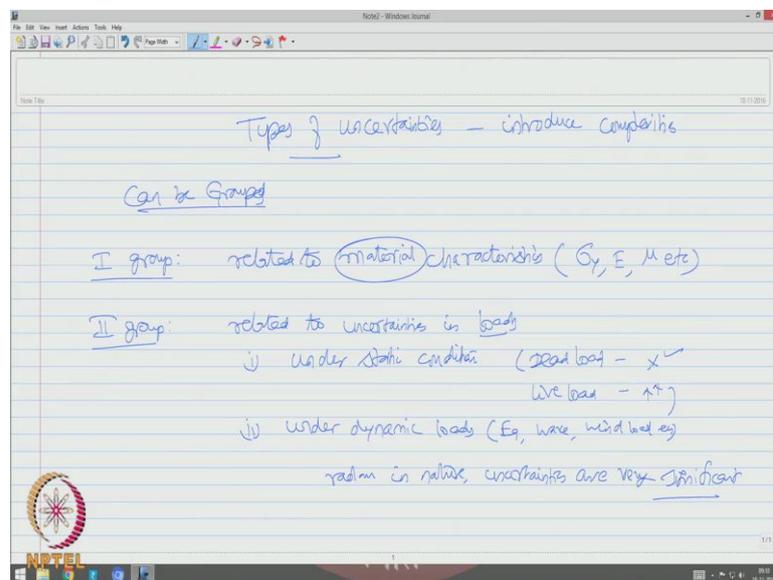


Offshore structures under special loads including Fire resistance
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Lecture – 16
Uncertainties

Friends, welcome to the 16th lecture under the NPTEL course on offshore structures and the special loads. In this lecture we are going to talk about some uncertainties involved in the analysis and design very briefly.

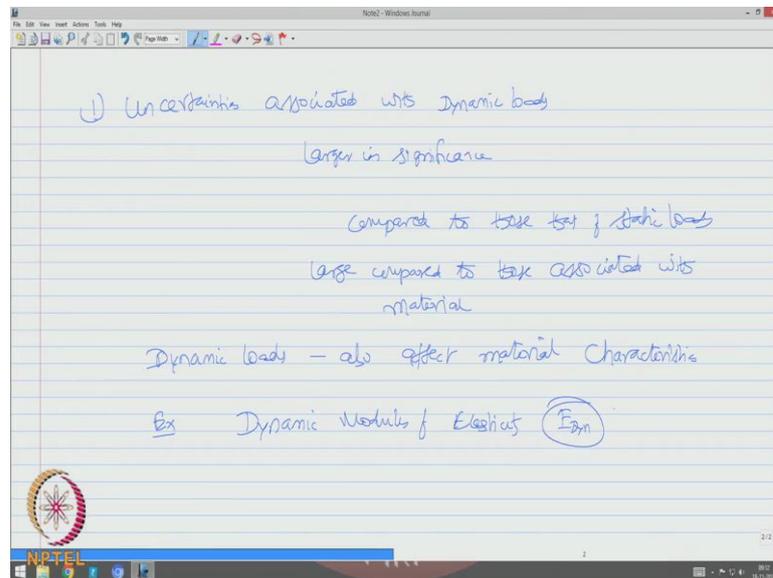
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There are different types of uncertainties which actually introduce complexities in analysis and design, these uncertainties can be grouped let us say the first level or the first group these are uncertainties which are related to material characteristics like (Refer Time: 01:28) young's modulus, poisons ratio etcetera. The second group of uncertainties are related to uncertainties in loads; one is material other is loads. This can be further sub divided in 2 routes, let us say under static conditions where we say for example, dead load, it may not have any major uncertainty so it can be omitted. Live load it may have large extent of uncertainty depending upon the designer choice and so on.

The second sub category under the second group could be under dynamic loads, under dynamic loads such as the earthquake load, wave load, wind loads etcetera, which are random in nature uncertainties are very high or I should say very significant.

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Now, uncertainties associated with dynamic loads or generally larger in significance compared to those that of static loads that is a first observation, they are also larger compared to those associated with material; interestingly dynamic loads also affect material characteristics that is a very interesting statement, let us say for example, will talk about dynamic modulus of elasticity $E_{dynamic}$.

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E_{dyn} is ratio of σ to ϵ under vibratory conditions
- This is computed either from free/forced vibrates test
in shear
Compression (ϵ_s)
Elongation
This property is more prominent for Visco-elastic materials
In purely elastic materials, σ - ϵ occurs in phase
So that the response to σ (i.e. strain)
occurs simultaneously with each other
In Visco-elastic materials, σ - ϵ will have a phase lag
- strain usually lags stress by $(\pi/2)$ rad

$E_{dynamic}$ is ratio of stress to strain under vibrations or vibratory conditions. How they are computed? This is computed either from free forced vibration test in shear compression or elongation. Generally this property is more prominent for Visco-elastic material. In purely elastic materials, stress and strain occurs in phase, so that the response to stress that is strain occurs simultaneously with each other, but that is true only the material is purely elastic, but in Visco-elastic material, stress and strain will actually have a phase lag, strain usually lags stress by $\pi/2$ radian; so ϕ which is the phase lag is actually $\pi/2$.

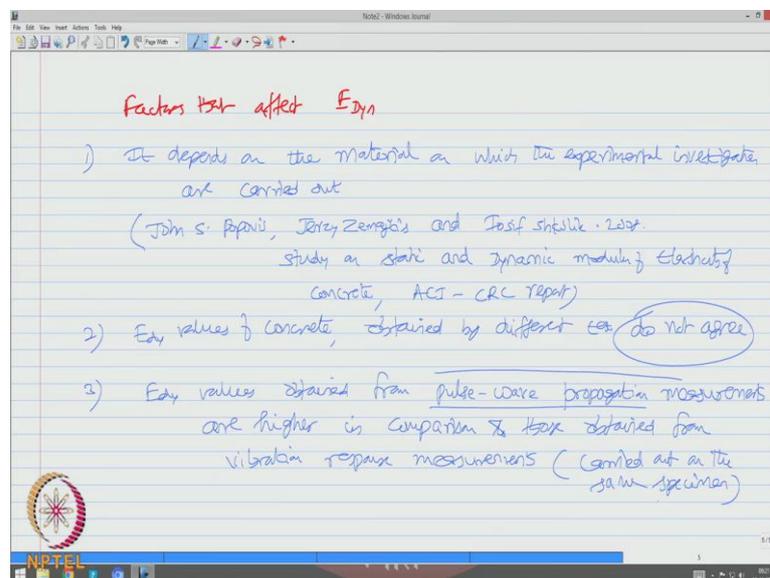
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ϵ lags σ by $(\pi/2)$ rad
 ϕ (phase lag) = $\pi/2$
 $\epsilon = \epsilon_0 \sin(\omega t)$
 $\sigma = \sigma_0 \sin(\omega t + \phi)$ (1)
 $\omega = 2\pi f$ f = frequency of oscillator
 ϕ = phase lag
Modulus of Elasticity - describes/defined as σ - ϵ behavior under static loads
Dynamic modulus of Elasticity describes behavior under cyclic (ϵ_s) vibratory loads

So, strain is $E_0 \sin \omega t$, where as stress is $\sin \theta \sin \omega t + \phi$ and we know ω is $2 \pi f$, where f is a frequency of oscillation and ϕ is called as phase lag. Generally modulus of elasticity is described or let say defined as stress strain behavior under monotonic loading, where as dynamic modulus of elasticity describes behavior under cyclic or vibratory loading.

So, friends in a case where the load process is highly dynamic in natural random, which has got lot of uncertainties, we should have a thorough idea about the dynamic modulus of elasticity of the material we should used in the design.

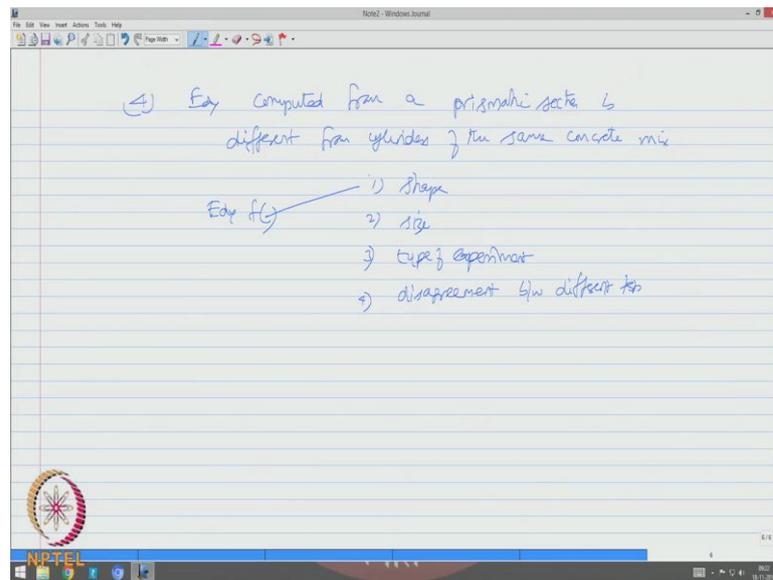
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Let us ask a question, what are the factors that affect dynamic modulus of elasticity? One, it actually depends on the material on which the experiment or experimental investigations are carried out, as very clearly stated by John. S. Popovics, Jerzy Zemajtis and Iosif Shkolniku. 2008 study on static and dynamic modulus of elasticity of concrete, which is an ACI - CRC report.

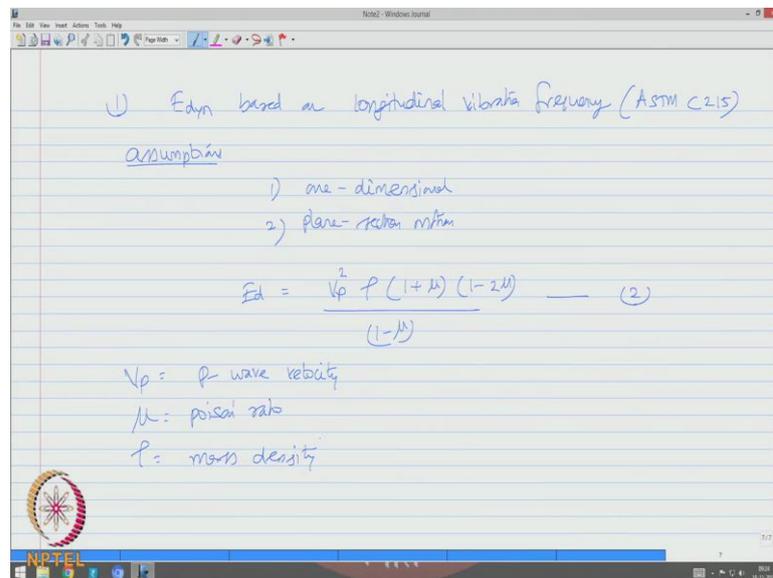
The second factor is that the dynamic modulus of elasticity values of concrete, obtained by different tests do not agree there is a wide variation. The dynamic modulus of elasticity values obtained from pulse-wave propagation measurements are higher in comparison to those obtained from vibration response measurements. In fact, dynamic modulus value also depends on the type of experiment you conduct, when even carry out on the same specimen, the test methods give you different results.

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The fourth factor dynamic modulus of elasticity computed from a prismatic section is different from cylinders of the same concrete mix. So, E_{dy} is a function of 1) shape, 2) size, 3) type of experiment and fourth a strong disagreement between different tests. So, there is a wide variation. Let us see one by one by different tests how E_d can be calculated.

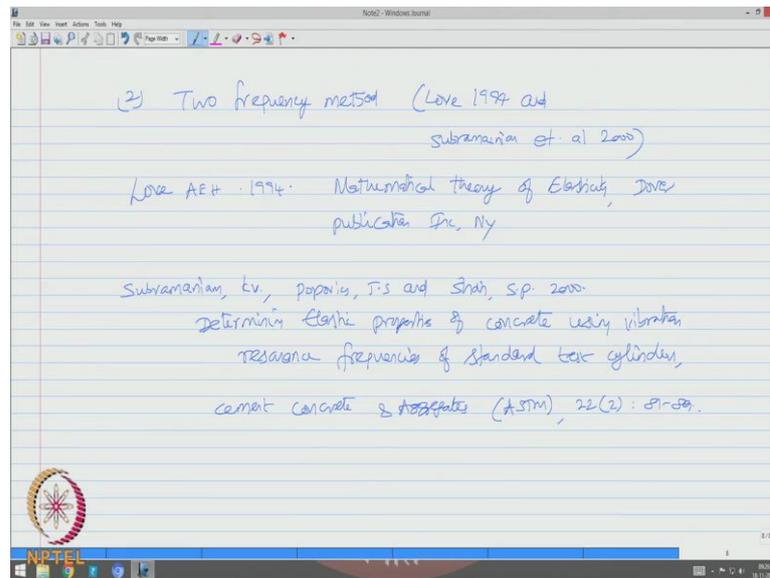
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Let us say E_{dyn} based on longitudinal vibration frequency. Please understand dynamic modulus depends very strongly on the type of (Refer Time: 15:06) test you perform. This is as per ASTM C215, it has got some basic assumptions, the test is performed on one-

dimensional specimen; the test shows plane section motion only no bending. So, E dynamic is given by $V_p^2 \rho (1 + \mu) / (1 - 2\mu)$ by $(1 - \mu)$. I call this equation number 2, where V_p is called the P wave velocity, μ is called Poisson's ratio and ρ is the mass density.

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There is another method by which can compute dynamic modulus of elasticity; this is two frequency methods, which is proposed by Love 1994 and Subramanian et al 2000; so Love AEH 1994, Mathematical theory of Elasticity, Dove publication, New York. Subramanian K V, Popovics J.S and S.P shah 2000, determines elastic properties of concrete using vibration resonance frequencies of standard test cylinders, cement concrete and aggregates ASTM, 22 2: 81-89.

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Acc to Two-frequency method,
dynamic poissons ratio is estimated

$$\mu_d = A_1 \left(\frac{f_2}{f_1}\right)^2 + B_1 \left(\frac{f_2}{f_1}\right) + C_1 \quad \text{--- (3)}$$

μ_d = dynamic poissons ratio
 f_1 = first longitudinal freq (Hz)
 f_2 = second resonance freq (Hz)
 A_1, B_1, C_1 = constants - dimensions of the cylinder

So, according to two-frequency method, dynamic poissons ratio is estimated, which is given by $A_1 \cdot f_2^2 / f_1^2 + B_1 \cdot f_2 / f_1 + C_1$ where μ_d is called dynamic poissons ratio; f_1 is the first longitudinal frequency in hertz, f_2 is the second resonance frequency in hertz A_1 , B_1 and C_1 are actually constants, which depends upon the dimensions of the cylinder.

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$$E_{dyn} = 2 (1 + \mu_{dyn}) P \left(\frac{2\pi f_1 R_0}{f_n}\right)^2$$

where R_0 = radius of the cylinder
 $f_n = A_2 (\mu_{dyn})^2 + B_2 (\mu_{dyn}) + C_2$
 A_2, B_2, C_2 = constants, depend on dimension of the cylinder
 (Subramanian et al. 2009)

$$E = 0.83 E_{dyn}$$

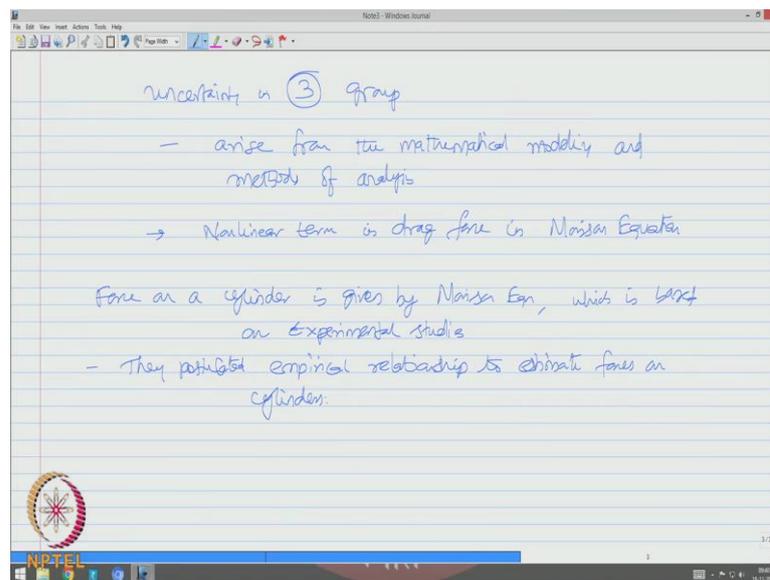
$$E = 1.25 E_{dyn} - 19 \quad \text{--- (Neville, 1997)}$$

Neville A.M. 1997. Properties of Concrete, 4th Ed. John Wiley, NY.

Once I know the dynamic poissons ratio, I can find dynamic modulus of elasticity as twice 1 plus μ_d dynamic rho, $2 \pi f_1 R_0 / f_n$ dash square where, R_0 is the radius of

the cylinder and f_n dash is given by $A^2 \mu$ dynamic square, plus $B^2 \mu$ dynamic, plus C^2 where A^2 , B^2 , C^2 are constants and they depend on dimensions of the cylinder as given by Subramanian et al in 2000. Now interestingly let us have a relationship between the conventional Young's modulus and the dynamic Young's modulus which is connected by this relationship or $1.25 E$ dynamic minus 19 is E as given by Neville 1997; so Neville A.M 1997 properties of concrete, 4th edition John Wiley, Newyork.

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Having said this, let us now understand uncertainty in third group. Third group essentially arise from the mathematical modeling and methods of analysis. So, interestingly let us talk about the non-linearity or non-linear term in drag force in Morison equation. Morison equation is used to determine the hydrodynamic force on offshore members, force on a cylinder is given by Morison equation which is based on experimental studies, they actually postulated an empirical relationship to estimate forces on cylinders given by Morison J.R O' Brain, M.P, Johnson, J.W and Schaaf, S.A in 1950.

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Morris, J.R., O'Brien, M.P., Johnson, J.W. and Schaaf, S.A. 1950.
 Force exerted by surface waves on pile, petroleum transactions,
 AIME, 189.

$$F(z,t) = C_I \dot{v}_x + C_D v_x |v_x| = F_I + F_D \quad (1)$$

This force will be acting in the direction of propagation of wave
 water particle velocity and acceleration are evaluated (Airy's theory)
 @ the centroidal axis of the cylinder

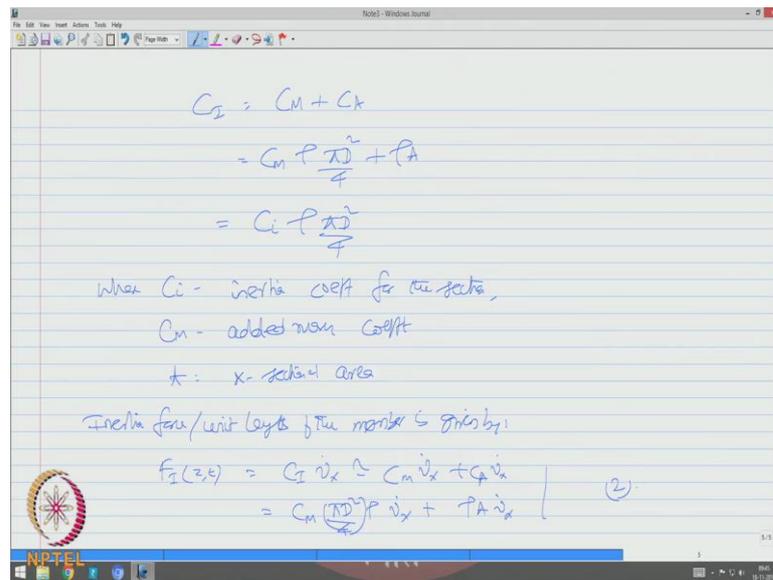
C_I consists of 2 terms

- 1) hydrodynamic mass contribution
- 2) due to variation in pressure gradient within the accelerating fluid.

Force exerted by surface waves on pipes petroleum transactions AIME 189. So, according to them F of z t is $C_I \dot{v}_x$ plus $C_D v_x |v_x|$ which is inertia term plus (Refer Time: 26:52) term. This force will be acting in the direction of propagation of wave that is in the forward direction.

Water particle velocity and acceleration are evaluated may be for example, using Airy's theory at the centroidal axis of the cylinder. C_I consist of 2 terms; one is the hydrodynamic mass contribution and the other term is arising due to variation in pressure gradient between of let us say within the accelerating fluid. The C_I has got 2 terms C_A which is $C_m \rho \pi D^2$ by 4, plus ρa which can be said.

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$$C_D = C_M + C_A$$

$$= C_M \rho \frac{D^2}{4} + \rho A$$

$$= C_i \rho \frac{D^2}{4}$$

where C_i - inertia coeff for the section,
 C_M - added mass coeff
 A - cross-sectional area

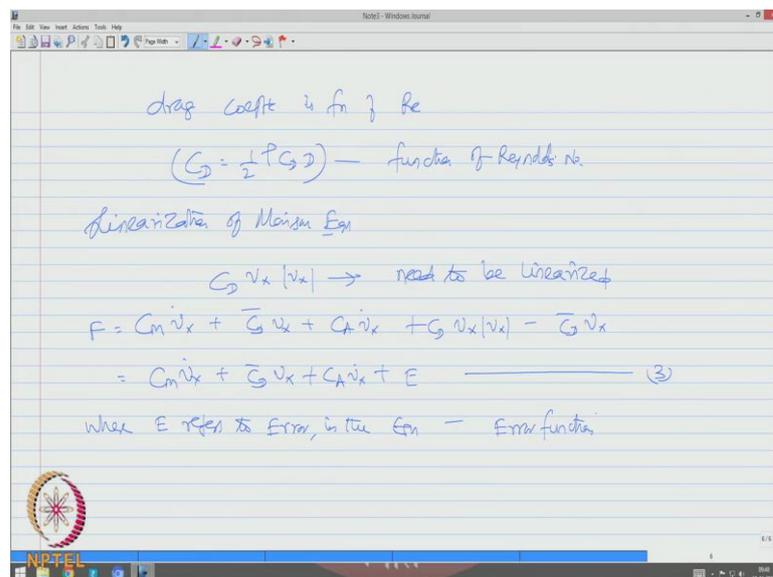
Inertia force/unit length of the member is given by:

$$F_I(z,t) = C_i \dot{v}_x = C_M \dot{v}_x + C_A \dot{v}_x$$

$$= C_M \left(\frac{\rho D^2}{4} \right) \dot{v}_x + \rho A \dot{v}_x \quad (2)$$

As C_i is $\rho \pi D^2$ by 4 where, C_i is the inertia coefficient for the section, C_M is the added mass coefficient, A is the cross sectional area therefore, inertia force per unit length of the member is given by F_I , again a specific location z it is $C_i V \dot{x}$ is approximately $C_M V \dot{x}$, plus $CA V \dot{x}$, which can be $C_M \pi D^2$ by 4 $\rho V \dot{x}$, plus $\rho A V \dot{x}$ - let us call this as equation number 2.

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drag coeff is fn of Re

$$(C_D = \frac{1}{2} \rho C_D D^2) - \text{function of Reynolds no.}$$

Linearization of Moirise Eqn

$$C_D v_x |v_x| \rightarrow \text{need to be linearized}$$

$$F = C_M \dot{v}_x + C_D v_x + C_A \dot{v}_x + C_D v_x |v_x| - C_D v_x$$

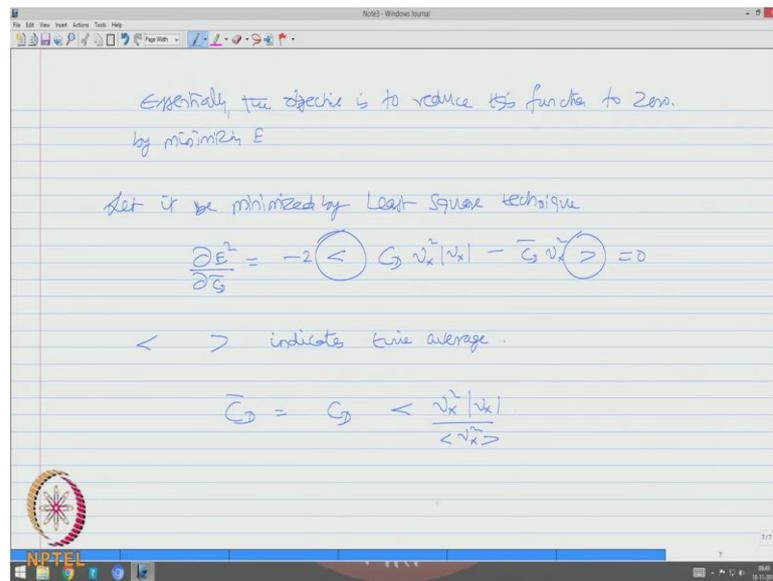
$$= C_M \dot{v}_x + C_D v_x + C_A \dot{v}_x + E \quad (3)$$

where E refers to Error, in the Eqn - Error function

We also know that the drag coefficient is function of Reynolds number. So, C_D half rho C_D dia is actually also a function of Reynolds number.

When you apply this in Morison equation, it gets squared off. So, let us talk about linearization of the Morison equation itself because Morison equation as a term $C D V x V x$. So, this need to be linearize, let say F is $C m V x \dot{}$, plus $C D \bar{V} x$, plus $C A V x \dot{}$, plus $C D V x$ minus $C D V x$, which can be $C m V x \dot{}$ plus $C D \bar{V} x$, plus $C A V x \dot{}$ plus E where E refers to error in the equation which is nothing but a specific error function.

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So, essentially the objective is to reduce this function to zero by minimizing E .

Let it be minimize by least square technique E square by dou $C D$ should be minus 2 of less than let say $C D V x V x$ minus, $C D V x$ square. So, the symbol indicates time average, $C D \bar{V} x$ is $C D V x$ square $V x$ by $V x$ square.

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For a Gaussian process, with zero mean

$$|U_x| = \sigma V_x$$

$$\langle |U_x| \rangle = \sqrt{\pi} \sigma V_x$$

$$\langle V_x^2 |U_x| \rangle = \sqrt{\pi} \sigma^3 V_x$$

Hence $\bar{C D} = \frac{C D \sqrt{\pi} \sigma^3 V_x}{\sigma V_x} = C D \sqrt{\pi} \sigma V_x$

It is necessary that distribn of V_x is to be determined (σV_x) then, $\bar{C D}$ can be computed

Hence for a Gaussian process is zero mean V_x square, is σV_x square and V_x time average will be $\sqrt{\pi} \sigma V_x$ and time average of V_x square V_x can be $\sqrt{\pi} \sigma V_x$ cube. Hence $\bar{C D}$ is $\bar{C D} \sqrt{\pi} \sigma V_x$ cube by σV_x square which can be $\bar{C D} \sqrt{\pi} \sigma V_x$. So, it is important and necessary that distribution of V_x is to be determined because we need σV_x , then only $\bar{C D}$ can be computed.

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For a surface elevation, η be Gaussian distribute

v (water particle vel) will also follow Gaussian distribute

Hence linearized Moirson En is given by

$$F(z,t) = \zeta V_x + C D \sqrt{\pi} \sigma V_x \quad (E)$$

For surface elevation η be a Gaussian distribution, V that is water particle velocity will also follow Gaussian distribution. Hence linearized Morison equation is given by F_z of t , $C_i V_x \dot{\eta}$, plus $C_D \sqrt{8 \pi \sigma} V_x$, V_x equation 4. So, that is the linearized term what we have, which is again an approximation of one important drag non-linearity coming down from the equation.

So, friends in this lecture we discussed about 2 levels of different uncertainties, one from the load side which is the drag non-linear term in the Morison equation. The second one is the dynamic modulus of elasticity, which is actually the relationship between stress and strain under vibratory conditions. So, we have seen how we can estimate them more appropriately by the given empirical relationship of different researchers in this lecture.

Thank you very much.