by A e h o plus e by R o, positive sign indicates the stress is tensile.

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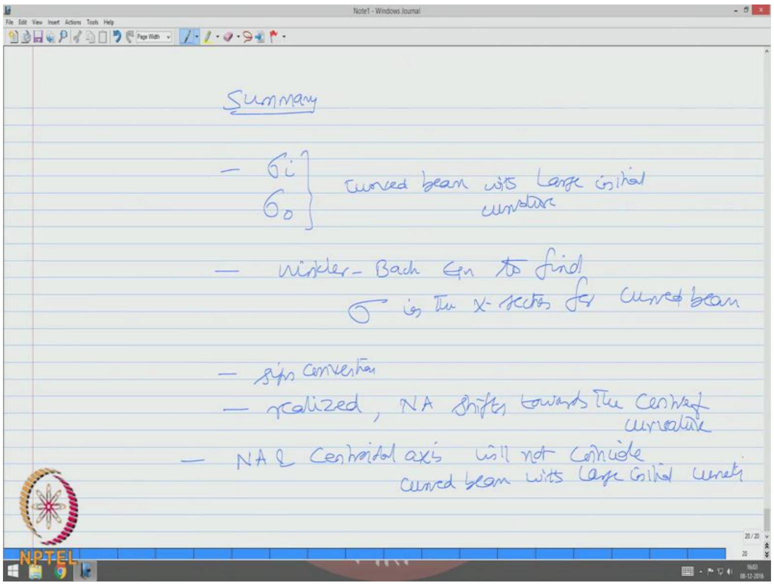

Friends, in this lecture we discussed how to find stresses at intrados and extrados for a curved beam with large initial curvature. We have also derived the famous Winkler back equation to find stresses in the cross section for a curved beam. We have understood the sign convention, we have realized how the neutral axis shifts towards the center of curvature; we have also understood that the neutral axis and the centroidal axis will not coincide for curved beams with large initial curvature.

We will extend the discussion in the next lecture, do some derivations to estimate the geometric property of the cross section that is small m and then apply it on problems and see how we can estimate stresses at the extreme fibers that is, the convex extreme and the concave extreme of a given curved beam.

Thank you.