

**Offshore structures under special loads including Fire resistance**  
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**Module – 02**  
**Advanced Structural Analyses**  
**Lecture - 25**  
**Unsymmetrical Bending-III**

Friends, welcome to the lecture number 25 which is continuation of the previous lecture in unsymmetrical bending, these are lectures in module 2, title advance structural analysis, under NPTEL course title, offshore structures under special loads including fire resistance. We have already understood and we are solving in example in between, we are interested to find the bending stress for a section which is subjected to unsymmetric bending.

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$$\sigma_x = - \left( \frac{M_u v}{I_u} - \frac{M_v u}{I_v} \right) - W$$

$$M_u = +M_z \cos \alpha_1 = 4 \cos(70.36) = +1.344 \text{ kNm}$$

$$M_v = -M_z \sin \alpha_1 = -4 \sin(70.36) = -3.767 \text{ kNm}$$
 Substituting,
 
$$\sigma_x = 0 \text{ for neutral axis.}$$

$$\frac{M_u v}{I_u} - \frac{M_v u}{I_v} = 0$$

$$\frac{1.344 \times 10^4}{1.575 \times 10^4} (v) - \left( \frac{-3.767 \times 10^4}{10.457 \times 10^4} \right) (u) = 0$$

$$\frac{v}{u} = -0.422 = \tan \beta \quad \left( \begin{array}{l} \beta \text{ is the inclination of} \\ \text{u to NA} \end{array} \right)$$

$$\beta = -22.88^\circ$$

We said that stress at any point is given by minus  $M_u v / I_u$  minus  $M_v u / I_v$ . We said for this being the z axis and these being u and v axis as located at alpha 1 using right hand screw rule, we said that the movement will be Mz which is now resolved as Mu which is Mz cos alpha 1 positive and Mv when it resolves, it is in opposite direction whereas, or Mu the moment is mark this way and Mv mark this way, but it is resolved in the opposite quadrant therefore, minus Mz sin alpha 1 which we said this is 4 cos 70.36, which

amounts to plus 1.344 kilo Newton meter and this is minus 4 sin 70.36 degrees, which is 3.767 kilo Newton meter.

Substituting sigma x will be said to 0 for locating the neutral axis because in neutral axis the stress values going to be 0. So, setting equation 1 to 0, we can say Mu by Iu into v is minus Mv Iv into u is said to 0. So, 1.344 by Iu 1.575 into v minus Mv is negative. So, minus 10 power 6 by 10.457 10 power 4 of u is said to 0, which shows that v by u will be minus 0.422. So, which we say is actually equal to tan beta, where beta is angle between the positive principle axis uu with the neutral axis.

Beta is the inclination between uu axis and neutral axis which says that beta is actually equal to minus 22.88 degrees. So, try to plot this neutral axis now is going to be minus. So that is the neutral axis which is neutral axis and minus therefore, we plot this angle as 22.88 degrees having located the neutral axis. Let us now get the section back and mark all these axis to find the stresses.

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point A will be compressive  
 point B will be tensile

A, B (coordinates)  
 (A)B in terms of (u, v)

converted the unsymmetric bending problem to uni-planar bending  
 - use classical flexural eqn.

$u_A = z \cos \alpha + y \sin \alpha$   
 $= (-7.5) \cos(70.36) + (17.5) \sin(70.36) = 13.96 \text{ mm}$

$v_A = -z \sin \alpha + y \cos \alpha$   
 $= -[-7.5 \sin(70.36) + 17.5 \cos(70.36)] = 12.95 \text{ mm}$

This is the given section, we have the z axis located here and y axis located here, uu axis located 70 degree, this angle is 70.36. We also located the neutral axis which is 22.88 degrees.

If you want really find out the stress at this point, let us say this point A, I must know the distance of this from the neutral axis measured along v, if you want to locate the stress at

this point B, I must know the distance of this point measured from the neutral axis which I call as  $v_b$ . Obviously, for the given section subjected to a load upper of P, the section is going to bend this way. So, the top members will be in compression, bottom or intension. So, it is expected that point A will be compressive and  $v_a$  will give me the maximum distance of this fibre and point B will be tensile and  $v_b$  will give me the maximum distance of this.

Our job is to know find for A and B, the coordinates I may wonder A and B coordinates of known to us in terms of the dimension we are marking now these are all given to us, but we need to find this A and B coordinates in terms of u and v because we are going to measure the stresses or the distance of fibre from the neutral axis.

We have converting or we have converted the classical unsymmetric bending problem to uni-planar bending problem, where I can now use the classical flexural equation that is advantage. So, I need to find u A which will be  $z \cos \alpha + y \sin \alpha$ , we already derived this equation earlier, let us substitute the values of we need the distances already known to us. So, z is minus 7.5  $\cos 70.36$  plus y is measured positive. So, 17.5 multiplied by  $\sin 70.36$  which I get as 13.96 millimetre, v A minus  $z \sin \alpha + y \cos \alpha$  which will be minus of minus 7.5  $\sin 70.36$  plus 17.5  $\cos 70.36$  which gives me 12.95 millimetre.

For the point A, I got the coordinates in terms of u and v axis, I can now find the stress at A.

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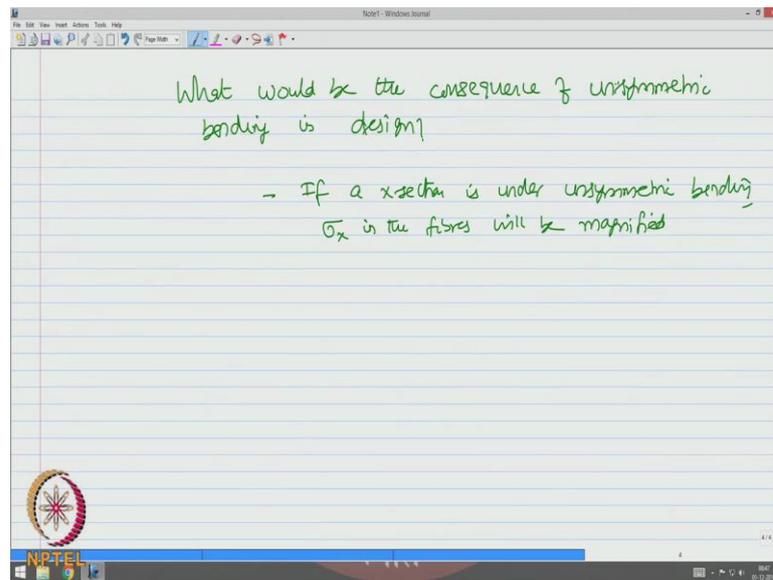
The image shows a handwritten derivation on a digital notepad. The first part calculates the stress at point A,  $\sigma_A$ , using the formula  $\sigma_A = -\frac{M_u}{I_u} v_A + \frac{M_v}{I_v} u_A$ . The values substituted are  $M_u = 1.344 \times 10^6$ ,  $I_u = 1.574 \times 10^8$ ,  $v_A = 12.95$ ,  $M_v = 3.767 \times 10^6$ , and  $I_v = 10.457 \times 10^8$ . The result is  $\sigma_A = -1602.66 \text{ N/mm}^2$ , noted as compression. The second part finds the coordinates for point B:  $u_B = z \cos \alpha + y \sin \alpha = (+2.5) \cos(70.36) + [-50 - 17.5] \sin(70.36)$  and  $v_B = -z \sin \alpha + y \cos \alpha = -(2.5) \sin(70.36) + [-50 - 17.5] \cos(70.36)$ . A note indicates that  $\sigma_B$  can be found by substitution, resulting in positive stress.

Stress at A is now minus Mu by Iu into v A plus Mv by Iv into u A, let us substitute this. One can see here Mu v is negative for the given problem. So, I get this stress as minus 08 Newton per mm square negative indicates it is compression, which is also inferred by us earlier.

Let us try to find similarly for point B, u B z cos alpha plus y sin alpha which will be minus 2.5 cos 70.36, you can see here is the point we are looking at (Refer Time: 14:15) plus 2.5 cos 70.36 plus minus 50 minus 17.5 of sin 70.36, similarly v B can be computed as minus sin alpha plus y cos alpha; so minus 2.5 sin 70.36 plus minus 50 minus 17.5 cos 70.36. Sigma B can be computed by substitution. So, I want you to do this and check, you will get tensile stress at the bottom.

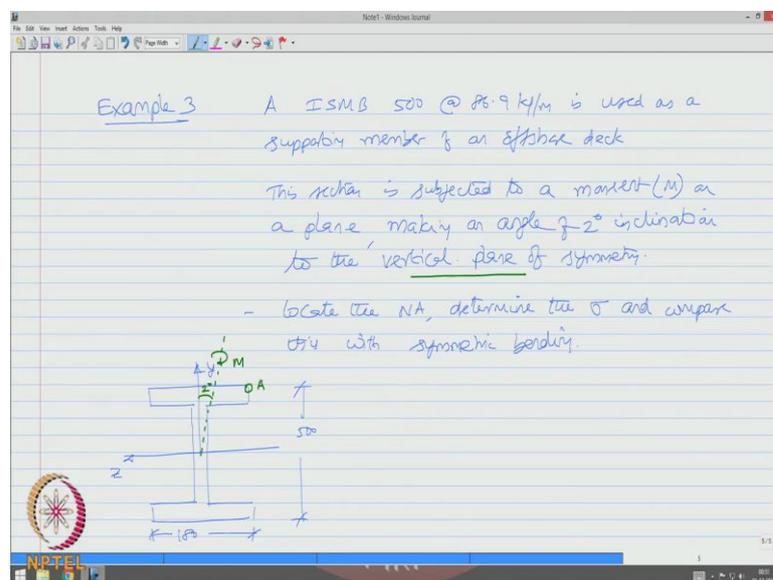
So, interestingly the problem of unsymmetrical bending is converted to simple uni-planar bending by locating the principle axis of inertia and the neutral axis and measuring the distances of extreme fibres from the neutral axis and then using the conventional flexural theory.

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To answer a very interesting question which generally people have intuitions to ask what would be the consequence of unsymmetric bending in design - the answer is very simple if a cross section is under stresses in the fibres will be magnified.

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To understand this we will take an example, let us say example 3, A ISMB 500 at 86.9 kg per meter is used as a supporting member of an offshore deck this section is subjected to a moment  $M$  on a plane which is not matching with the axis symmetry, but making an angle of 2 degrees inclination to the vertical plane of symmetry. So, locate the neutral

axis, determine the stresses and compare this with symmetric bending that is the issue. So, essentially there is an I section which has got 2 axis of symmetry, y axis and z axis, it is ISMB 500, the properties are given in the steel tables. It says that the loading is acting on a plane which is just 2 degrees to the vertical plane of symmetry and the moment is acting about this (Refer Time: 20:11).

We need to find the stresses; obviously, we need to know the maximum value, let us compare this at point A and see what is the difference in the stress.

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ISMB 500 @ 86.9 kg/m

$$M_u = M \cos 2^\circ$$

$$M_v = -M \sin 2^\circ$$

$I_z = 45218.3 \times 10^4 \text{ mm}^4$   
 $I_y = 1369.8 \times 10^4 \text{ mm}^4$   
 z, y planes of axes of symm  
 $\therefore I_{zy} = 0$

To find the  $\sigma_A$

$$\sigma_A = -\frac{M_u v}{I_u} + \frac{M_v u}{I_v}$$

$$= -\frac{(M \cos 2^\circ)(250)}{45218.3 \times 10^4} + \left\{ \frac{(-M \sin 2^\circ)(-90)}{1369.8 \times 10^4} \right\}$$

$$= M \left[ -0.55 \times 10^{-11} + 0.23 \times 10^{-11} \right]$$

$$= 0.32 \times 10^{-11} \text{ N/mm}^2$$

$\sigma_A$  (no induced load)  $\frac{M y}{I_z} = \frac{M}{45218.3 \times 10^4} \times 250 = -0.55 \times 10^{-11} \text{ N/mm}^2$   
 $\therefore \phi = \frac{0.55 - 0.32 \times 100}{0.55} = 41.8\%$

pt(A) will have Comp  $\sigma$

For a given section ISMB 500 at 86.9 kg per meter  $I_z$  is given 45218.3,  $I_y$  is 1369.8 10 power 4, zy planes are axis of symmetry since  $I_{zy}$  is 0, they are all principle planes of symmetry,  $M_u$  I want to resolve which is  $M \cos 2$  degrees and  $M_v$  is minus  $M \sin 2$  degrees.

Now we have the properties of the section, we have the moments, let us find the stress at point A, the point A is located here. So,  $\sigma_A$  is minus  $M_u$  by  $I_u$  into  $v$  plus  $M_v$  by  $I_v$  into  $u$ , I can now straight away say this minus because of this value. So,  $M \cos 2$  degrees that is  $M$  value that is  $M_u$  and  $I_u$  is as same as  $I_z$  because they are principle axis of inertia 2 18.3 into 10 power 4. Let us assume that  $M$  is in Newton millimeter and  $v$  is 250 because this section is 500 and this distance is 180. So, from this  $A_g$  it is going to be positive 250 plus  $M_v$  is minus  $M \sin 2$  degrees by  $I_v$  that is  $I_y$  1369.8 10 power 4 and  $u$  is minus 9 which makes this equation as  $M$  of minus 0.55 10 power minus 11 plus 0.23

10 power minus 11 which makes 0.22 M into 10 power minus 11 Newton per mm square.

Let us now consider sigma A dash that is no inclined load which is simply M by I into y which I can say M by 45218.3 10 power 4 into 250 that is instead of the load acting at an inclination the load acts vertical. So, I get this as minus 0.55 M 10 power minus 11 Newton per mm square. So, there is a percentage increase because of the inclined load as 0.55 minus 0.32 by 0.55 into 100 which is about 41.8 percent. So, even in the case of symmetrical axis sections when the load is not acting on the plane of symmetry, it causes unsymmetrical bending which enhances the stress by about 40 percent.

Now, let us locate the neutral axis for this problem.

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To locate Neutral axis

$$\sigma_x = -\frac{M_x(u)}{I_u} + \frac{M_y(v)}{I_v} = 0$$

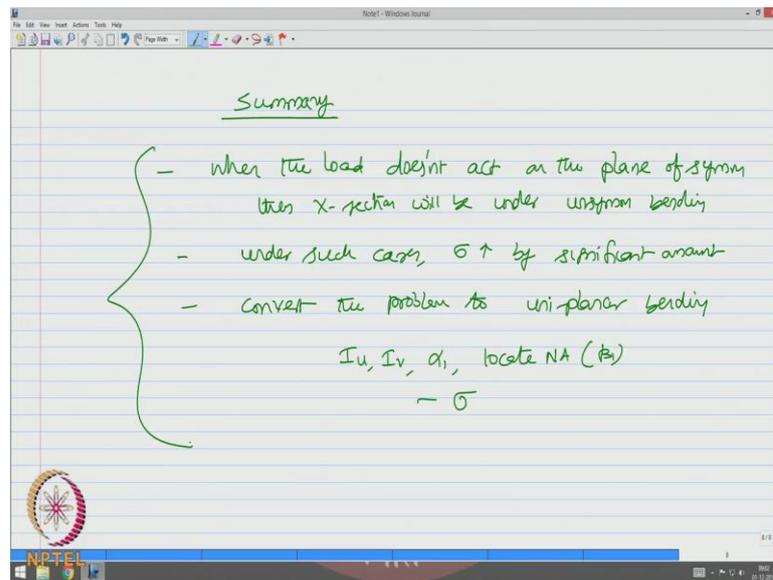
$$-\frac{M \cos^2 2^\circ}{45218.3 \times 10^4} (v) + \frac{-M \sin^2 2^\circ}{1369.8 \times 10^4} \times u = 0$$

$$\frac{v}{u} = -1.153 = \tan \beta$$

$$\beta = -49.06^\circ$$

We know to locate neutral axis, we need to equate the stress to 0. So, let us say sigma x is minus Iu into v plus Mv by Iv into u set as to 0. So, minus M cos 2 degrees by 45218.3 10 power 4 into v plus minus M sin 2 degrees by 1369.8 10 power 4 into u set as to 0, to locate the neutral axis which tells me by simplifying v by u comes to let us say M goes away v by u comes to let us say, this is negative, this also negative, this stays here which comes to minus 1.153 which we say as tan beta. So, which say beta is minus 49.06 degrees. So, locating back in the figure, they can mark the neutral axis as under an angle 49.6 degree. So, for the given load point, A is above a neutral axis. So, this will cause compression. So, point a will experience compressive stress.

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Friends, to summarize when the load does not act on the plane of symmetry, then the section will be under unsymmetric bend. Under this case, stresses are enhanced by significant amount. To solve the problem, convert the problem to uni-planar bending by locating the principle axis, find  $\alpha_1$ , then locate neutral axis by finding  $\beta_1$ , then find stresses so which we explained with couple of examples.

Thank you.