Offshore structures under special loads including Fire resistance
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Module - 2
Advanced Structural Analyses
Lecture – 24
Unsymmetrical Bending – II

Friends, in lecture 24 in module 2, we will continue with the discussion on Unsymmetrical Bending.

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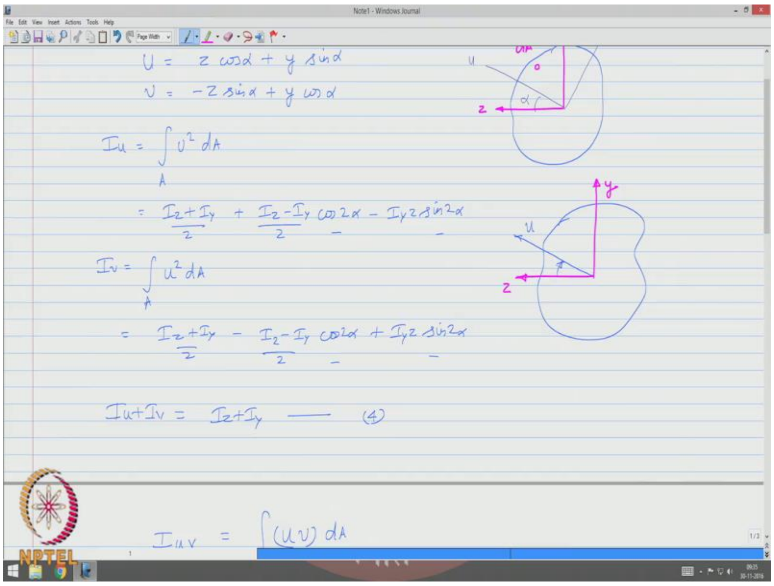

So, for a typical cross section whose z axis and the y axis are shown like this, we picked up sample area dA, we try to locate the principle axes of inertia u and v with angle alpha we said that U is z cos alpha plus y sin alpha and V is minus z sin alpha plus y cos alpha, we found I u as simply second moment of area of v square dA which amounts to I z plus I y by 2 plus I z minus I y by 2 cos 2 alpha minus I y z sin to alpha and I v is integral for the entire area with second moment of area u square dA, which was derived as I z plus I y by 2, minus I z minus I y by 2 cos 2 alpha plus I y z sin 2 alpha.

By looking at the above 2 equations of I u and I v, one can state away right a simple observation that adding these two I u I v, because a negative cos 2 alpha terms cos 2 alpha terms and sin 2 alpha terms will cancel, I will get simply I z plus I y which we said equation 4.

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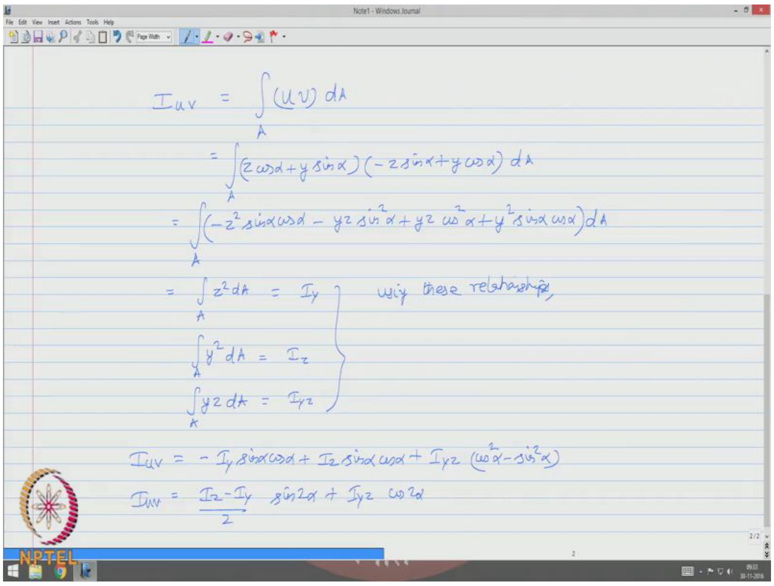

We need to also work out I x I z y are, let say I u v which is integral for the entire area U V dA, which is now z cos alpha plus y sin alpha into minus z sin alpha plus y cos alpha of dA. Let us do this multiplication for the entire area a minus z square sin alpha cos alpha, minus y z sin square alpha, plus y z cos square alpha, plus y square sin alpha cos alpha of dA integral for the entire area A.

We know integral z square dA will give lead to I y, integral y square dA will lead to I z, integral y z dA will lead to I y z, using this relationship for the entire area we can now write I u v will be minus I y sin alpha cos alpha, plus I z sin alpha cos alpha plus I y z cos square alpha minus sin square alpha, which can be now said as I z minus I y by 2 sin 2 alpha, because 2 sin alpha cos alpha is sin 2 alpha and cos square alpha minus sin square alpha is cos 2 alpha. So, plus I y z cos 2 alpha that is I u v.

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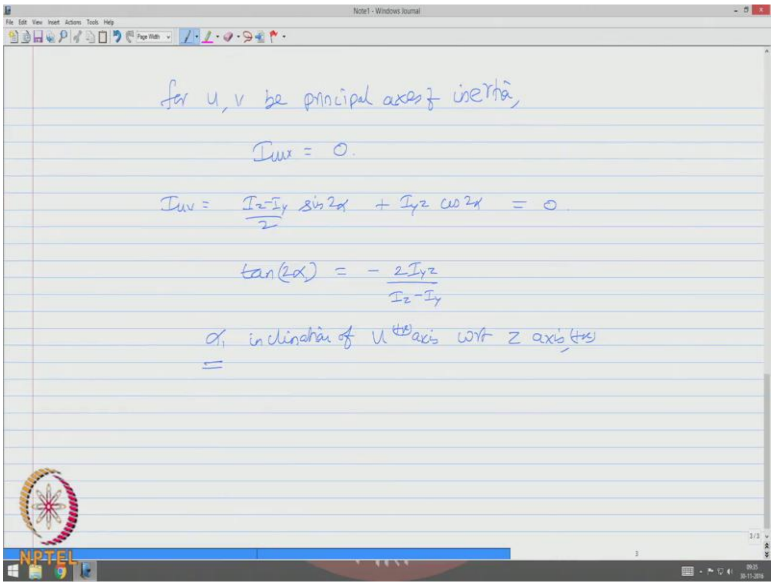

For U and V be principal axes of inertia, I u v should be 0 that is I z minus I y by 2 sin 2 alpha, plus I y z cos 2 alpha should be said to 0.

So, we rearranging you will know that tan 2 alpha is minus 2 I y z by I z minus I y; for our clarity because is alpha 1. So, alpha 1 is inclination of positive u axis with respect to z axis which is also positive. So, let us go back to this figure and mark alpha. So alpha once again explain, if this is my z axis this is my positive y axis and I want to mark the u positive, if this is my positive u axis this becomes my alpha 1, where alpha 1 is inclination of the positive u axis measured from positive z axis.

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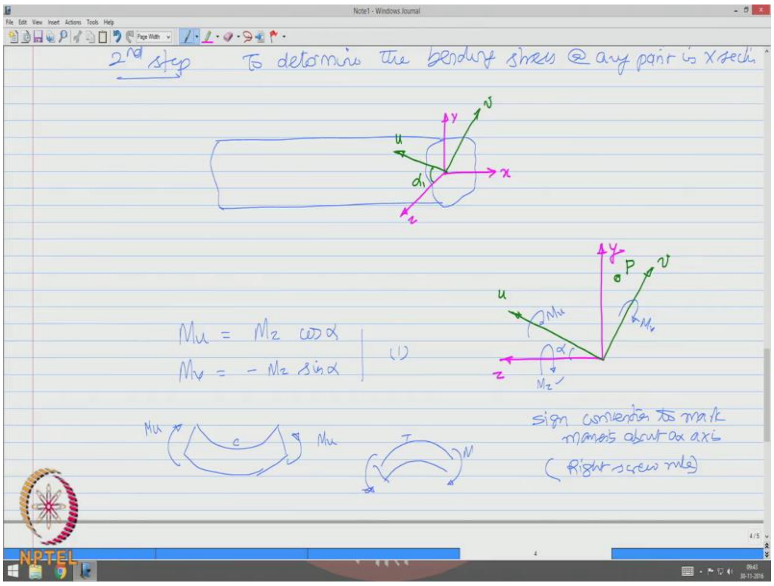

Having determined the principal axes moment of inertia, the next step is to determine the bending stresses at any point in the cross section.

So, let us say this is my being which has a cross section of some shape, we have the z axis and the y axis and the x axis mark like this, in this plane will also try to mark the u axis, we know this angle now is actually alpha 1, this may u axis and this becomes my v axis. So, let us pick this cross section and type mark only the axis which is y, which is z, let us also mark the u and v which is the point p, where I want to find the bending stress let us use to mark the moments sin convention, to mark the moments about an axis we will use right screw rule.

So, let a (Refer Time: 11:35) phase the positive direction of axis and try to screw along the axis, like driving a screw with the help of a screw driver, you will notice that this will become my direction of M z, this will become my direction of M v and this will become direction of M u and so on. We already have this angle, we call this for our understanding is alpha because alpha 1 is for our understandings clarify this between 2 positive axes actually the angle between the z and the u axis is alpha. So, from this figure one can write that M u is M z cos alpha and M v if you try to resolve this moment about v you get this as minus M z sin alpha.

Because this M z when you try to resolve along the v axis; comes to be in the negative sin. So, equation number let say 1.

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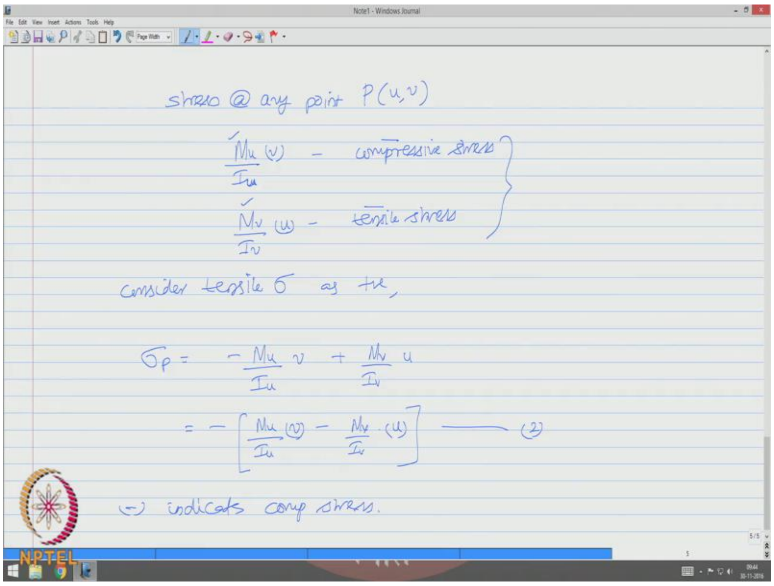

So, stress at any point which is described with the coordinates u v we are not working on x z y any more, your transforming the problem unsymmetrical bending, the uniplanar bending, so instead of z y axis you will work out with u v. The advantage is if I am able to locate the principal axes of inertia u v and try to find the stress about this with the coordinates, I can use a conventional flexure formula, which can be M u by I u into v, which will cos compressive stress and M v by I v into u which will cos tensile stress.

Let us try to understand how we look these natures the stresses. So, let us look at M u which will create a bending of this nature. So, this will create bending of this nature which creates compression at the point in the positive coordinates. We look at M v; M v is applying moment of this order, which will create tension yeah of M v. So, M u will result in compression and M v will result in tension. Now let us consider tensile stresses as positive. So, stress that point p is actually minus M u by I u into v plus M v by I v into u, instead I can even say M u by I u into v minus M v by I v into u equation number 2.

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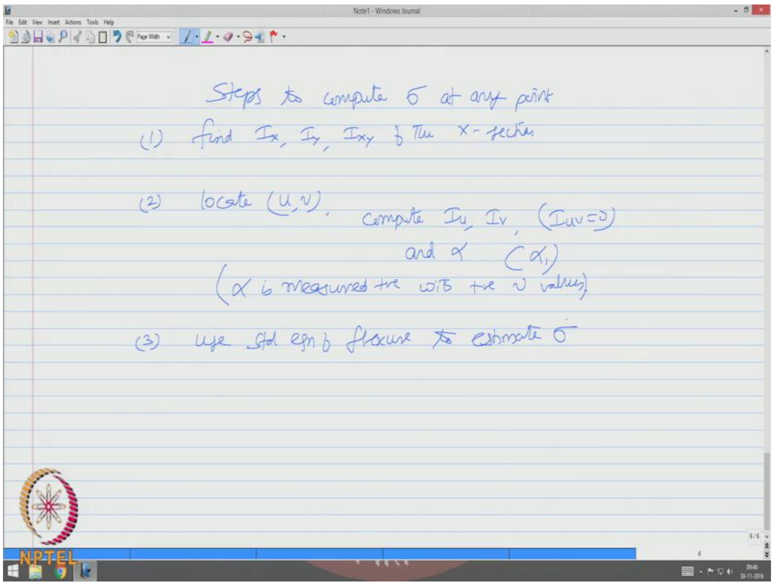

Negation indicates compressive stress; let us summarize and then do a problem what are the steps to compute stress at any point under unsymmetric bending, when the cross section is subjected to unsymmetric bending one can follow the flowing steps to estimate the stresses; find I x, I y and I x y of the cross section, which are from the standard equations for a given cross section, to locate the principal axes of inertia u and v. So, compute I u, I v and we know I u v is 0 and alpha to be very clear alpha 1, please understand alpha is measured positive with positive v values.

Then once you locate the principal axes of inertia, use the standard equation of flexure to estimate the bending stress. So, friends let us take an example problem to understand how to locate these principle axes of inertia, so that by locating this axes of inertia and marking neutral axis with respect to this I can use the conventional equation of flexure to obtain bending stress and any point in the cross section.

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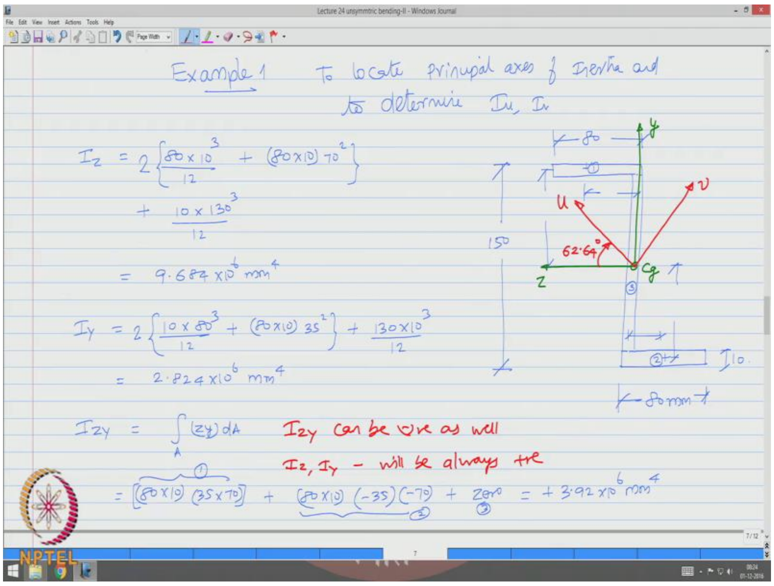

So, we will taken example; the example is to locate the principal axes of inertia and to determine I u, I v for the given section. You will take this is an example, we will take a z section, the dimension section are given like this is 80 millimeter or dimensions or a millimeters; the thickness is 10 millimeter.

The overall depth of the web is 150, the section as uniform thickness through and through of 10 millimeter and the tough range is also of 80 millimeter. So, this section has 2 axis symmetry that is locate them and say this is my Z axis and this is my Y axis and this becomes my Cg both these are axis of symmetry, but they are not principle axis of inertia we need to locate them for this given problem. Let us first try to find the moment of inertia of the section about these two axes I Z we can do this by using the conventional principal axes theorem. So, I call this as piece 1, this as piece 2 and the wave as piece 3, each one of them have the local C g which you need to estimate the coordinates to find the moment of inertia about the Z and Y axis.

So, I Z for member 1; 80, 10 cube by 12, plus area into k square. So, this distance is not we are not interested in which will be 75 minus 5 which is 70 square, this is for member 1 or that of flange plus let us do it for the bottom flange as multiplying this by 2, plus let us do it for the web which is member 3 which is 10, 130 cube by 12. So a substitute this and get this value as 9.684, 10 power 6. Similarly I can find I y for member 1, 10, 80 cube by 12 plus 80 into 10. Now I would like to find this distance which will be 40 minus 5 which is 30 phase square this is for member 1 and 2.

So, multiply this by 2, plus for the web it is 130 10 cube by 12, which now gives me 2.824 10 power 6, m m 4. Now the tricky part we would like to get the product moment of inertia I z y. I z y by definition is z y dA integrating for the entire area. So, we are divided the section into 3 components, we would like to find the z n y coordinates of all the 3 components and some them up to get I z y. Please note I z y can be negative as well, but I z and I y will be always positive. Now let us try to find I z y. So, I z y now going to be the top flange 80 into 10, the z and y coordinates of this flange, the z coordinate will be positive for this case, it is plus 35, y coordinate is also positive 70, this for member 1.

For the bottom flange area will be again same, but the z coordinate will be the dimensional same, but it is minus 35 because it is in opposite direction of z and similarly this distance is opposite of y so minus 70, this is for the member 2 and for the web the z wave coordinates will match and therefore, it is 0 for member t3. Summing up we get I z y as positive 3.92 10 power 6 mm 4.

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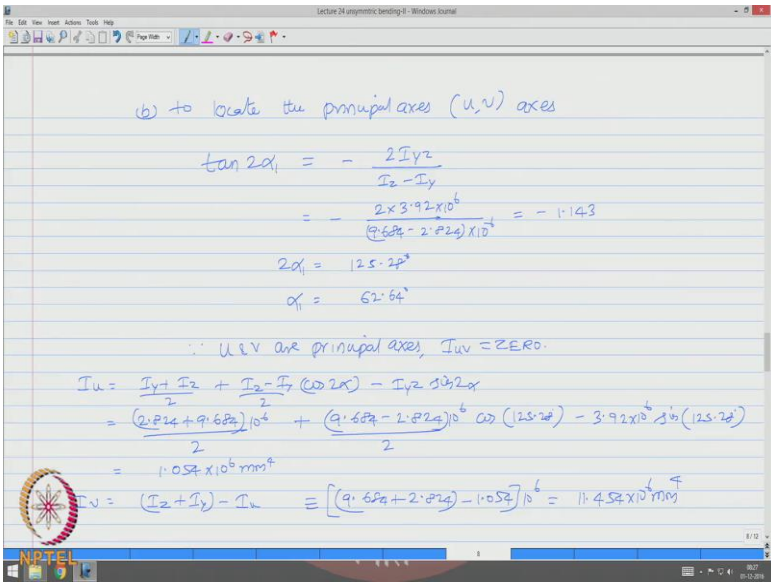

Now, we need to locate the principal axes of inertia, that is u and v axes. So, using the same equation what we derive, we know that tan 2 alpha 1 is minus 2 y z by z minus y, substituting we get 2 of 3.92 10 power 6 by 9.684 minus 2 .824 of 10 power 6.

Which gives me minus 1.143, which now tells me 2 alpha 1 is 125.28 degrees or alpha 1 is 62.64 degrees. So, let us go back to this figure I am locate the principal axes, this is my 62 degrees, this is my U and this becomes my V and this angle becomes my 62.64 degrees. Say is expected that since u and v are principal axes, I u v will be 0, let us try to find I u from the equation I y I z by 2, plus z minus y by 2, cos 2 alpha minus I y z sin 2 alpha, substituting 2.824 plus 9.684, 10 power 6 by 2, plus 9.684 minus 2.824, 10 power 6 by 2, cos 125.28 degrees, minus 3.92 10 power 6, sin 125.28 degrees, which gives me the net value as 1.054 10 power 6 mm 4.

Similarly, I v this can be easily solved by sampling saying I z plus I y minus I u. So, which we say 9.684 plus 2.824, minus 1.054 of 10 power 6, which gives me 11.454 10 power 6 mm 4. So, for a given section which has axes of symmetry, we can easily locate the principal axes and find the moment of inertia about this principal axes, that is example 1 as described.

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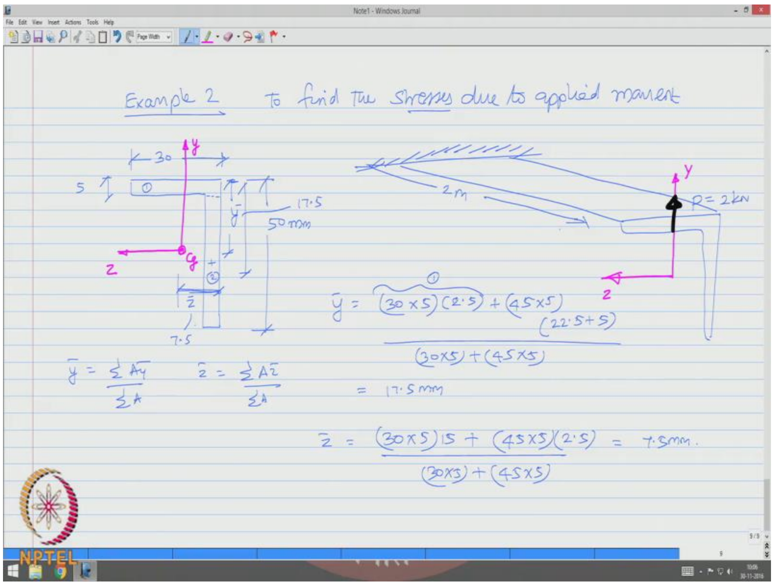

So, let us talk about example 2; to find the stresses due to applied moment, let us take an example of you cantilever section, whose cross section is an angle which is fixed is what 2 axis, let us marked them as Z and Y; we will also apply there force which is P equal to 2 kilo newton, which is over a span of 2 meters.

Let us draw this cross section separately on write down the geometric properties, let say this dimension is 50 all in millimeters, this dimension is 30 and the thickness through 1 through is 5 millimeter, let us mark these axis as Z and Y and this is my C g. So, from the first principals, one can easily calculate the value of Y bar and Z bar. So, let us do that quickly to understand. So, y bar is simply sigma A y bar by sigma A and z bar a simply sigma A z bar by sigma A; applying this equations let us try to find y bar. So, let us talk about the top flange. So, let us say the top flange is taken like this. So, 30 into 5 that is the area, we are measuring it from the extreme end therefore, the distance is 2.5 millimeter, plus let us talk about this flange let say this is piece number 1 and this is piece number 2, this is for the first piece, the second piece let say 45 into 5 is the area and the distance of this from here.

So, 45 is 22.5 plus 2.5. So, I should say 22.5 plus 5 divided by area 30 into 5 plus 45 into 5, which gives me y bar as 17.5 millimeters. Now to locate z bar we do the same thing, let us measure z bar from the extreme right that is my origin, extreme right I am measuring z bar from here from here. So, the top flange 30 into 5 is the area, when the distance will be 15 and the vertical plan that is piece number 2. 45 into 5 is the area and the distance is 2.5, the total area 30 into 5 plus 45 into 5, which can be 7.5. So, y bar is 17.5 and z bar is 7.5. So, with this data let us compute I z and I y it is doing the cross section again.

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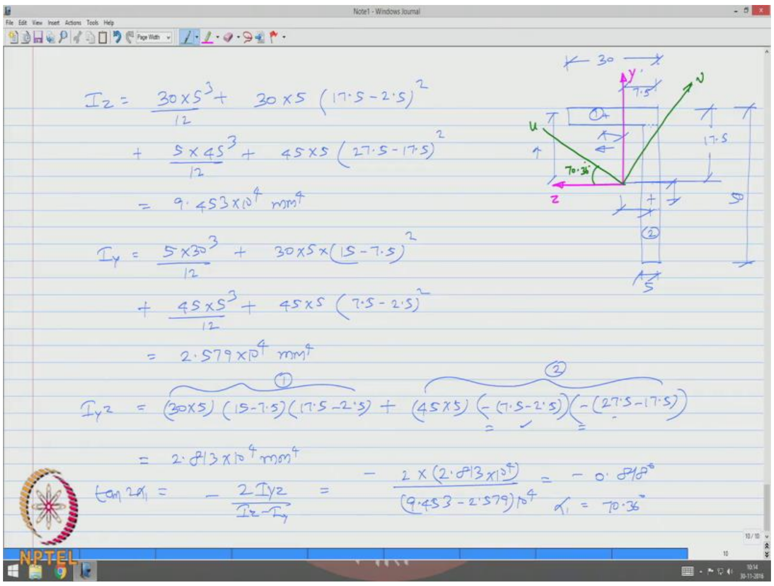

Let us locate the centre over axis z and y, this is 17.5 and this y is 7.5, on the dimensions this is 50, this is 10 on this is 13. So, from first principles I can find I z that is 30 into 5 cube by 12 plus parallel axis theorem, which will be 17.5 minus 2.5 whole square, plus 5 oh this is 5 (Refer Time: 38:30) 5 into 45 cube by 12, plus again parallel axis theorem 45 into 5 into 27.5 minus 17.5 the whole square.

Which gives me 9.453 into 10 power 4 mm 4. I y 530 cube by 12, now the piece divided like this plus 30 into 5 into 15 minus 7.5 the whole square, plus 45 into 5 cube by 12, plus 45 into 5 into 7.5 minus 2.5 the whole square, which gives me 2.579 into 10 power 4 mm 4; let us find out I y z, which will be let say for the top flange piece number 1; 30 into 5 the positive z coordinate, which is 15 minus 7.5, which is positive in other (Refer Time: 40:28) likes here, which is positive into 17.5 minus 2.5, that is this distance which is also positive in y, that is for the top one piece number 1, plus for piece number 2. 45 into 5 is the area see is somewhere here I am looking for this distance, which is negative in z axis.

So, I should say 7.5 minus 2.5 that negative similarly I am looking for this distance which is again negative (Refer Time: 41:35) y, which is minus of 27.5, minus 17.5. So, that is for the piece number 1; and this is for the piece number 2. This is piece number 2. So, piece number 2 has got z value negative and y value negative. So, this is 7.5, this is 7.5 and this distance is 2.5, that is what we have done here and this distance is 45 by 2 and from here is 27.5 and the 17.5, this distance minus this is what we have here.

So, I y z have to calculation comes to 2.813, 10 power 4 mm 4. So, now, I can quickly locate tan to alpha 1, which is minus of 2 y z by z minus y, which is minus of 2 of 2.813 10 power 4 by 9.453, minus 2.579 10 power 4, which is actually minus 0.818 degrees, which amounts to alpha 1 as 70.360 degrees, they u and v this (Refer Time: 43:43) is 70.36 degrees.

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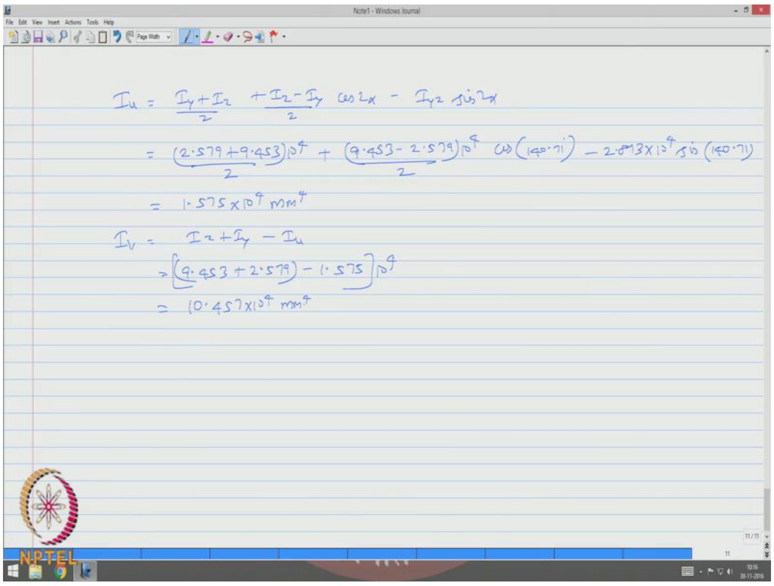

Now, 1 can find I u as I y I z, I z I y cos 2 alpha, I z sin 2 alpha; 1 can substitute for the values cos 2 alpha. So, 140.71 degrees, minus I y z that is 2.813 sin 140.71, which gives I u as 1.575 10 power 4 and I v is I z plus I y minus I u. So, its 9.453 plus 2.579, minus 1.575 of 10 power of 4, which gives me 10.457 10 power 4 mm.

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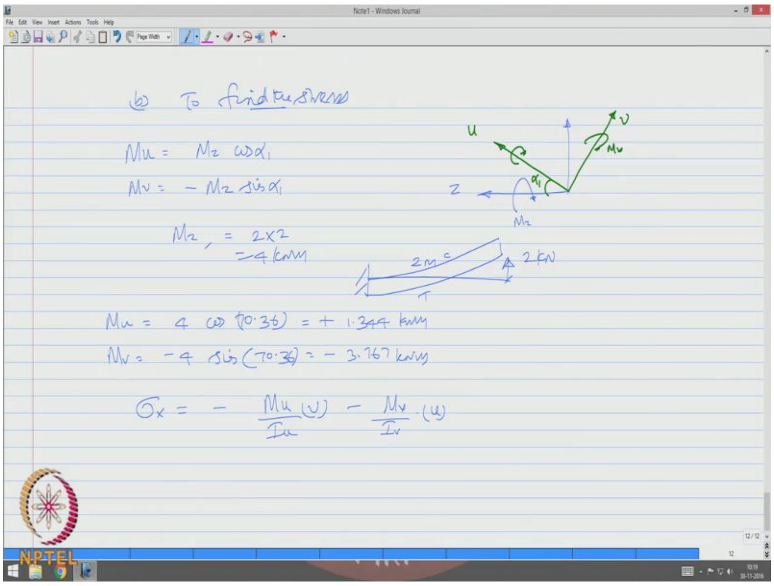

So, now I want to find the stresses. So, we have M z which is creating moment, we have u and v axis which is alpha, which will have M u and M v. So, now, I can write M u will be M z cos alpha 1 and M v will be minus M z sin alpha 1. So, let say M u is going to be

M z; let say what is M z? Go back to this equation the original problem. So, cantilever with 2 kilo newton for a span of 2 meters, it will make the beam to bend this way. So, tension and the bottom and compression the top, the moment is going to be actually 2 into 2 which is 4 kilo newton meter. So, now, M u it is actually 4 of cos 70.36, which comes to plus 1.344 kilo newton meter and M v is minus 4 sin 70.36 which is minus 3.767 kilo newton meter.

By a transform the entire problem from the z y plane to u v plane. So, the stress at any point x should be generally given by M u by I u into v, minus M v by I v into u, let us try to extend this discussion for the next lecture and estimate the stresses within example and one more example which will continue.

Thank you very much.