Offshore structures under special loads including Fire resistance

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Module - 02
Advanced Structural Analyses
Lecture - 30
Curved Beams- I

Friends welcome to the 30th lecture, we are going to discuss now in these set of lectures details about Curved Beams.

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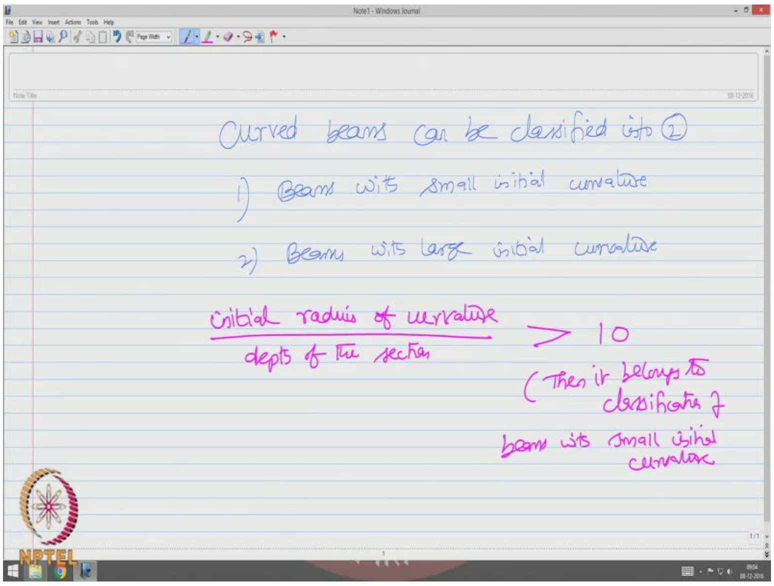

Curved beams can be classified into 2 sections; one beams with small initial curvature, two beams with large initial curvature, though there is no clear distinct difference in the definition of 1 and 2 sections; however, literature says when the ratio of initial radius of curvature to the depth of the section. If this ratio exceeds number 10, then it belongs to classification of beams with small initial curvature, if it is the other way then it can be classified as beams with large initial curvature.

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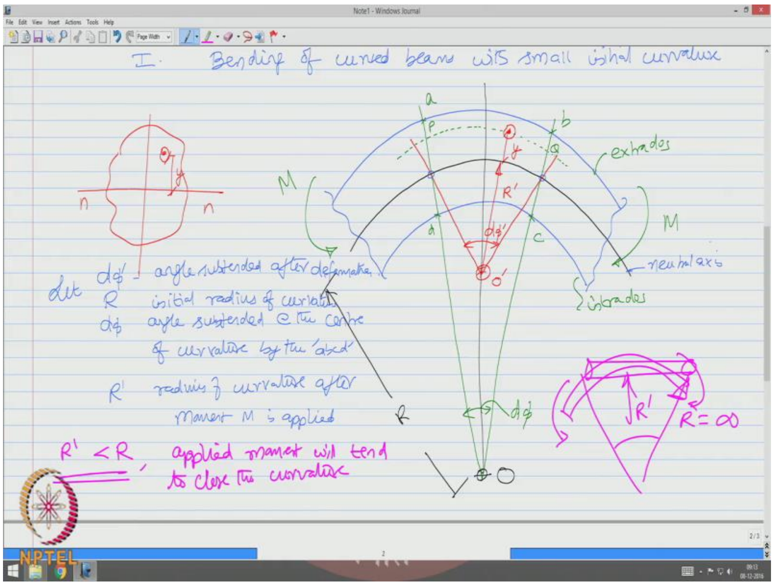

Our interest is now to study bending of curved beams with small initial curvature. Let us take a beam, let us say you have to define certain geometric parameters of this beam, let us choose a layer whose radius is (Refer Time: 03:33) from here, this point is o. Now let us select a fiber, which initially as an angle d phi, let us say this fiber intersects of this cross section intersects at a b at the extrados, and c and d at the intrados. Let us say this is my intrados, this is my extrados, when I now apply a moment M, the curvature will now change therefore, the points P and Q will now shifted until to shift the new center which is called as O dash and the new angle is d phi dash, and the new radius is R dash and the fiber distance is y. So, I have a typical cross section, which is got n n axis and (Refer Time: 06:06) a point and that point distance y, which corresponds to any point on this let us say the point this one.

Now, let R be the initial radius of curvature, d phi be the angle subtended at the center of curvature by the element a b c d, R dash be the radius of curvature, after moment M is applied interestingly, one can note that R dash will be lesser than R, because the applied moment will tend to close the curvature, I can give an example take a straight bar I would say the curvature is infinity, when we bend the bar it has some curvature whose value will be R dash and R dash will lower than R, because any moment which closes the curvature will always reduce radius of curvature, and d phi dash is the angle subtended by element after deformation.

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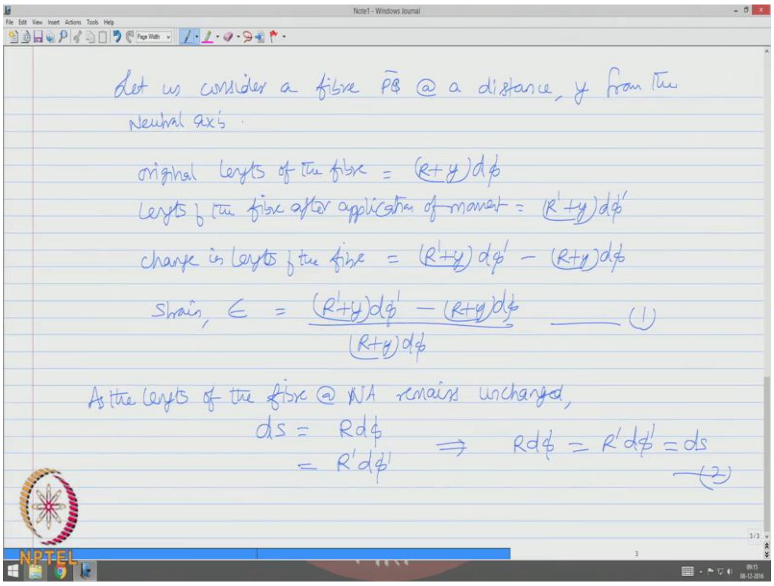

Having said this let us now consider a fiber PQ, at a distance y from the neutral axis, why neutral axis? You can see here at this point there is no change in therefore, the black line what you see here is the neutral axis, which is n n, which can see in this section also n n. So, now, original length of the fiber is actually R plus y into d phi. Now length of the fiber after application of moment is R dash plus y into d phi dash therefore, one can easily find change in length of the fiber which will be R dash plus y d phi dash minus R plus y d phi. So, the strain of this fiber will be change in length by the original length, which I call equation 1. Interestingly as the length of the fiber at the neutral axis remains unchanged because it does not deform, I call that length as d s which will equal to both R d phi as well as R dash d phi dash, which implies that R d phi is R dash d phi dash is d s.

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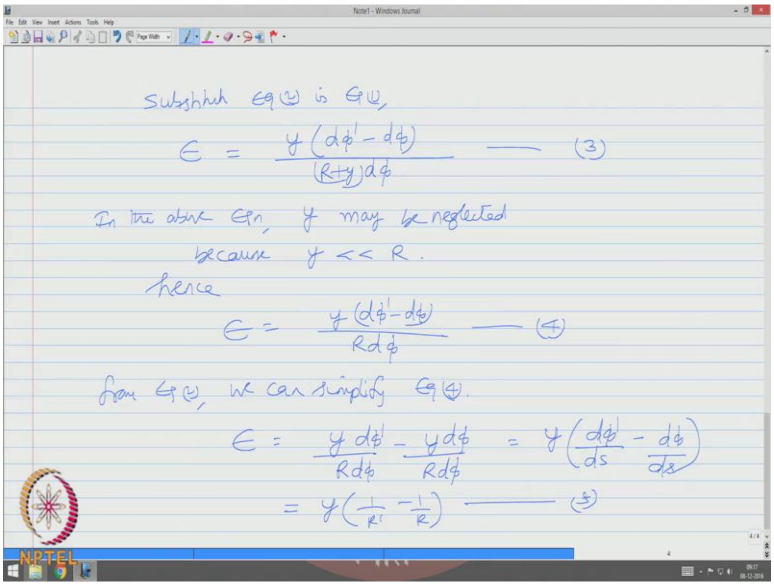

Let us substitute equation 2 in 1; we get strain now as y of d phi dash minus d phi by R plus y d phi this is equation 3. In the above equation y may be neglected because y will be very very small compared to R, hence strain is now y d phi dash minus d phi by simply R d phi; I call this equation number 4. Now from equation 2 we can simplify equation 4; epsilon now can be y, I should say d phi dash by R d phi minus y d phi by R d phi, which I can say as y d phi dash by d s minus d phi by d s, which can be simply y 1 by R dash minus 1by R, I call equation number 5.

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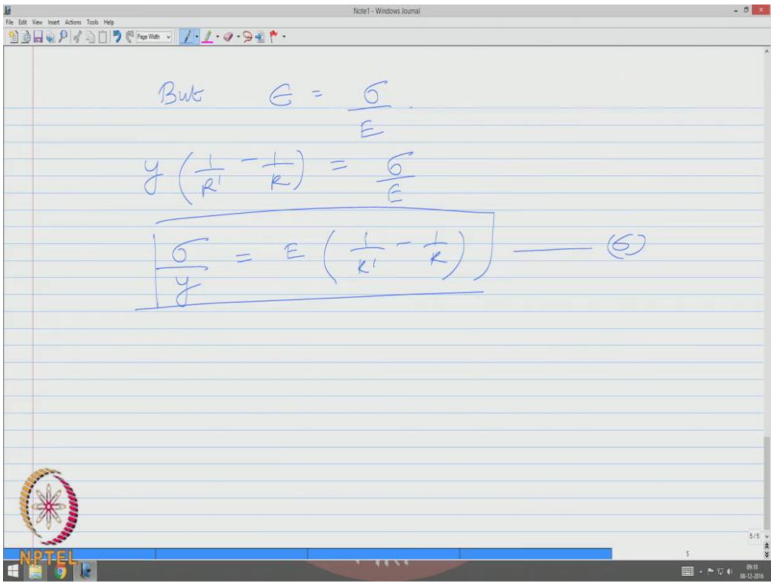

But we know strain is actually stress by youngs modulus therefore, y 1by R dash, minus 1 by R is strain by this. It means sigma by y will be e times of 1 by R dash, minus 1by R equation number 6, which is a classical equation of bending for curved beams.

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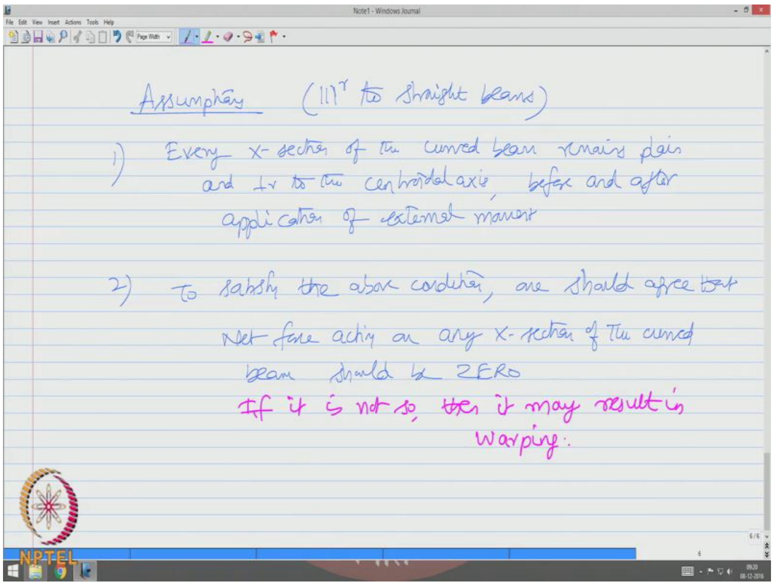

There are some assumptions in this equation, which are actually similar to straight beams, let us see what are these assumptions, 1 every cross section of the curved beam remains plain and perpendicular to the centroidal axis before and after application of external moment, the second assumption is very interesting. Now to satisfy the above condition, we need to impose one condition that, one should agree that net force acting on any cross section of the curved beam should be 0, then only the above condition can be meaningful; if this condition in is violated if it is not so, then it may result in warping.

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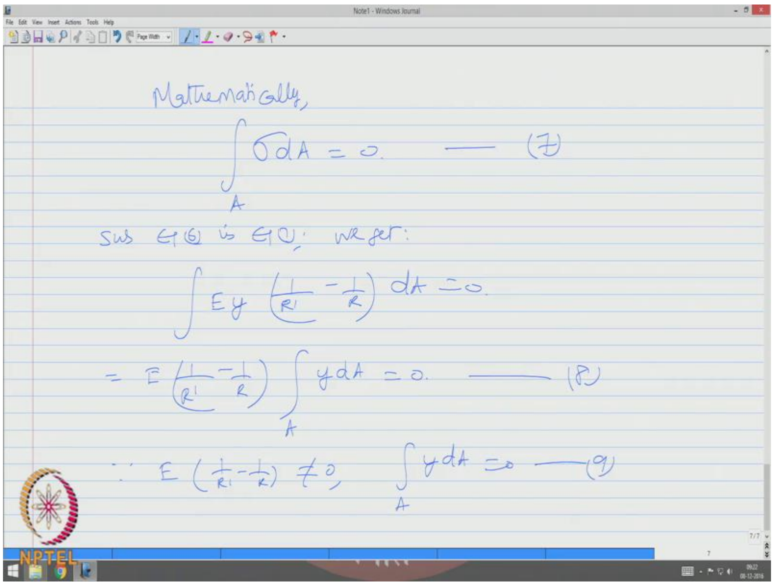

So, mathematically d A the force should be said to 0. So, now, let us substitute equation 6 in equation 7, we get integral EY 1 by R dash by R of d A should be 0, which we say E 1 by R dash minus 1 by R integral A, y d A should be 0, since E by R cannot be 0 integral y d A should be said to 0.

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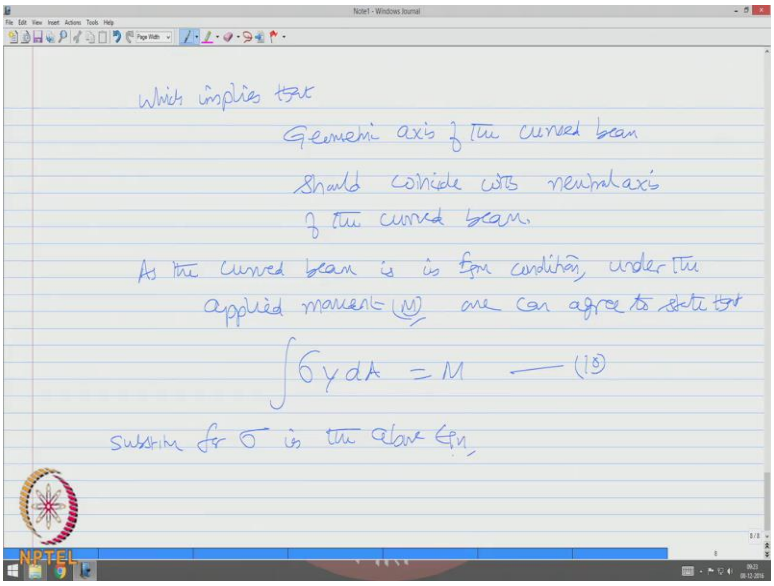

Which means that, which implies that the geometric axis of the curved beam should coincide with neutral axis of the curved beam. So, that is a very interesting statement, we derive from this understanding. Further as the curved beam is in equilibrium condition, under the applied moment M, one can agree to state that sigma y d A should be actually equal to this moment.

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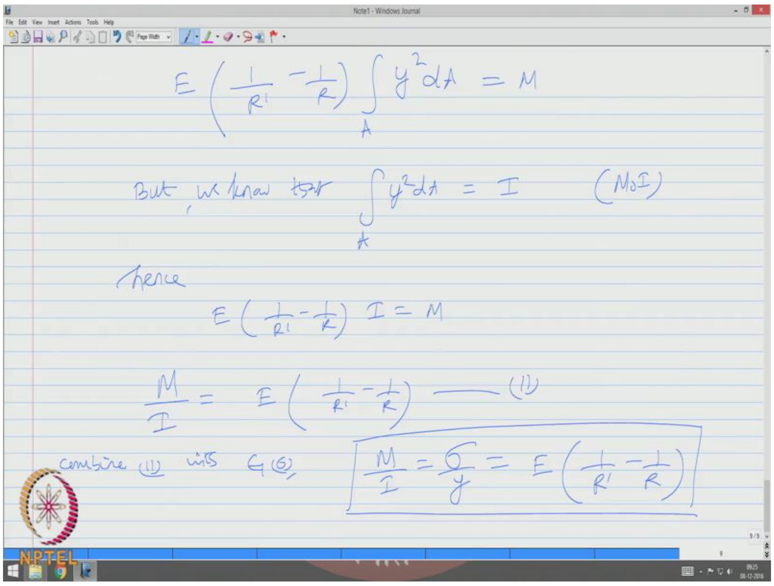

Now, substituting for sigma in the above equation because we already have an equation for sigma you can see here, we already have an equation for sigma substituting this in the above equation E 1 by R dash by R, y square d A should be M, but we know that integral y square d A second moment of area is actually moment of inertia and hence E 1 by R dash by R into I is M, that is M by I is E times of 1 by R dash, minus 1 by R. If we combine this equation with equation 6, which is this equation, sigma by y is equal to this. So, now, we can say M by I is also sigma by y, which is E by R instead of that it is R dash minus R, that is my classical equation for bending of curved beams.

So, friends interestingly based upon the assumption that the geometric axis of the curved beam, will align with the neutral axis of the curved beam, while the moment applied decreases radius of curvature, one can easily say that this equation is valid for curved beams with small initial curvature.

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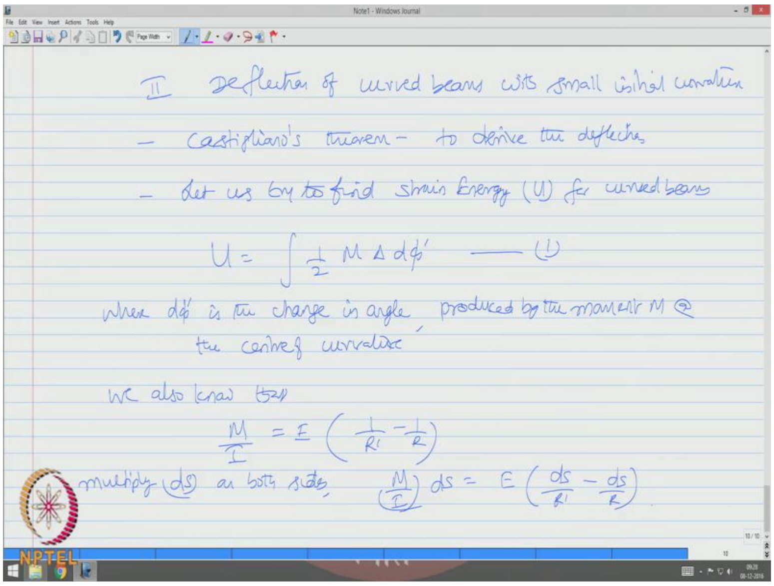

Let us now proceed to find deflection of curved beams with small initial curvature; we use Castilians theorem to derive the deflection, should use this theorem we must have an equation of strain energy. So, let us try to find the expression for strain energy. U is generally given by half M delta d phi for curved beams equation 1; where d phi is the change in angle, produced by the moment M at the center of curvature. I think it should be d phi dash, we also know that M by I is E of 1 by R dash, minus 1 by R let us multiply d s on both sides. So, M by I d s is E d s by R dash, d s by R.

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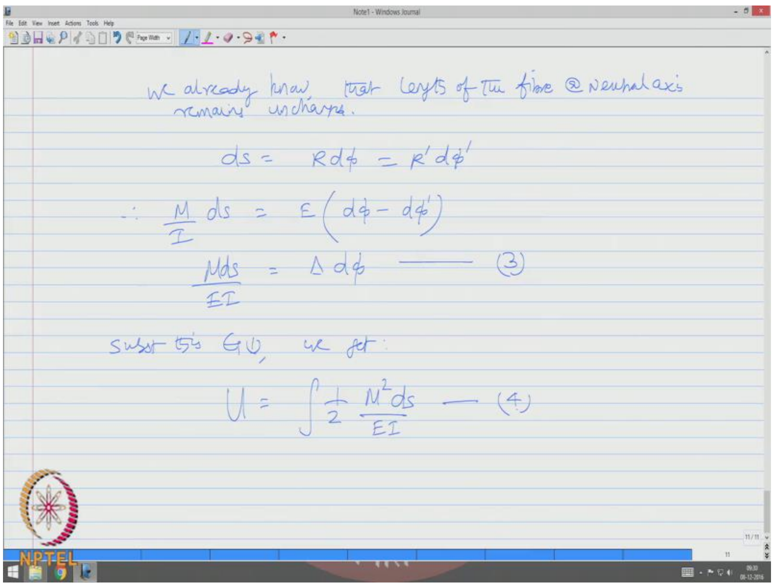

Look at equation 2, we already know that length of the fiber at neutral axis remain unchanged and we said d s is as same as R d phi, which is same as R d phi dash hence M by I d s can be now said as E times of d phi minus d phi dash, that is M d s by E I is delta d phi equation 3. Now substitute this in equation 1, we now get strain energy is integral half M square d s by EI.

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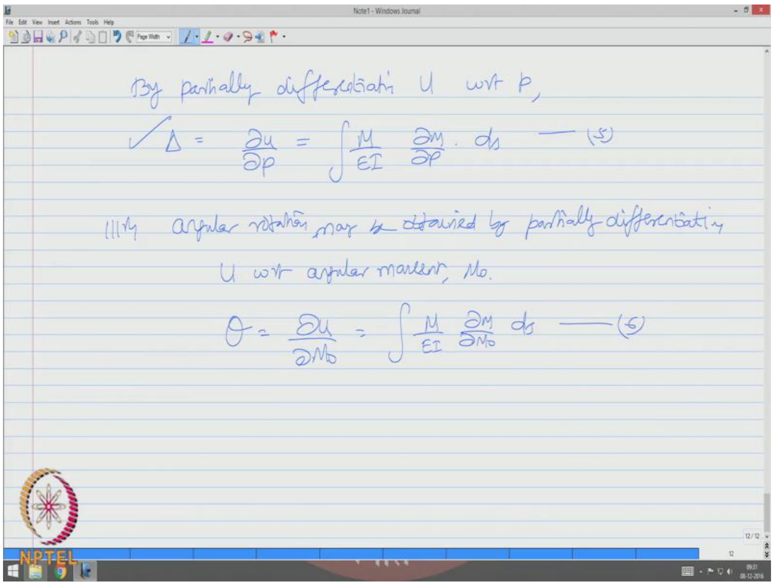

By partially differentiating U with respect to P, we get delta which will be dou u by dou p, which will be M by EI, dou M by dou p d s equation 5 that is the deflection. Similarly to get the angular rotation may be obtained by partially differentiating U with respect to angular moment M 0. So, I get theta which is this by M 0, which is integral M by E I, dou M by dou M naught of d s.

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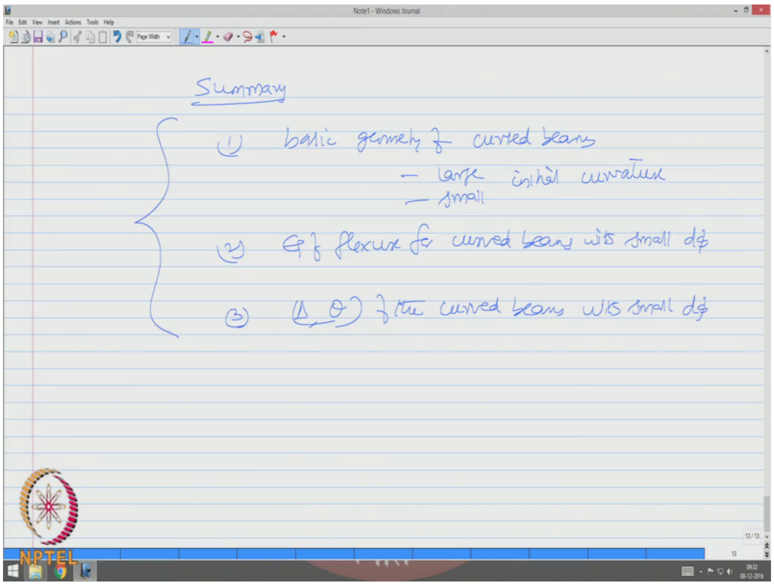

So, friends in this lecture we understood the basic geometry of curved beams with large and small initial curvature, we derive the equation of flexure for curved beams with small initial curvature, we also derived equation for deflection and angular rotation of the curved beams with small initial curvature. We will extend this discussion for beams with large initial curvature then extend this to find out the stresses at the intrados and extrados of the curved beams, which is very important assessment in terms of curved beams used very commonly in offshore structural platforms.

Thank you.