Offshore structures under special loads including Fire resistance
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Lecture – 15
Response Spectrum- II

Friends, we will continue with the 15th Lecture where we will discuss more in detail about the response spectrum under the NPTEL course titled Offshore Structures under special loads including Fire Resistance.

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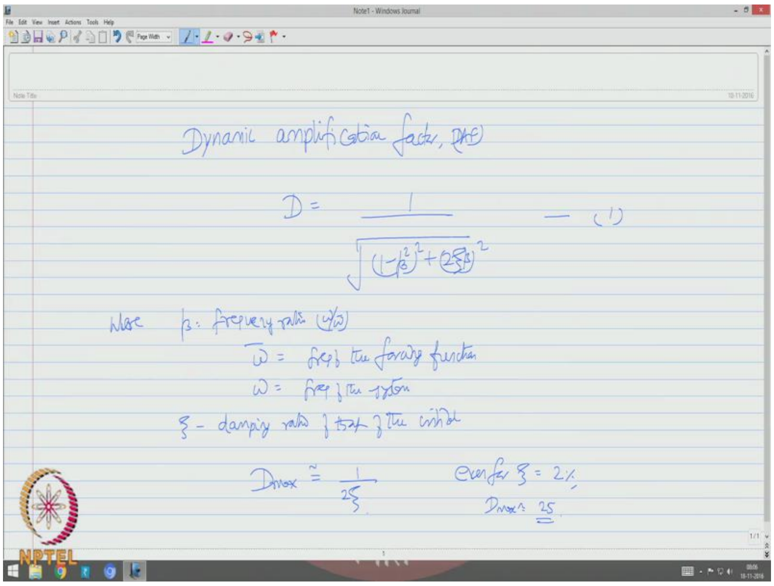

In the last lecture we said that the dynamic amplification factor DAF is given by 1 by root of 1 minus beta square square plus 2 zeta beta the whole square. Where, beta is the frequency ratio that is omega by omega bar; where omega bar is a frequency of the forcing function and omega is the frequency of the system, and zeta is the damping ratio of that of the critical. We also said that the maximum amplification can happen at 1 by 2 zeta. So, even for zeta of 2 percent the amplification can be as high as 25.

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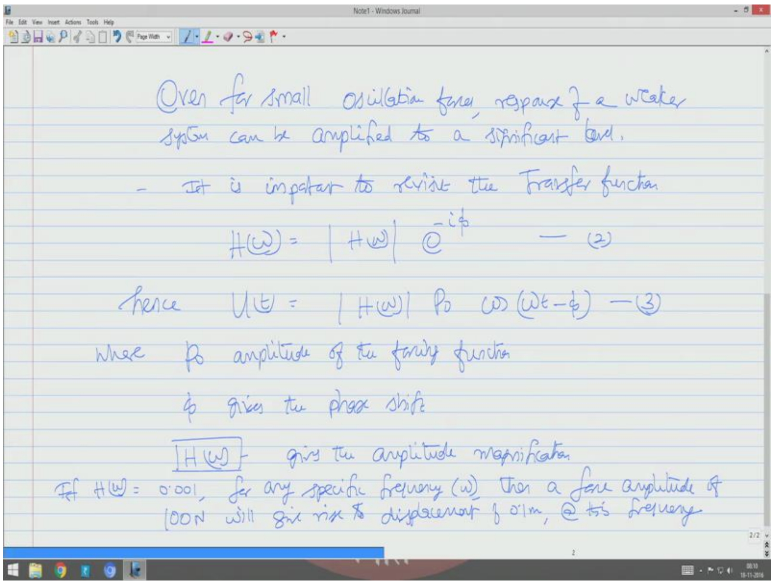

So, this implies a very important understanding to ask that even for small oscillations or let us say very specifically even for small oscillation forces response of a weaker system can be amplified to a significant amount. Hence, it is important to revisit the transfer function which we discussed in the last lecture. We said the transfer function H omega shall be now defined with a complex valued function e i phi. There is specific reason why where using e i phi and explain that once we continue with the derivation.

In this case the response of the system in time domain now can be written as P 0 cos omega t minus phi. Where, P 0 is the amplitude of the forcing function, and phi gives the phase shift. And of course, as we agree now H omega gives the amplitude magnification. Let say if H omega is let say 0.001 for any specific frequency omega, and then force amplitude of 100 Newton will give rise to displacement of 0.1 meter at this frequency.

We also have to recollect that the transfer function or the impulse response function is not no dependent, but it is system dependent. We would like to connect H omega closely with the dynamic amplification factor in the consecutive sections of this derivation.

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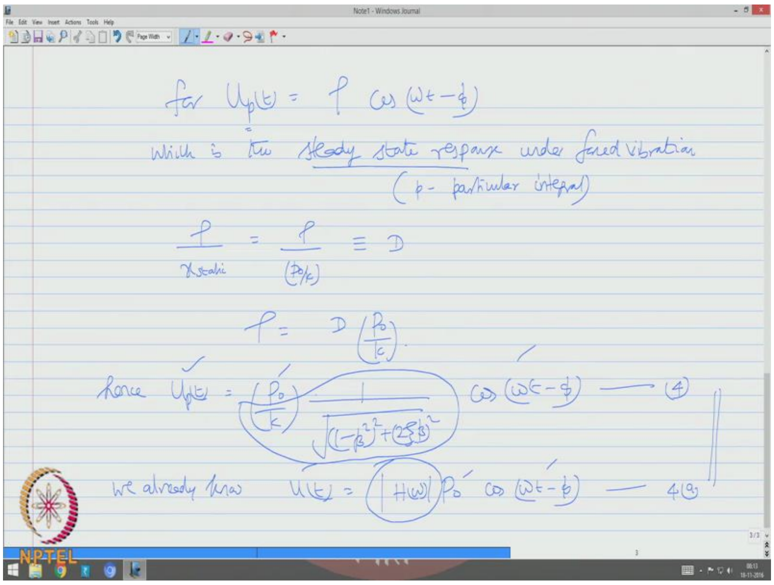

Therefore, for let say u p of t we know that is going to be rho cos omega t minus phi, which is the study state response of the given system under forced vibration that is why using suffix p, p is stands actually form particular integral. On the other hand we call that as a study state response. So, let say rho by x static is rho by P 0 by k that is what x static is we call this as D, therefore rho actually D times of P 0 by k. And hence u p of t now will be P 0 by k, D we already know is 1 by root of 1 minus beta square square plus 2 zeta beta square of cos omega t minus phi. Let me calls equation number 4.

We already have one equation earlier which we said we already know that u of t is H omega P 0 cos omega t minus phi; let me call this equation number 4 a. Now let us compare these two equations 4 and 4 a, because both of them are for the response history. Once we compare this by close of derivation one can easily find H omega is related to this term because P 0 cos omega t minus phi, so H omega related to this term which we will write now as.

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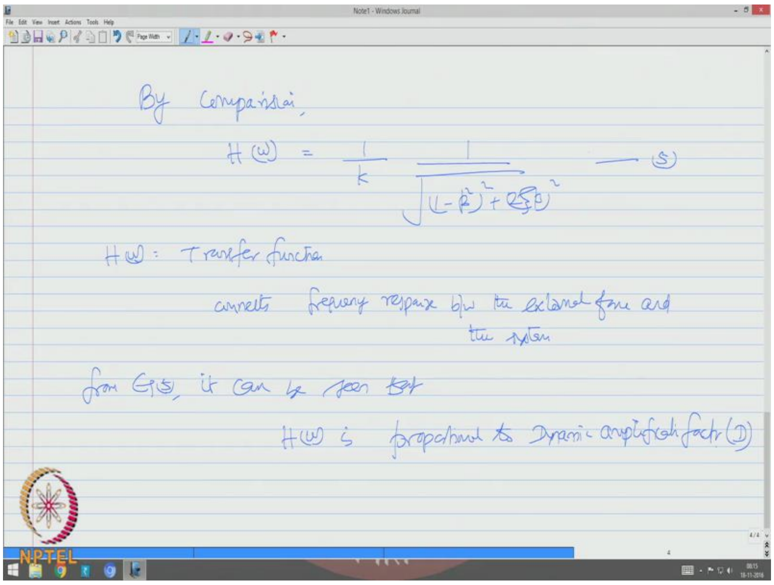

So, we should say by comparison H omega is 1 by k of root of 1 minus beta square square plus 2 zeta beta square. H omega is called transfer function. What actually it transfers? It transfers or it connects frequency response between the external force and the systems pass. One can easily see from the above equation 5, it can be seen that H omega is proportional to dynamic amplification factor D.

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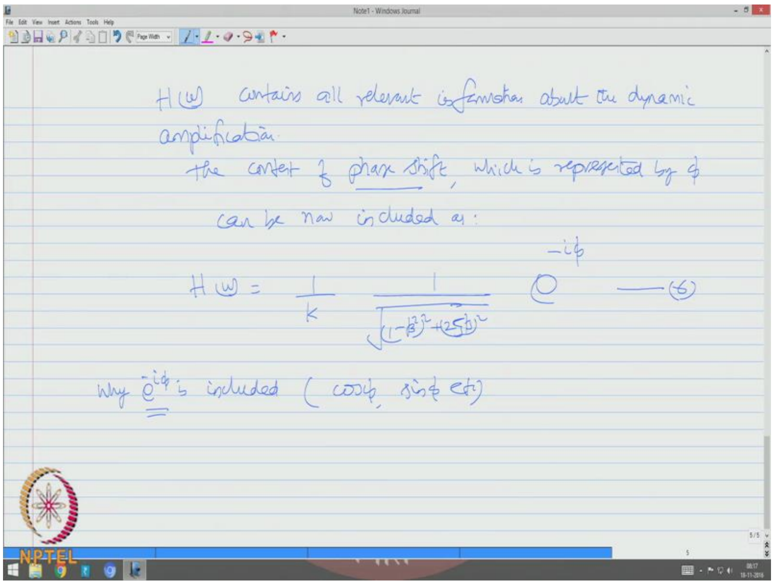

Interestingly, H omega contains all relevant information about the dynamic amplification. The content of phase shift, because it is important to realize that the response and the force may not be on the same phase. There can be always a possibility that the response can be of a different phase; phase lag or phase a head of that of the forcing function.

So, we must have some component of phase shift to be introduced in bridging the response and the frequency. So, the content of the phase shift which is represented by phi can be now included as follows. Let H omega be 1 by k of 1 by root of 1 minus beta square square plus 2 zeta beta square into e to the power of minus i phi. We have a specific reason why e to the power of i phi is included, what is the speciality about this exponential function. We could have used cos phi, we could have use sin phi etcetera; why e power i phi?

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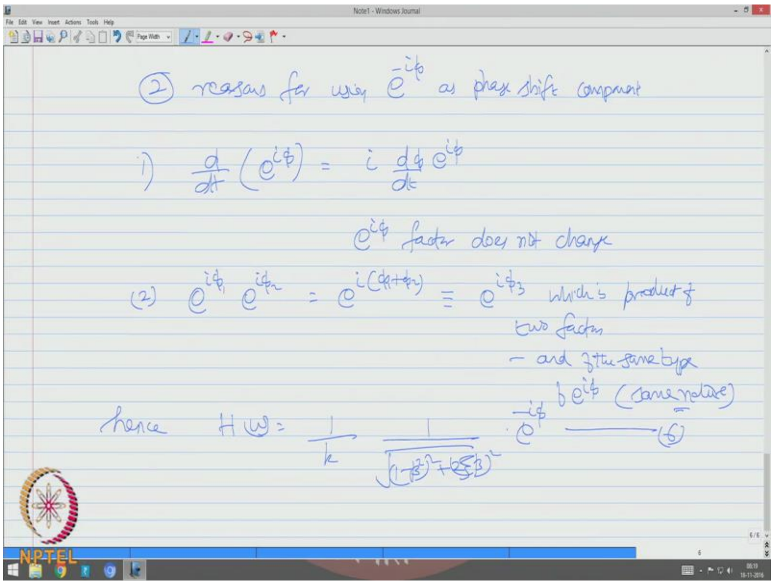

There are two reasons for this there are two reasons for using e power minus i phi as phase shift component. One you say d by dt of e power i phi is actually i of d phi by dt of e i phi. What does it mean is, e i phi factor actually does not change when under derivation or when then the differential term is included is only becoming i of the same factor.

The second advantage is e i phi 1 e i phi 2 etcetera can be simply e i phi of phi 1 plus phi 2 which can be said as; let say e i phi 3 which is product of two factors and of the same type of e i phi, it is of the same nature. So, the products also do not change the nature of the phase. Hence, we now agree that H omega can be now said as 1 by k of 1 by root of 1 minus beta square square plus 2 zeta beta the whole square of e minus i phi.

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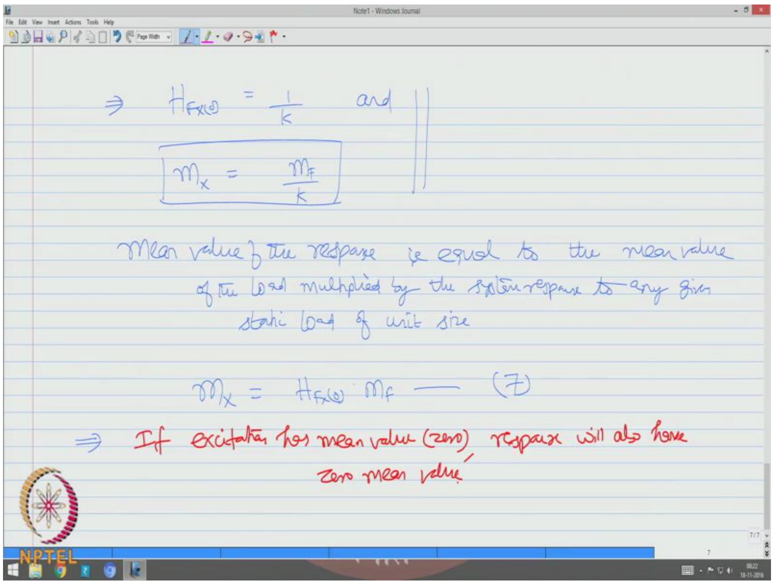

Let us extent this argument from the last lecture that H F x 0 can be now 1 by k and m x mean value is m f by k, we are extending this discussion from the last lecture. Therefore, we can say mean value of the response is actually is equal to the mean value of the load multiplied by the system response to any given static load of let us unit size or unit amplitude. So, that is a very interesting advantage we gain by saying this statement.

So, one can now say m x is H F x 0 of m f, because H F x 0 is 1 by k so this 1 by k substituted as F x 0 and m f. So, that is a very interesting advantage we get. So, this implies a very important observation if excitation has a mean value zero response will also have zero mean value.

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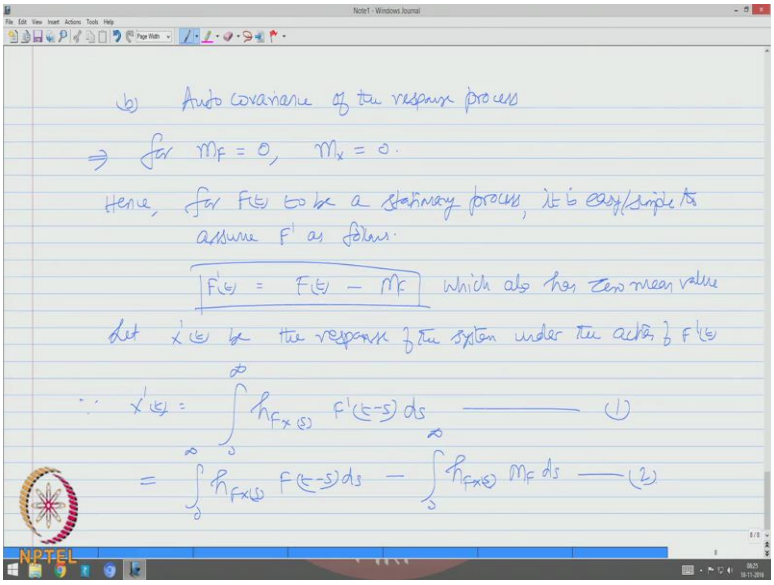

Let us extends discussion further to elaborate more on the transfer function. For this let us try to now find the auto covariance of the response process. We have just now seen that for mean value of the forcing process being 0 m x mean values the response is also 0, both are zero mean process. In such situation for F of t to be a stationary process it is easy and simple to assume F dash as follows. Let us assume F dash as F of t minus m f 1 or let say m f which also has zero mean value. Let X dash of t be the response of the system under the action of F dash of t.

Therefore, X dash of t is actually infinity 0 impulse response function F dash t minus s ds. This equation is borrowed from the earlier definition what we discussed in the last lecture. Let us extend this as 0 infinity h F x s F of t minus s ds minus, because we already know F dash of t can be F of t minus m of f therefore, minus 0 infinity h of F x s m F ds.

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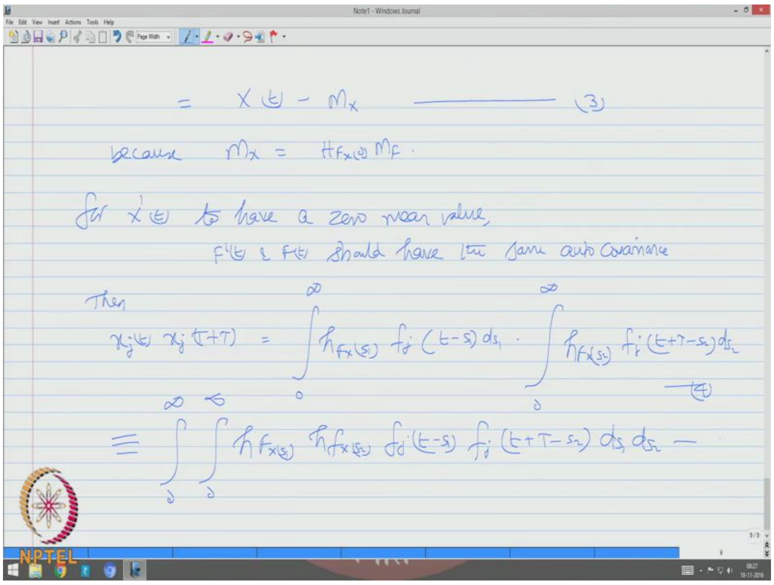

Which now can be said as simply X of t minus m of x, because we already know m of x is H F x 0 m of f; we call this equation as let say 3. Now for X dash of t to have a zero mean value, F dash of t and F of t should have the same auto covariance, because that is a statistical condition required to fulfil this requirement.

In that case x j of t x j of t plus tau is simply the auto covariance 0 infinity H F x s 1 f j t minus s 1 ds 1 multiplied by 0 infinity H F x s 2 f j t plus tau minus s 2 ds 2, which can be now said as 0 to infinity 0 to infinity H F x s 1 H F x s 2 f j t minus s 1 f j t plus tau minus s 2 ds 1 ds 2.

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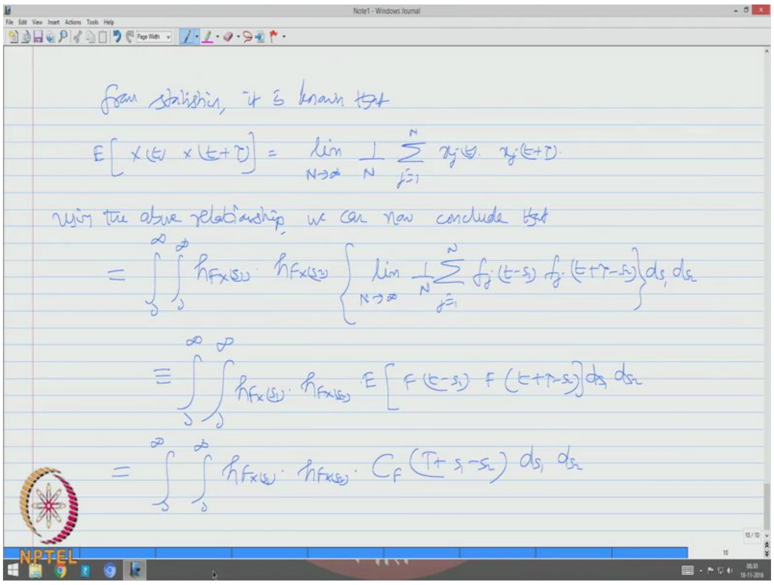

From statistics it is known that expected value of a function with a function and an increment product can be expressed as limit N tends to infinity 1 by N summation j equals 1 to N x j of t x j of t plus tau, this is known establish result. Using this, the above relationship we can now conclude that double integral of H F x is 1, H F x is 2, limit N tends to infinity 1 by N N summation j equals 1 to N f j t minus s 1 f j t plus tau minus s 2 of ds 1 ds 2.

Can be slightly rewritten as 0 to infinity 0 to infinity H F x 1 H F x s 2 expected value of F t minus s 1 F t plus tau minus s 2 ds 1 ds 2, which can be further simplified as 0 to infinity 0 to infinity H F x s 1 H F x s 2 C F tau plus s 1 minus s 2 of ds 1 ds 2.

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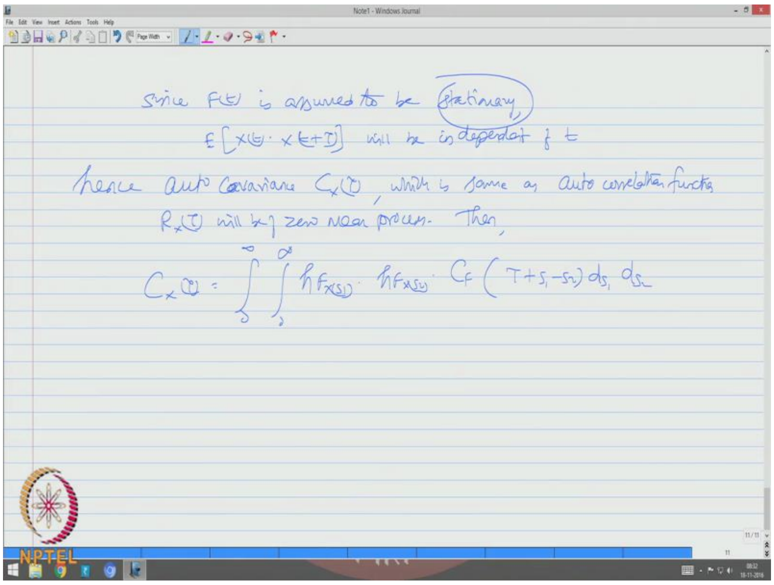

Since, F of t is assumed to be stationary expected value of X of t X of t plus tau will be independent of t, because there is a classic definition of assumption what we have in the stationary process. Hence, the auto covariance function C x of tau which is same as auto correlation function R X of tau will be of zero mean process. Then C x of tau the auto covariance function can be expressed as double integral h of F x s 1, h of F x s 2, C F T plus s 1 minus s 2 ds 1 ds 2.

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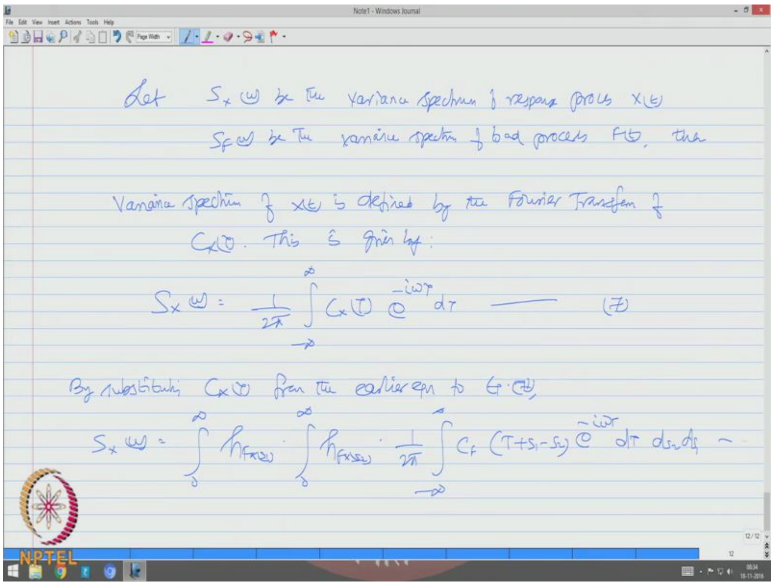

Having said this, let S x omega be the variance spectrum of the response process X of t and S F omega be the variance spectrum of load process F of t, then variance spectrum X of t is defined by the Fourier Transform of C x of tau which is the auto covariance function. This is given by S x omega is 1 by 2 pi minus to plus C x of tau e minus i omega tau d tau, call this as 7th equation.

Now by substituting C x of tau from the earlier equation to equation 7; S x of omega now will be 0 to infinity h F x s 1 0 to infinity h F x s 2 1 by 2 pi minus to plus C F tau plus s 1 minus s 2 e minus omega tau d tau ds 2 ds 1.

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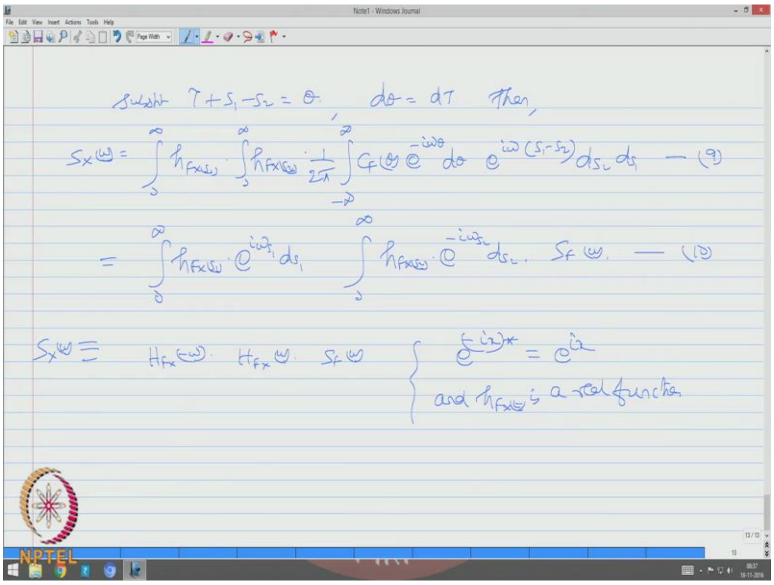

Let say substitute tau plus s 1 minus s 2 as theta, also d theta is d tau then, S x omega will be 0 to infinity h F x s 1 0 to infinity h F x s 2 1 by 2 pi minus to plus C F theta e minus omega theta d theta e i omega s 1 minus s 2 of ds 2 ds 1. Which can be reduce to 0 to infinity h F x s 1 e i omega s 1 ds 1 0 to infinity h F x s 2 e minus i omega s 2 ds 2 into s of omega; which I can now say is s f x minus omega H F x omega S F omega, that is my S x omega. This is due to the reason that e minus I x of star is as same as e i x and h F x t is actually a real function.

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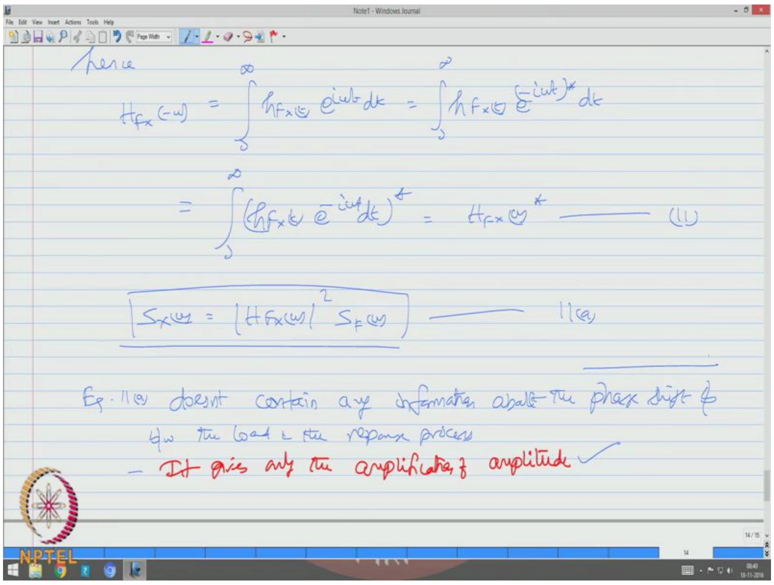

Hence, H F x minus omega is 0 to infinity h F x t e i omega t dt which is 0 to infinity h F x t e minus i omega t star of dt, then we can say 0 to infinity h F x t e minus i omega t dt of star is h F x omega star. Having said this, we can say now S x omega is H F x omega square of S F omega. That is the relationship between in the response spectrum and the force spectrum by connecting them to a transfer function and transfer function is actually proportional to the dynamic amplification factor it is independent of the load and its a system dependent characteristic, we calls equation number 11.

There is one important comment about equation 11, since we have squaring it here on let say 11 a. Equation 11 a does not contain any information about the phase shift phi between the load and the response process. What it gives me? It gives only the amplification of amplitude. Now we understand that the transfer function gives information about amplification of amplitude but no information from the phase shift.

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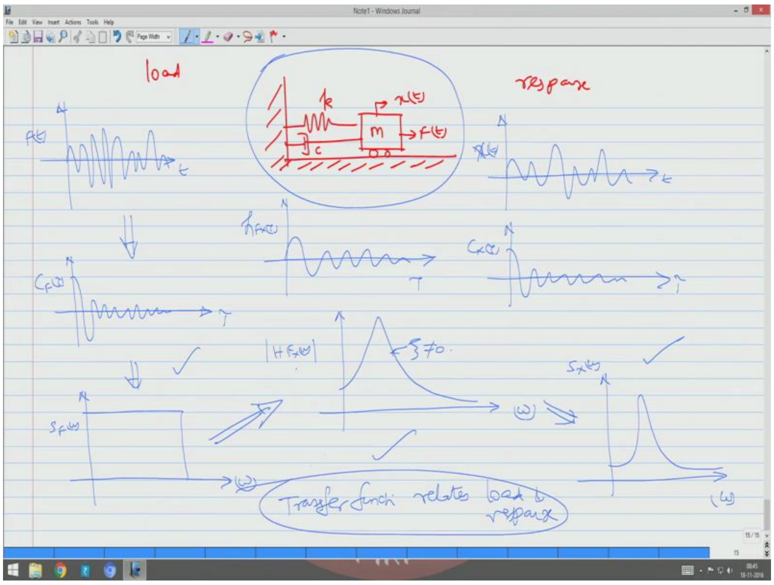

Having said this, let us try to understand this in graphical terms. Let say this portion I will talk about the load, this portion I will talk; let say I have a forcing function, function of time which is F of t which has some relationship. What it do actually is, I try to find the auto correlation function as a function of tau I find C F of tau and it looks like this.

When apply this to the system I will get the force in frequency domain which is S F omega let say which comes like this. Now I apply this to a system, let say the system is a single degree spring (Refer Time: 35:35) has some stiffness in some mass (Refer Time: 35:42) of t as a response and F of t is applied to the system here. The system has an impulse response function which looks like this, it also has the response history which is going to come or which attempts to look like this, which can be seen in the traces of the auto correlation function.

So, when you try to apply the transfer function I get a plot which is going to be similar to that of the dynamic amplification factor, but I am going to plot let say H F x omega which may typically look likes this for any specific damping not equal 0. Ultimately, when I apply the transfer function to the load process I get the response process simply in the frequency domain as S x omega which looks like this. So, the transfer function applied on to idealized model which depends on the system properties, which does not depend on the load characteristics. Hence, we to connect the response spectrum to the input load spectrum.

Therefore, I can say the transfer function transfers or relates load and response.

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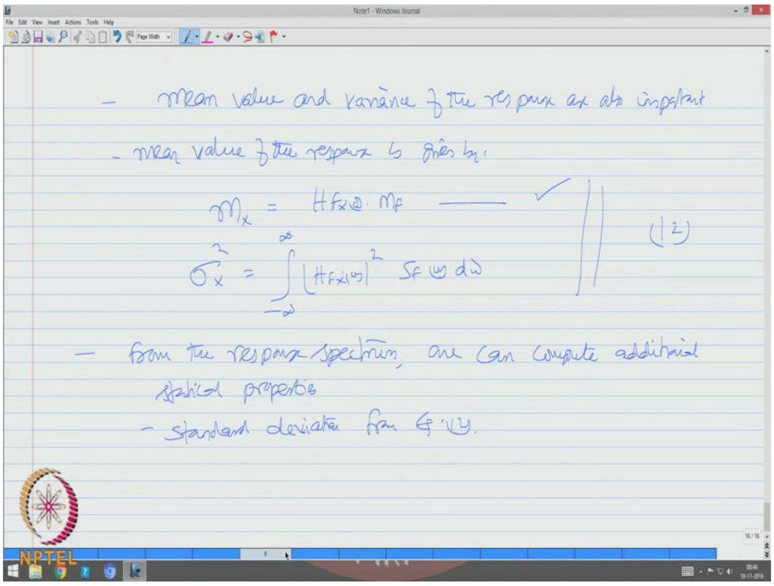

Statistically, mean value and variance of the response are also important. Therefore, mean value of the response is given by m x which is H F x 0 of m f. you already have this equation with us. And the variance is explained by like let us call this as equation number 12. So, it is interesting that from the response spectrum one can compute additional statistical properties. For example: standard deviation can be computed from equation 12.

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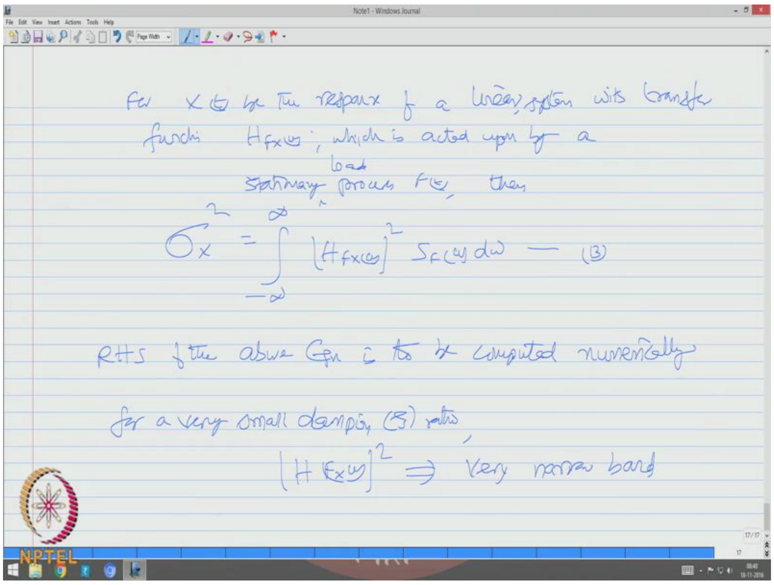

For X of t be the response of a linear system with transfer function H F x omega which is acted upon by a stationary process; I should say stationary load process F of t. then sigma x square is minus to infinity H F x omega square S F omega d omega; let us say equation 13. Right hand side of the above equation is actually to be computed numerically, but for a very small damping ratio H F x omega square will lead to a very narrow band.

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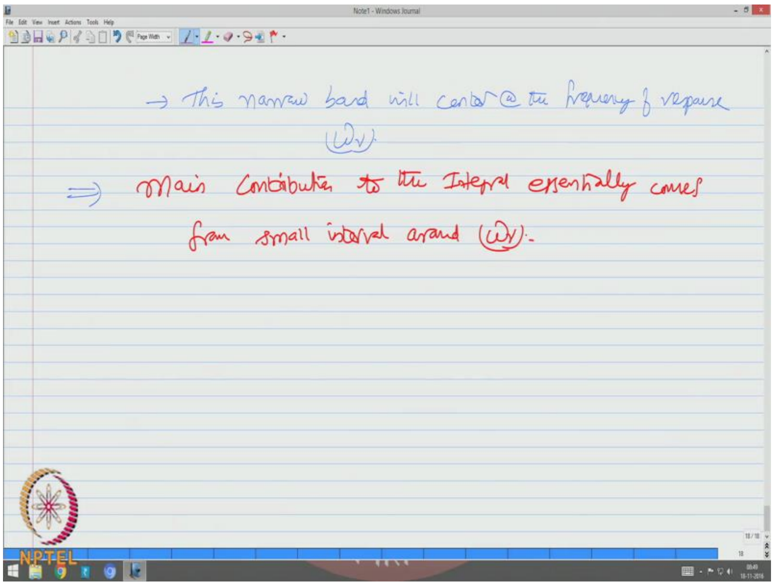

This narrow band will centre at the frequency of resonance which I call as omega r. What does it mean? This means that main contribution to the integral essentially comes from small interval around resonance frequency omega r.

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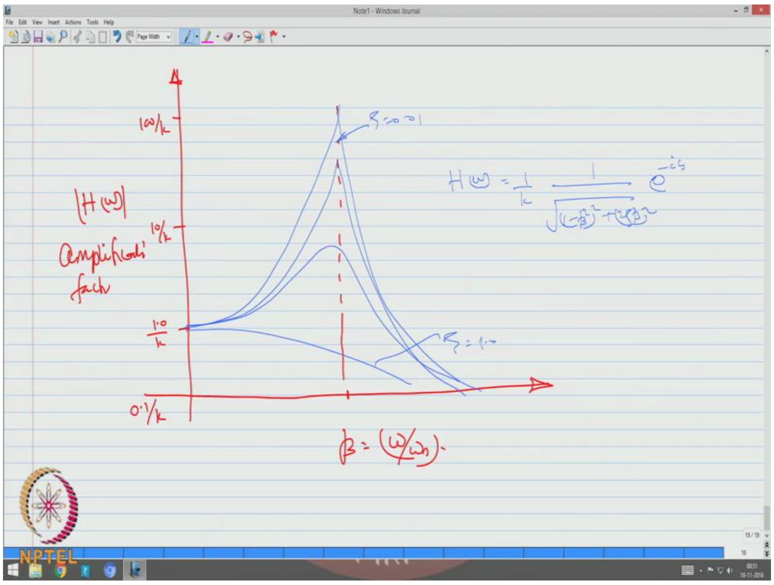

Let us try to plot how H omega looks like. So, this is going to be H omega which I call as amplification factor, and of course this is beta which is omega by omega n. If I try to plot this value at 1, that is a resonance then these values are all let say 0.1 by k; you remember H omega as said denominator of k so 1.0 by k, let say 10 by k, let say 100 by k. All typical plots for different zeta starts from here goes to this peak and comes down vertically to 0. Now example this could be of zeta of 0.01, and this could be of zeta 1.0. In the above equation we should say that H omega is 1 by k of 1 by root of 1 minus beta square square plus 2 zeta beta square e to the power of minus i phi; that is what we have used.

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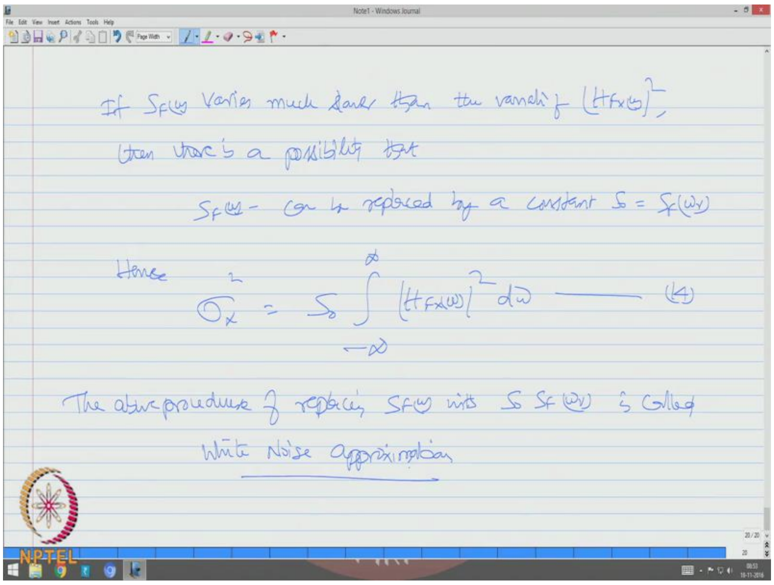

If the frequency response varies much slower than the variation of H F x omega square, then there is a possibility that S F omega can be replaced by a constant S 0 which is actually equal to S F omega r. Hence, sigma x square can be now said as S 0 minus infinity to plus infinity H F x omega square d omega. I call this equation number 14. The above procedure of replacing S F omega with S 0, S F, omega r, is called White Noise approximation.

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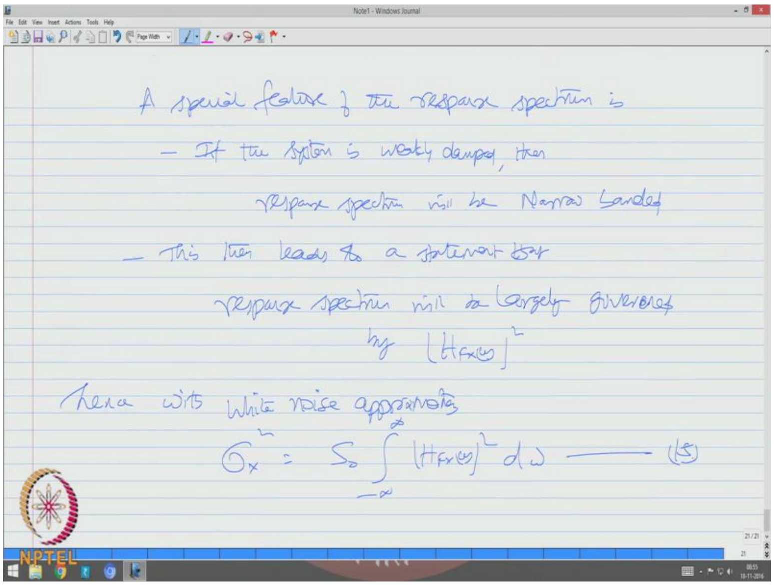

It is going to be very typical feature. One of the special features of the response spectrum is; if the system is weakly damped then the response spectrum will be narrow banded. This then leads to a statement that response spectrum will be largely governed by H F x omega square. Hence, with white noise approximation one can say sigma x square is S naught minus to plus H F x omega square d omega; equation number 15.

So friends, in this lecture we discussed some important characteristics of response spectrum and the transfer function. And we understood how transfer function remains independent of the load process, but strongly depends on the characteristics of the given system which is of course very obvious but still is very interesting to understand the white noise approximation and advantage of this approximation in estimating the response spectrum under the given load spectrum; if the process by enlarge remains stationary and Gaussian distribution.

Thank you very much.