Offshore structures under special loads including Fire resistance
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Lecture – 14
Response Spectrum

Friends, welcome to the 14th Lecture titled Response Spectrum. Whichever important lecture were we tried to discuss certain important terminologies which we will using later in case of analysis of offshore platforms or offshore structures under special loads.

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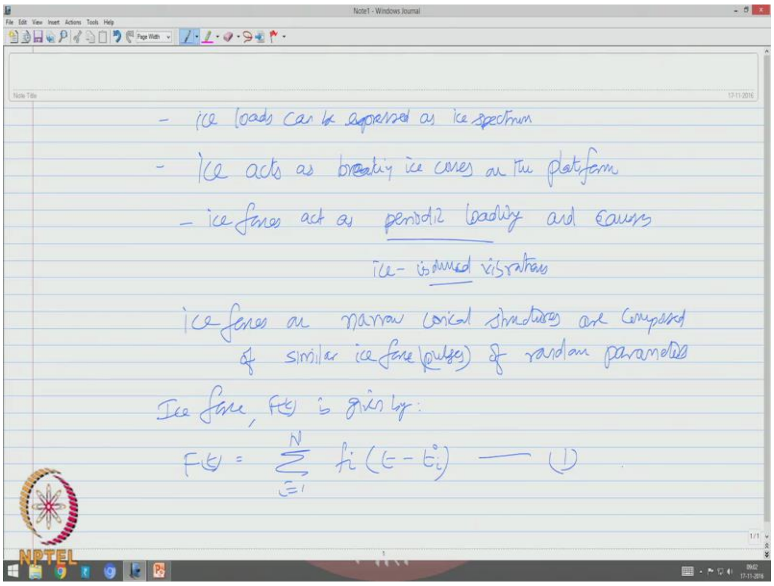

In the last lecture we said that ice loads can be expressed as ice spectrum it depends on couple of parameters, one of them influences the response or the ice flow computation very significantly the other one does not. We also said that ice acts as breaking ice cones on the platform. Broken ice pieces are cleared up, but this ice forces acts. Therefore ice forces act as periodic loading and sets or and causes ice induced vibrations.

Ice forces on narrow conical structures are actually composed of similar ice forces or I should say ice force pulses of varying random parameters, therefore ice force which is expressed as F of t is given by F of t is actually equal to summation of various pulses of t minus t i.

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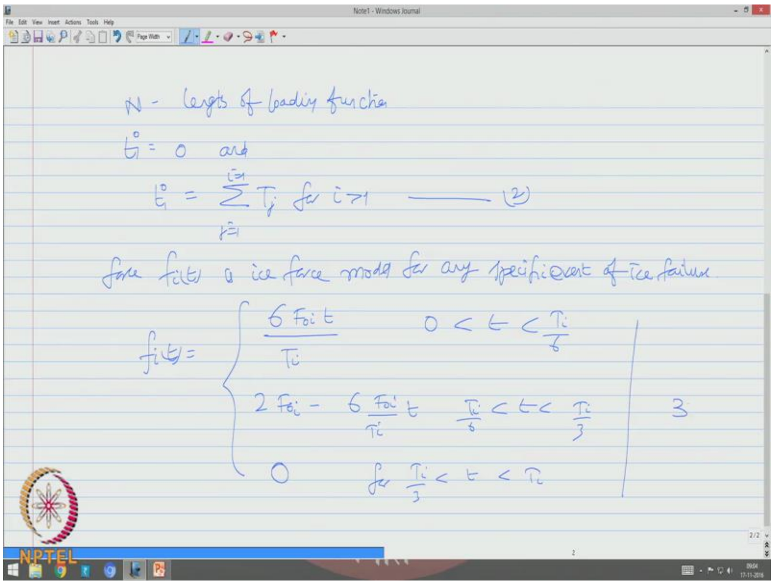

In this case n is what we called length of loading function and t 1 0 is 0, whereas t 1 0 is actually equal to j to 1 i to 1 J j or I should say T j for i greater than 1. The force f i of t in the equation is ice force model for any specific event of ice or I should say specific event of ice failure. So, that failure can be defined as 6 F 0i t by T i for t range in between 0 and T i by 6. Otherwise it is equal to 2 F naught i minus 6 F naught i by T i for t ranging between T i by 6 ti by 3 otherwise it is 0 for any value of t between T i.

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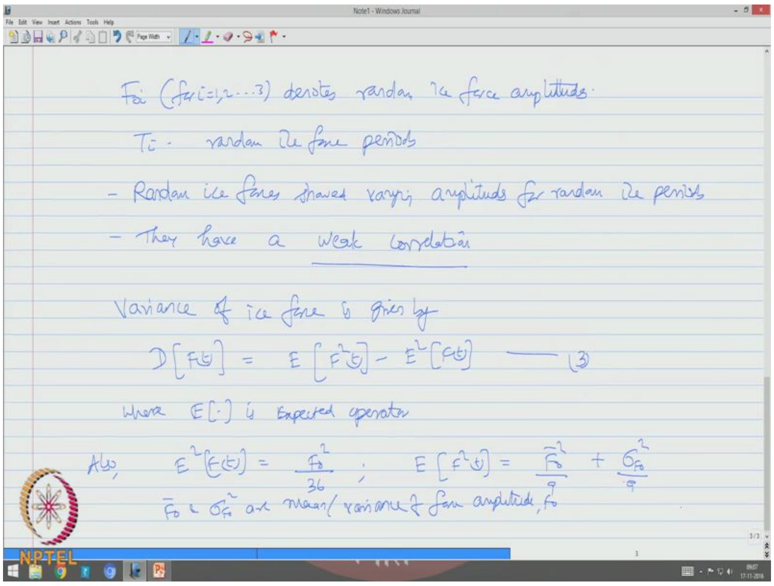

In the above equation F 0i for I equals 1, 2, etcetera denotes actually the random ice force amplitudes, and T i denotes the random ice force periods. Various researchers show the experimental investigations conducted on random ice forces or let us say ice forces show varying amplitudes for random ice periods, but they have a weak correlation that is the interesting news.

Therefore, variance of ice force is given by D F of t which is expected value of a square of t minus square of expected value of F of t, where we know that E of anything is expected operator. Also E square of F of t is actually F 0 square by 36. And hence E f square of t is F 0 square by 9 plus sigma F 0 square by 9, where F 0 and sigma F 0 square are actually the mean and the variance of force amplitude F 0.

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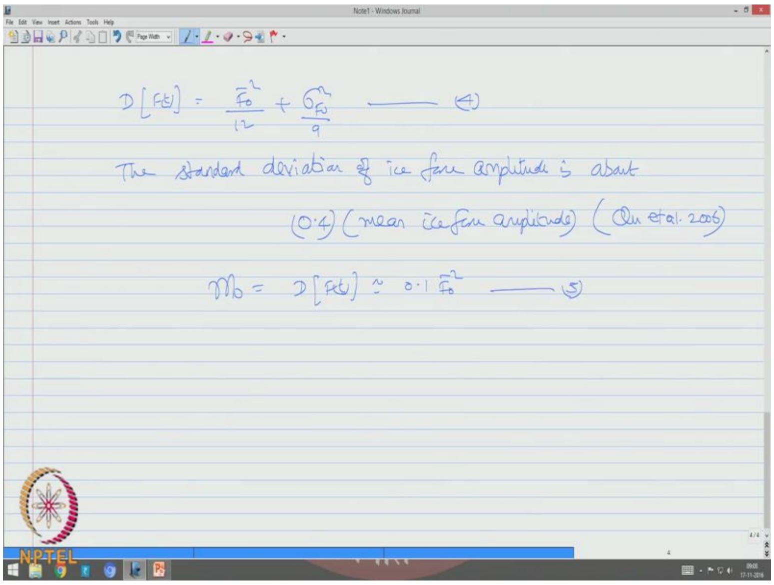

Therefore, D of F of t which is the variance of the ice force this is actually F 0 square by 12 plus sigma F 0 square by 9.

It is also said that the standard deviation of ice force amplitude is about 0.4 times of that of the mean ice force amplitude. We can see this from Qu et al. 2006; therefore the 0th moment which is actually D F of t is approximately 0.1 time of F 0 square.

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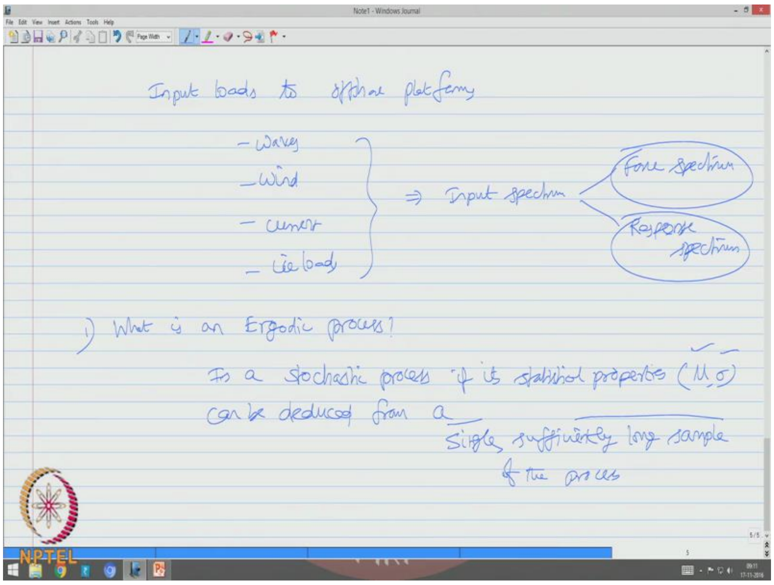

So friends, we have been discussing the input loading to offshore platforms be at waves, be at wind, be at current, be at ice loads. We will also further discuss earthquake loads extremely high severe loads etcetera in their coming lectures. All these are discussed as in the form of input spectrum. So, we have four spectrums which are considered as an input for the analysis. We have also discussed something about the response spectrum.

Interestingly, if you know the force spectrum of a given system you can always find the response spectrum of the system which is interacting with these loads by connecting the forced spectrum to the response spectrum. So, we will discuss that in this lecture in detail. Before looking into that let us ask series of fundamental questions to understand certain clarity. Let us first ask, what is an Ergodic process? Ergodic process is actually a stochastic process, if its statistical properties like mean and standard deviation can be deduced from a single sufficiently long sample of the process. So, the whole convenience is to identify that single long sample of the entire process whose mean and standard deviation will be used in computing the whole analysis.

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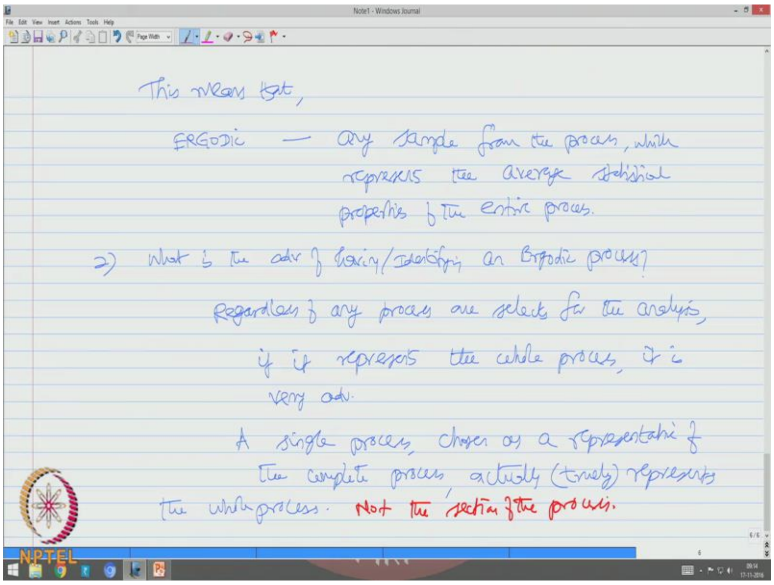

This means that Ergodic actually refers to any sample from the process which represents the average statistical properties of the entire process. The second question is comes in mind is what all the advantages or what is the (Refer Time: 13:20) advantage of having or let us say identifying an Ergodic process. Interestingly, regardless of any process you choose one selects for analysis, it does not matter what process you are selecting. If it represents the whole process, it is very advantages. That is a single process chosen as a representative of the complete process, actually and truly represents the complete process; not the section of the process that is interesting.

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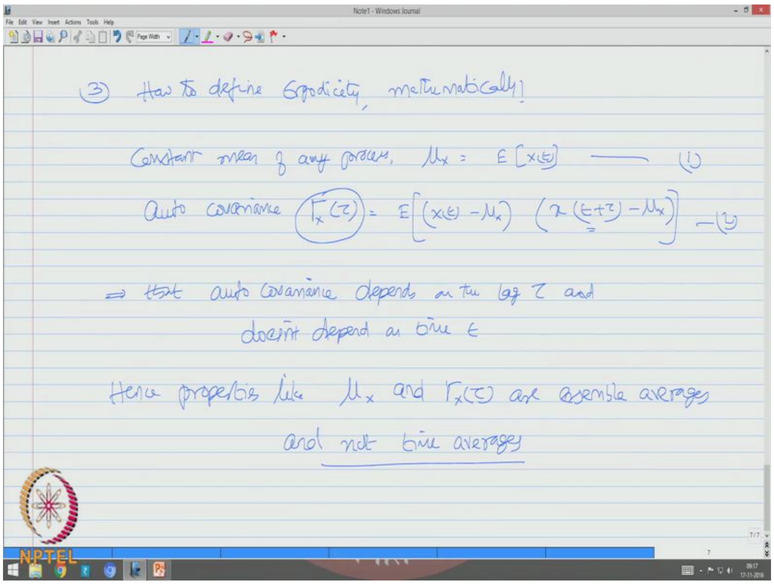

Let us define Ergodicity mathematically, so this is a third question. How to define Ergodicity mathematically? Let us say constant mean of any process is mu x is nothing but expected value of the variable. Auto covariance of the process is given by this equation which is nothing but the expected value of x of t minus mu of x into x of t plus tau minus mu of x. In fact, I should use the different symbol I should say this one mu of x, so that is let us say equation 1 and equation 2.

So, equation 2 implies that the auto covariance; which implies that the auto covariance depends on the lag tau because there is a lag tau and does not depend on time t. See here auto covariance depends on the lag tau does not depend on time t. Hence, statistical properties like mu x and gamma x of tau are ensemble averages and not time averages; that is very very interesting statement which you want to define Ergodicity mathematically.

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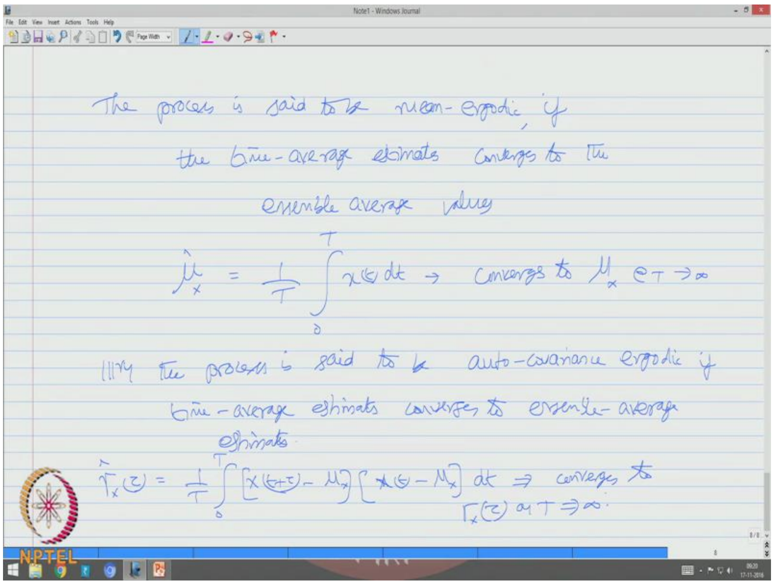

The process is set to be mean Ergodic if the time average estimate converges to the ensemble average values. That is mu hat of x is 1 by T of 0 to T of x of t dt which actually converges to the time average converges to mu x at T tends to infinity. Similarly, the process is said to be auto covariance Ergodic if the time average estimates converges to ensemble average estimates. Mathematically R x of tau hat is time average 0 to T which is x of t plus tau minus mu x multiplied by x of t x of t minus mu x dt which converges to gamma tau as t approaches infinity.

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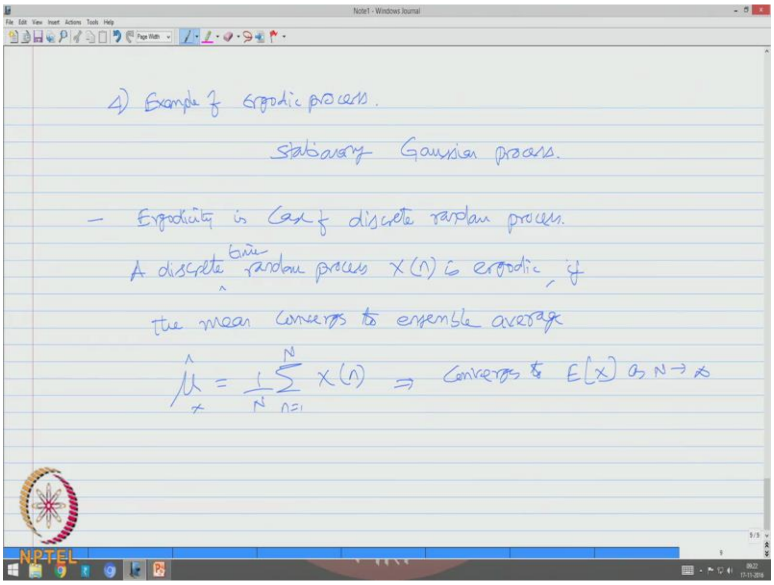

What is the classical example of an Ergodic process? So, that is the forth question which one is interested to know. For example friends, the stationary Gaussian process are generally seen as example of Ergodic process.

Suppose I want to know Ergodicity in case of discrete random process, then how do you define Ergodicity. A discrete random process, let us say to be very specific a discrete time random process x of n is Ergodic, if the mean converges to the ensemble average that is u hat which is now 1 by N of N equals 1 to n x of n which now converges to expected value of x as N tends to infinity.

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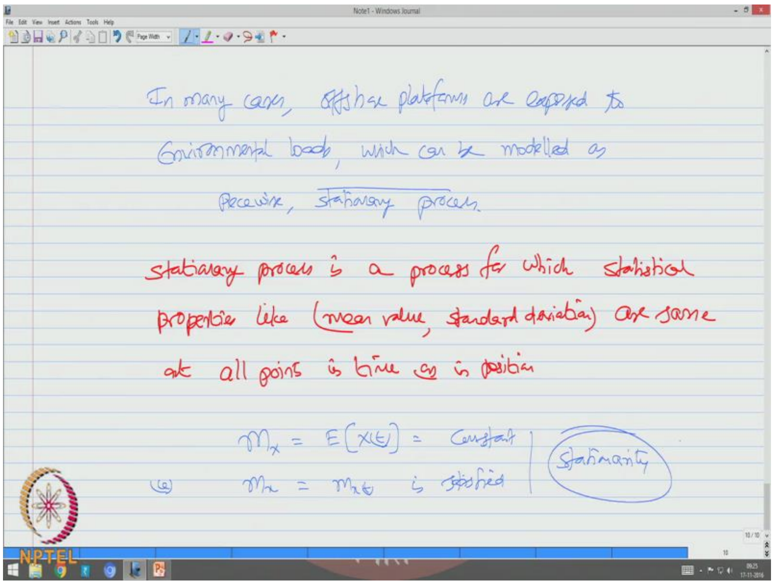

So friends, we have understood that the input forces define in terms of spectrum has certain parameters based on which the values or estimated. To estimate these spectrum we need parameters correlation characteristics, we generally assume this to be Ergodic and stationary process now we understood what is Ergodicity. And now we look further into what do we meant by response spectrum, and how do we actually bridge the response spectrum to the input force spectrum to actually get the response.

We now agree that in many cases offshore platforms are actually exposed to environmental loads which can be modeled as piecewise stationary process. Just for recollection, what this is stationary process? Stationary process is that process for which statistical properties like mean value and standard deviation, they are only examples there can be many are same at all points in time or in position. We explain this earlier as well just to recollect this idea.

Therefore, m x is nothing but expected value of x of t which is now a constant that is m x is also m x of t which is satisfied that stationarity.

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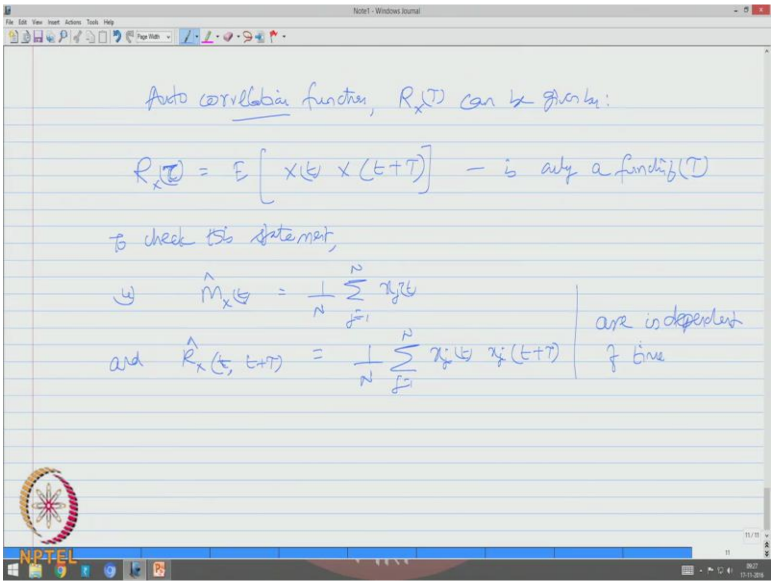

For a stationary process auto correlation function which is R x of tau can be defined as expected value of x of t x of t plus tau. Please understand the auto correlation function is only a function of tau; we need to check this statement. That is m hat of x of t, we know that it is going to be average of j equals 1 to n of x j of t and R hat of x of t comma t plus tau is actually 1 by n of summation of j equals 1 to n x j of t x j of t plus tau or independent of time this, what we need to check.

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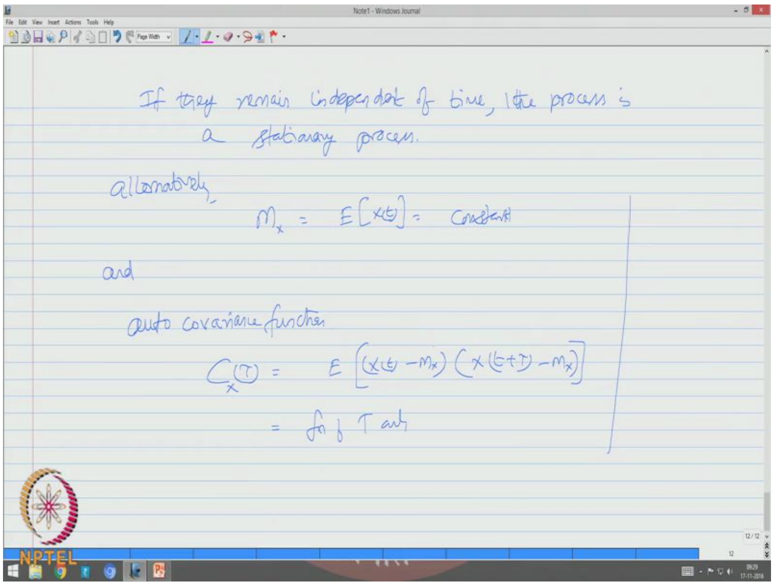

If they remind it independent time, then the process is stationary. Alternatively, m x which is expected value of x of t is also said to be constant and auto covariance function C x of tau can be said simple as E of x of t minus m x x of t plus tau minus m x which is function of tau only.

One can now ask me a question what is essential advantage of considering the loading or the process as a stationary process. That is a good question; let us try to answer this.

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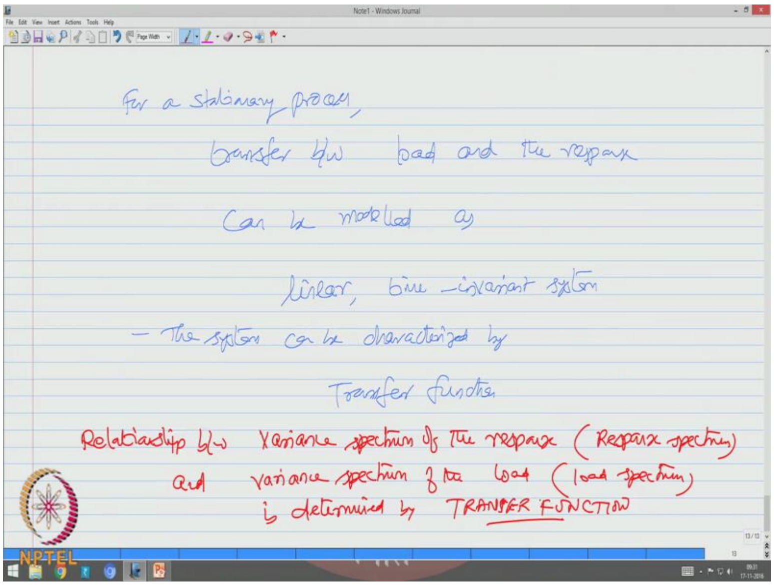

Friends, for a stationary process we have got a very serious advantage, let us highlight this and understand this for a stationary process. Transfer between the load and the response which we are interested can be modeled as; linear, time invariance system that is a very very interesting statement and one of the very serious advantage here. The system than can be characterized by transfer function.

So, transfer function is the one which is going to bridge the response to the load in a stationary process. Therefore relationship between variance spectrum of the response which we call as response spectrum and variance spectrum of the load which we call as load spectrum is determined by a transfer function.

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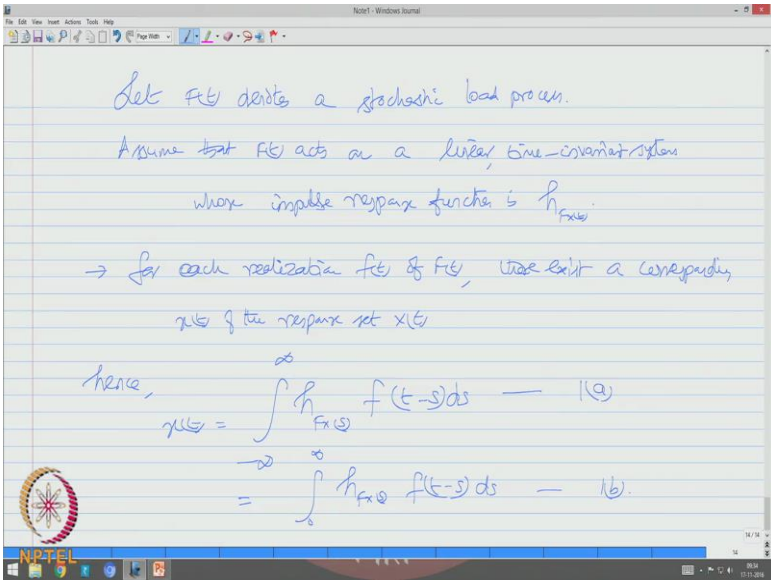

So, our job is now to understand this transfer function. So, let F of t denotes a stochastic load process. Assume that F of t acts as a linear time invariance system; acts on actually not as, acts on a line linear time invariance system whose impulse response function is given by h F x of t. What does it mean? It means that for each realization F of t of the sample capital F of t there exist a corresponding x of t of the total response set x of t. So, one to one link, one to one bridge, one to one corresponds will be there.

Hence, x of t is minus to plus infinity h F x of s F of t minus s ds, let say 1 here which can be said as 0 to infinity h F x s F of t minus s ds.

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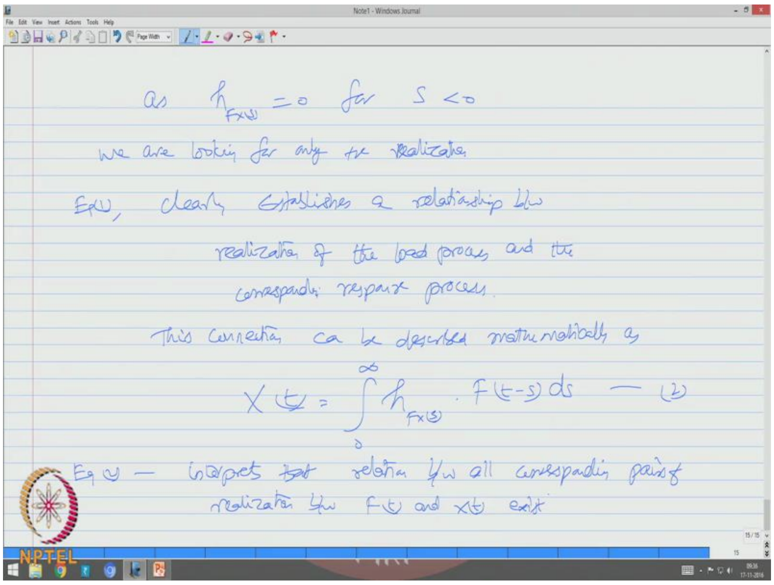

Because, h fx s is 0 for any value less than 0. We are looking for only positive realization. Therefore, equation 1 clearly establishes your relationship between realization of the load process and the corresponding response process. This connectivity or this connection can be described mathematically as X of t is 0 to infinity h F x s F of t minus s ds. Equation 2 interprets that, relationship between all corresponding pairs of realization between F of t and x of t exists.

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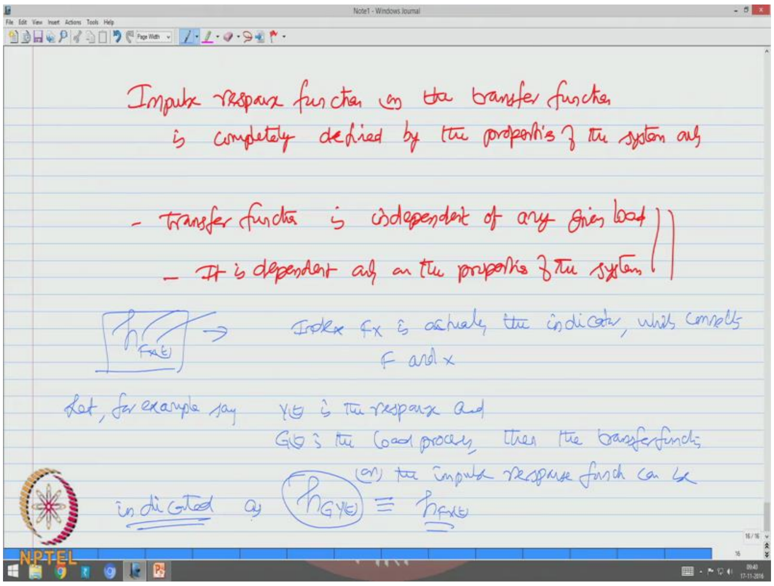

There is a very important ideology here, impulse response function or the transfer function which determines the connectivity between the load process and the response process is completely defined by the properties of the system only. That is, transfer function is independent of any given load. It is dependent only on the properties of the system. So that is a very interesting and strong statement we have which is very helpful in interpreting this transfer function which actually connects the input process which is the load spectrum to the output process which is in my case the response spectrum.

Let us try to understand one important terminology; h F x t let us try to understand this term. Here the index F x is actually the indicator which connects F and x. Let us say for example: y of t is the response and j of t is the load process just for understand, then the transfer function or the impulse response function can be indicated as; please understand indicated as not, defined h of g y of t which is as identical as h of F x of t. So, this is actually an indicator it actually connects the force and the response. If the force is g and the response is y then I write the impulse response function in this terminology (Refer Time: 39:52) actually an indicator for the whole analysis.

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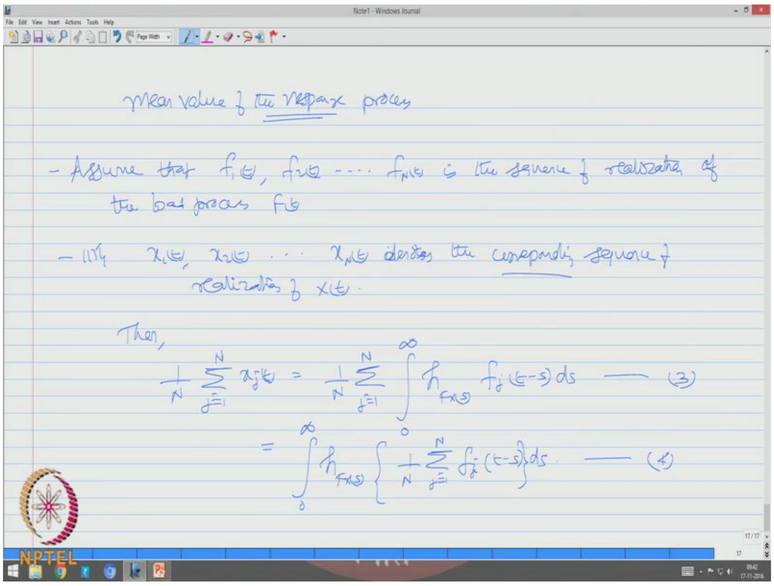

Now, let us try to extend the discussion for identifying the mean value of the response. Please understand we are now looking for the mean value of the response process. So, let us say assume that f 1 of t, f 2 of t with respect to f n of t is the sequence of realization of the load process F of t.

Similarly, x 1 of t, x 2 of t till x n of t denotes the corresponding; that is very important the corresponding sequence of realization of x of t. Then, 1 by N of summation of j equals 1 to N x j of t can be said as 1 by N of summation of j equals 1 to N of integral of h F x s; now we understand this indicator, f j t minus s ds; equation 3 which can be extended as this integral is on 0 to infinity. So, rewritten as 0 to infinity h F x s that is the integral value. I can simply say summation of j equals 1 to n of f j t minus s ds; equation 4 leads to very interesting understanding.

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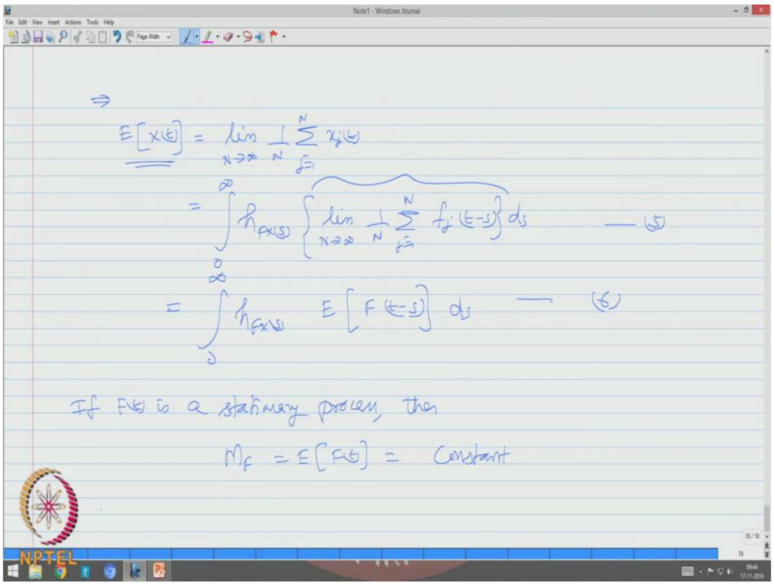

So, with this leads to expected value of the set of response processes is actually limit N tends to infinity 1 by N of summation of j equals 1 to N of all the corresponding realize set subset of x of t, which can now we expressed as integral of 0 to infinity of h F x s indicator impulse response function. Then I implement this limit N tends to infinity 1 by N of summation of j equals 1 to N of f j t minus s ds; equation 5.

Look at this carefully I can rewrite this as, 0 to infinity h F x s expected value of F of t minus s, because it is t minus s or the summation limit which is actually expected value by definition; so equation 6. If F of t the complete set of the load process is stationary processes then we agree that m F is actually equal to expected value of F of t, which is a constant we already said that in the beginning.

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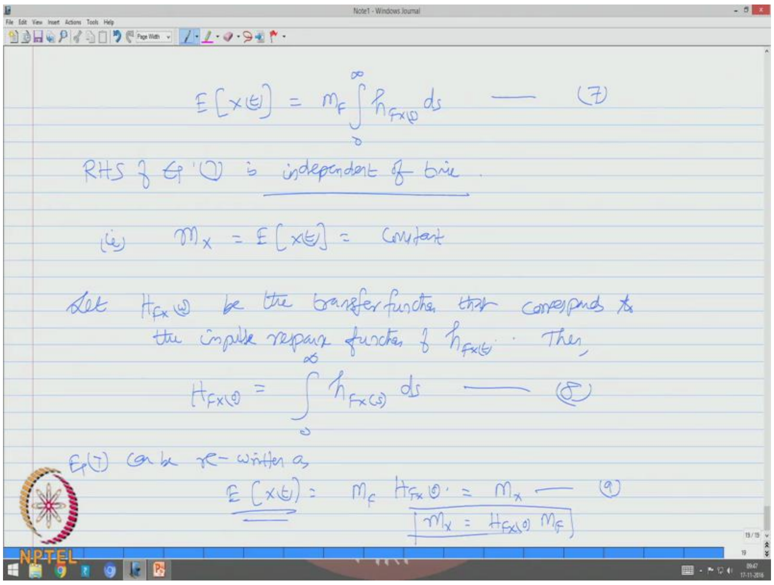

Hence, expected value of x of t which is going to be m F of 0 to infinity h F x s ds; so equation 7. From equation 7 looking at the right hand side of equation let say; right hand side of equation 7 is interestingly independent of time. That is which tells me is that m x which is E x of t is constant, that is a very important assumption, and characteristic of a stationary process which is also Ergodic.

Let capital H F x omega be the transfer function that corresponds to the impulse response function of h F x t. In that case h F x 0 is 0 to infinity h F x s ds. Now equation 0037 can be also rewritten as expected value of x of t is m F, because 0 to infinity ds is actually equal to h of F x 0- which is again m x was E x of t is m x. It means m x is actually F x 0 of m F.

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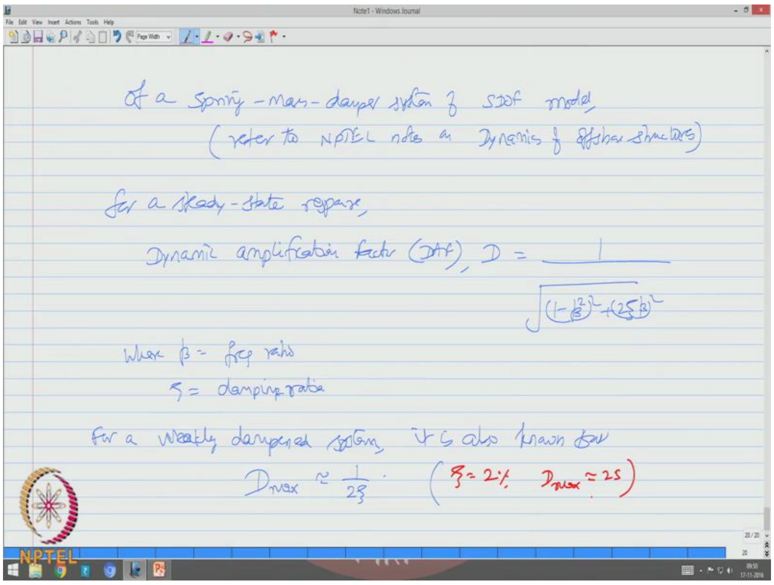

From the fundamentals of dynamics on single degree freedom system model of a spring mass system; let us say of a spring mass damper system of a single degree freedom model which I would request you to refer back to NPTEL notes on Dynamics of Offshore Structures lectures given by me. I borrow the derivation from the directly, I am not going to derive the derivation now in this lecture.

So, we know that for a steady state response dynamic amplification factor which DAF; which is indicated as D is given by 1 by root of 1 minus beta square square plus 2 zeta beta square, where beta is the frequency ratio and zeta is the damping ratio. For a weekly damped system it is also known that the dynamic amplitude factor maxim, will approximately 1 by 2 zeta. So just for understanding, if zeta is just 2 percent you will see D max is about 25 times.

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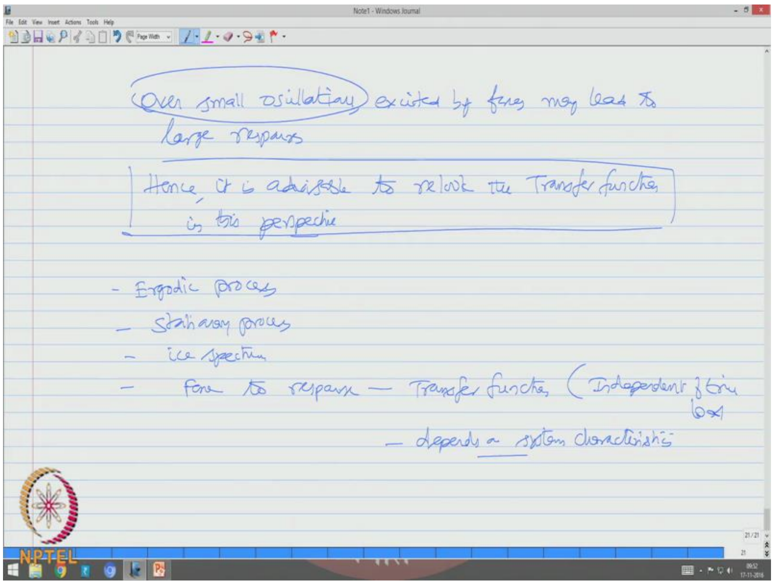

Which implies that even for small oscillations, excited by forces may lead to large responses. Hence, it is advisable to relook the transfer function in this prospective.

So friends, in this lecture we discussed about the Ergodic process, the stationary process, some extension of ice spectrum, we are in the process of connecting the force to response using impulse response function but I call this as a transfer function which is independent of time and load, but depends on system characteristics. We will extend this discussion in the next lecture. And try to revisit the transfer function with more detail, because the dynamic amplification is phenomenally high even for small oscillations when the system is weekly damped. We are looking for complete structures which are essentially weekly damped with large band width of frequencies and we want to know what would be the characteristic of the response spectrum under these situations.

Thank you very much.