Risk and Reliability of Offshore Structures
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Module - 01
Lecture - 05
Rules of Probability

Friends, welcome to the 5th lecture title Rules of Probability on the online NPTEL course title Risk and Reliability of offshore structures.

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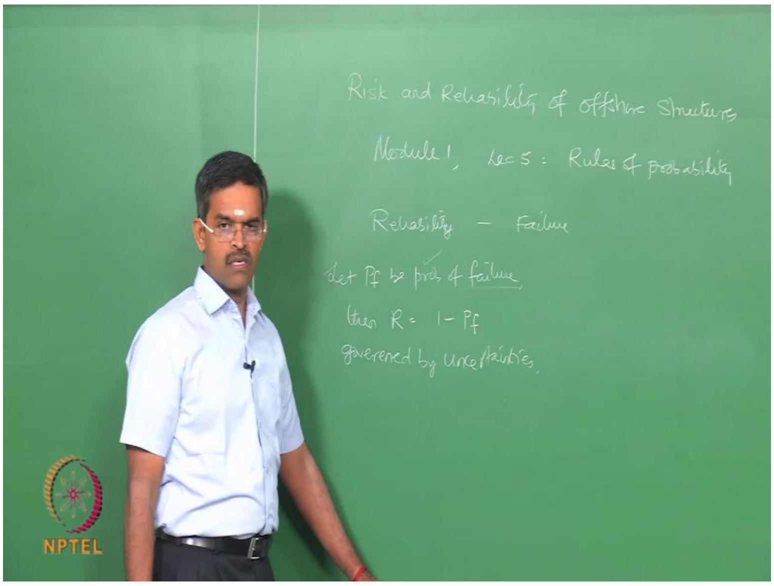

We are talking about the lectures on module 1. Today is the 5th lecture title Rules of Probability. In the last lectures, we connected reliability to failure and compare the difference of expression of interest between the two major terminologies involved in reliability analysis of offshore structures. Failure is a way of expressing nonperformance of an intended function of a given system over a specific period of time under given specific conditions. However, reliability expresses the positive side of the failure stating that the ability of the system to perform the intended function over a specific period of time under specific conditions.

Let p of f be probability of failure. Then, reliability is expressed as one minus probability of failure, which we discussed in the last lecture. All the around you will see that, failure cannot be ascertained with greater degree of accuracy. I can always associate failure with only in probabilistic terms. The main reason being reliability methods are governed by uncertainties, which we discussed in the last lecture. So, now, we clearly understand that, the reliability will be expressed only in probabilistic terms. You can also express this in frequency terms. We will talk about that slightly later, but predominantly reliability is expressed in probabilistic terms. Now, the question is if you want to express reliability in probabilistic terms, what is the major advantage probability has in expressing a concept of failure or converse of the concept of failure in a realistic world.

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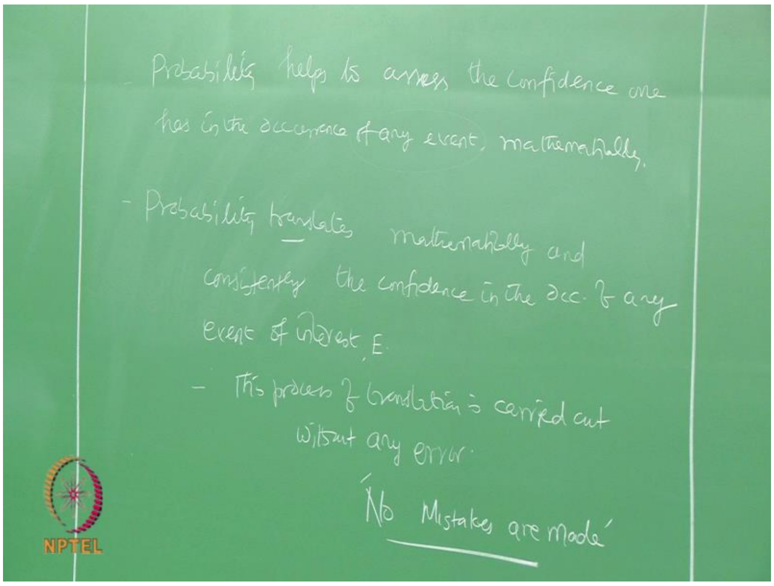

So, probability helps to assess the confidence one has in the occurrence of an event mathematically. So, probability helps to assess the confidence you have in occurrence of any event in a mathematically manner; that even could be a possible failure of a given structure. So, therefore, one can say probability translates mathematically and consistently the confidence in the occurrence of any event – event of interest. Let us say E. The great advantage what the probability rules have is that, this process of translation is carried out without any error; that is, no mistakes are made.

So, the method ascertains that, you do not make any mistake while translating your level of confidence in occurrence of any specific event in a mathematical manner, which is actually demanded in the reliability analysis. So, therefore, probabilities tools are very interesting and very helpful in expression of interest for us in offshore structural engineering. Let us now look at the rules around which the probability whirls around.

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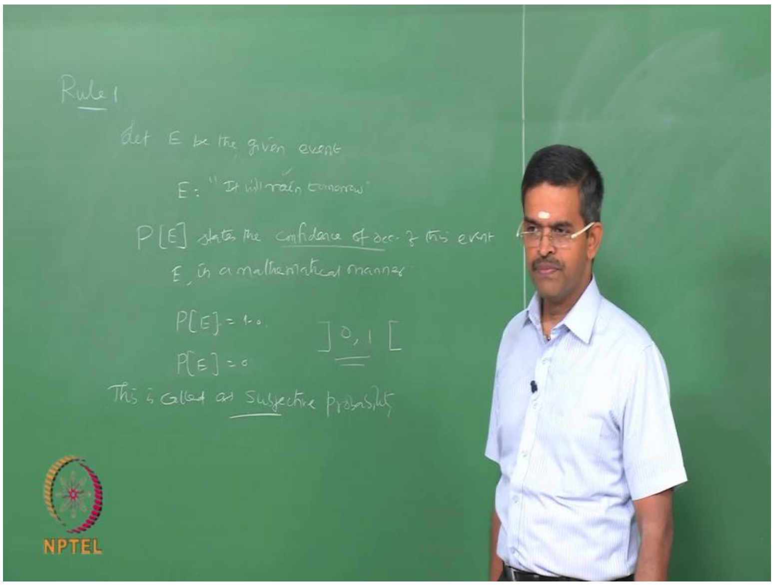

Let us say rule one. Let E be the given event. I will take a very simple example we will talk about complicated examples slightly later. A simple example – it will rain tomorrow; I will take an event. So, event E – it will rain tomorrow. I have to express this in terms of probabilistic terms. So, I should say P of E states the confidence of occurrence of this event E in a mathematical manner. For sure by any chance if you know it is going to rain tomorrow, then P of E is 1.0. For sure by any means and methods you know that, it will not rain tomorrow; then, probability of occurrence of that event is 0. It means the probability that it may rain tomorrow will vary between the brackets of 0 and 1. So, on the other hand, it is basically converting your level of confidence into a number. So, we call this as subjective probability.

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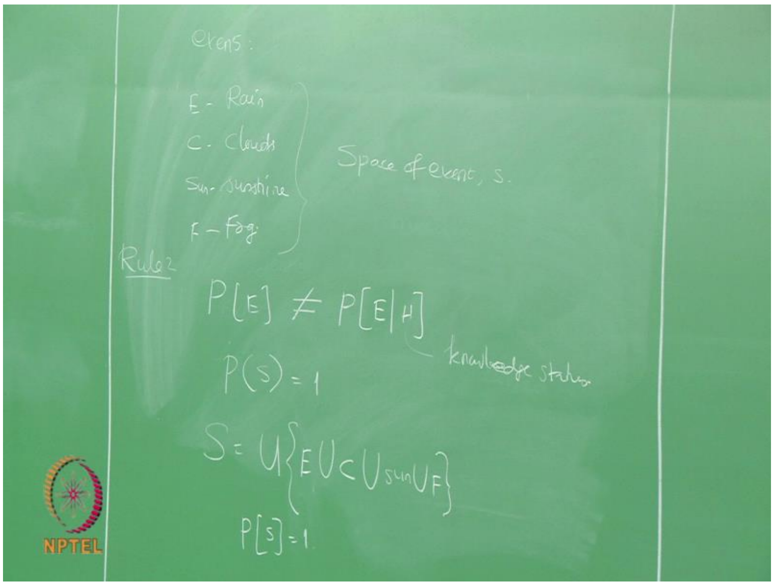

Let us expand this event example slightly larger. For the rain to occur tomorrow, there are various symptoms. For example, let us say there are various other events possible, which can confirm whether there will be rain tomorrow. What are those possible events? Let us say rain is one event, which we are trying to guess. Second event could be clouds. Third could be sunshine. And, fourth could be a fog. Let us say these are the possible interconnecting events, which can govern or improve or decrease or decline the level of confidence you have in expressing the occurrence of specified event; where, in our example, it will rain tomorrow.

So, for example, if you try to look at the knowledge status acquired by you through the metallurgical reports, this will improve the confidence in you in ascertaining the statement of P of E; that is, P of E, that is, probability of occurrence of this event will not be equal to the probability of occurrence of this event if the knowledge is improved upon; where, H talks about the knowledge status. This is what we call as rule 2. So, probability of occurrence of an event will not be of course equal to the probability of occurrence of the same event if the knowledge status on the occurrence of the event is improved. We have some clarity about the specific event.

Interestingly, all those knowledge status, which are going to contribute to the knowledge level of your understanding in confirming the occurrence of the event, is what we call space of events. We have to agree that, probability of occurrence of the space of event is always one. Essentially, S is unions of all the events, that is, let us say this is an event E. This I call as cloud; (Refer Time: 10:54) C. This I called as S say sunshine and this is call as F. So, E union C union sun union fog. So, probability of space of events will be expressed as (Refer Time: 11:17) because it is going to happen all of them together in one go. Let us try to understand slightly more the rule 2 graphically.

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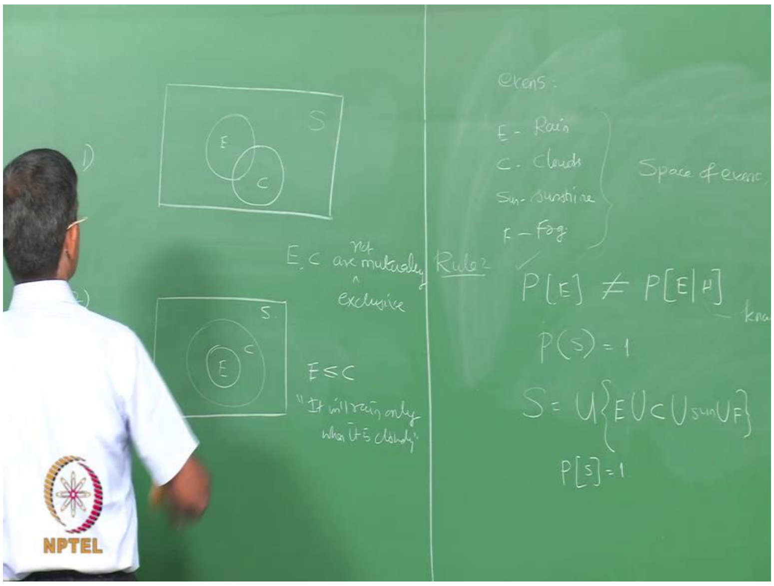

For example, if there is going to be a cloud, there is a possibility that it may rain. I can express this graphically like if this is my event E; if there is going to be a cloud, there will be rain certainly at least to some extent. Therefore, I will be able to overlap the other event C with that of E while the remaining could be the space of event, which can be sunshine and fog. In this case, please note that, the event C and the event E are mutually – let us say are not mutually exclusive, because one depends on the other.

One can also interestingly express in this manner. If you see a dark cloud, it is going to rain; it means E will be less than or equal to C; it will rain only when it is cloudy. On the other hand, it will not rain when there is sunshine; it will not rain when there is a fog. So, you can express this statement graphically like this. All will confirm to rule 2. So, the knowledge of occurrence of specific event or the confidence level in occurrence of an event will change if you know the more knowledge information about the occurrence of that event what the rule 2 states. That is what the rule 3 is.

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From rule two, I just extend rule 3 in estimating P of E. There can be other alternatives. All the alternatives put together is called space of events, which is union of all possible events. Therefore, one can say rule 3 – S is a union of all possible events like E is may rain fall; C is may cloud; S is may sunshine; and, F is may fog. So, I can call this as rule 3 and of course this as rule 2. Let us extend our discussion further.

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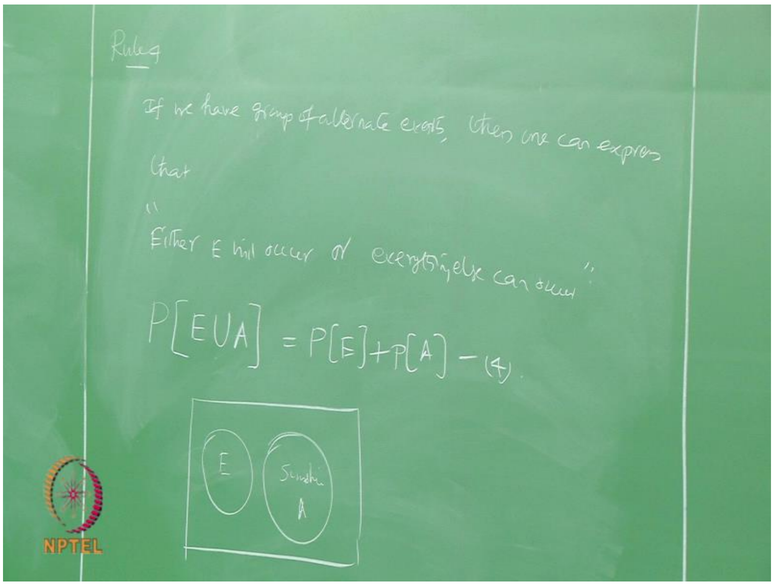

If we have group of alternative events; if we have group of alternative events; then, one can express that, either E will occur or everything else can occur. So, this can be expressed as probability of E union A, which can be expressed as probability of E plus probability of A. This can be expressed graphically in this fashion. Let us say this is my event E and this is my event A, which can be sunshine.

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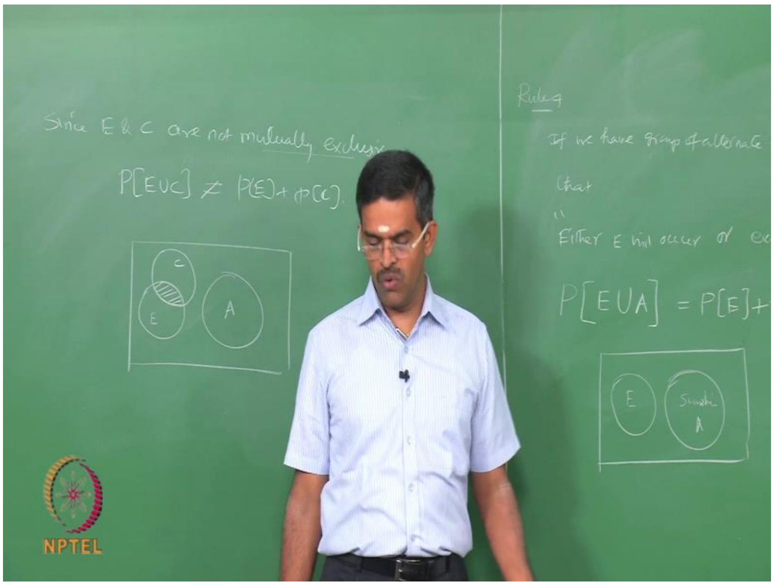

Now, interestingly, in this example, since E and C are not mutually exclusive; we can say that, probability of E union C will not be equal to probability of E plus probability of C. So, the statement of rule 4 will be applicable only to those events, which are mutually exclusive. If you try to plot this graphically and trying to find the reason why it is so; let us say E and C are not mutually exclusive. Therefore, you will plot the value of E here; you will plot the value of C here; and of course, you will plot the remaining factors A here. So, the intersection area between the E and C because they are not mutually exclusive, will be extra.

Therefore, this statement or rule 4 cannot be applicable to those events which are not mutually exclusive. So, while identifying the space of events related to or in support of expressing the confidence level of occurrence of any specific event of your choice, you have to be very careful in making a statement saying that, which events amongst the space or mutually exclusive within the considered event and which events in the given space are not mutually exclusive with respect to the considered event. So, you have to make two different sets of events saying that, which are mutually exclusive with that of the considering event and which are not mutually exclusive with that of the considered event, because the rule cannot be applied to all the space of events as a blind fold.

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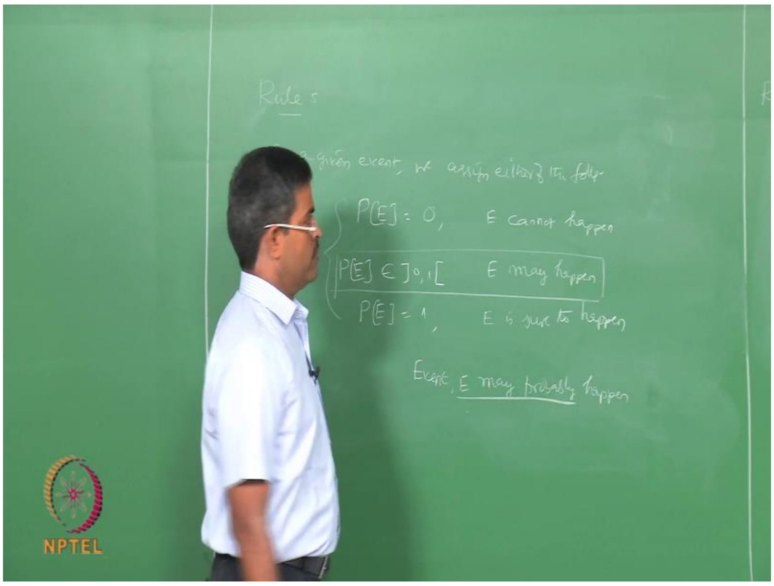

Now, let us talk about rule 5. For a given event, we assign either of the following. If we say probability of that event is equal to 0; then, we can say the event E cannot happen. If I say the probability of event is 1; then, we can say E is sure to happen. If I say E may happen; then, probability of E element of 0 and 1. So, this how we express. The probability of any specific event in this form, this particular statement expresses that, the event E may probably happen. So, that is what we say as rule number 5.

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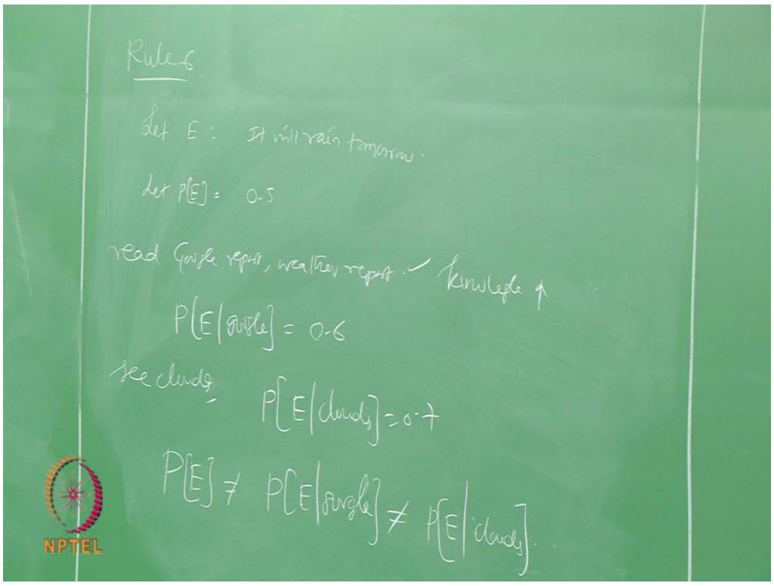

Let us talk the next rule of probability, which is rule number 6. Let E be the event; it will rain tomorrow. Let the guess what you make or expression of confidence that occurrence of event as far as you are concerned be 0.5 for a guess, because it is a number between 0 and 1. If we say 0; it is not going to rain; if we say 1, certainly going to rain; can make a guess in between 0 and 1. Let it be 0.5. Now, you read the Google report or for that instance any weather report. So, your knowledge level has increased.

So, now, your knowledge level given Google report (Refer Time: 21:23) certainly more than this. Let us say the Google weather report says there is a possibility of rain. So, your knowledge level in expressing the confidence slightly higher. You get up in the morning tomorrow; you see clouds. Now, your knowledge level after seeing clouds is going to be sure towards the occurrence of that event rain. So, on the other hand, probability of an event is never equal to probability of an event given some knowledge status, probability of an event of verification of knowledge status, etcetera. But, again express the whole interest in terms of conditional probability.

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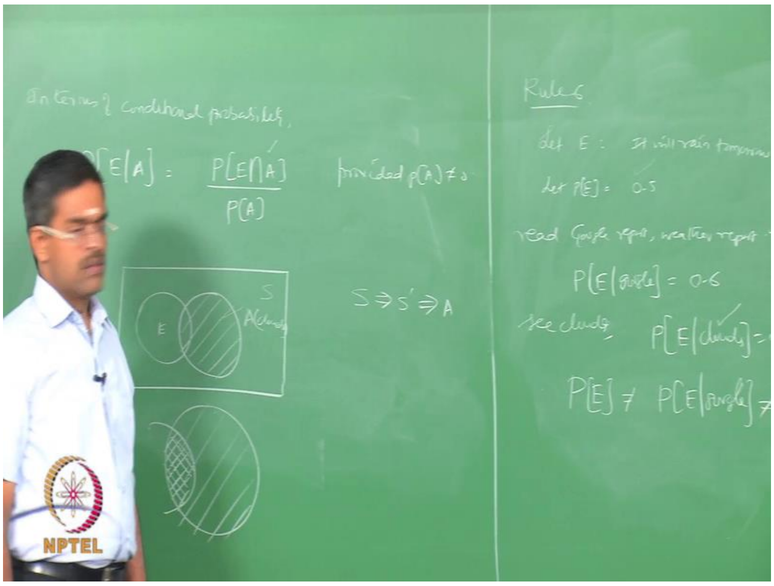

In terms of conditional probability, P of E with all other events A is simply given by P of E intersection A normalized with P of A provided P of A is not equal to 0. A is going to surely occurring. This A can be even clouds let us say. Now, the question comes why there is an intersection here. There is an intersection, not intersection. Let us try to plot this. I have space of events, which can be E, C, sunshine and fog. Let us plot E, which is going to be the rain. Let us say A is the cloud. And, we also know cloud and rain are not mutually exclusive. Let us say this is my A, which is cloudy, which you also saw when you get up in the morning. So, obviously, there is going to be an overlap in between this.

Now, the moment you saw the cloud, you have shifted your level of confidence from the greater space of event to a closer space of event. It means S has been modified to S dash, which actually becomes A; which is nothing but the cloud. So, the whole event A has now become to this extent; it is because of this reason, we are using intersection. And, since the whole exercise is dependent on the surety of occurrence of a specific event A in this case cloud; therefore, we are normalizing this understanding with this put to that event, because this event can never become 0, because that is that event, which has increased your confidence level say for example, from 0.5 to 0.7. This is what we call as rule 6; so, equation 6.

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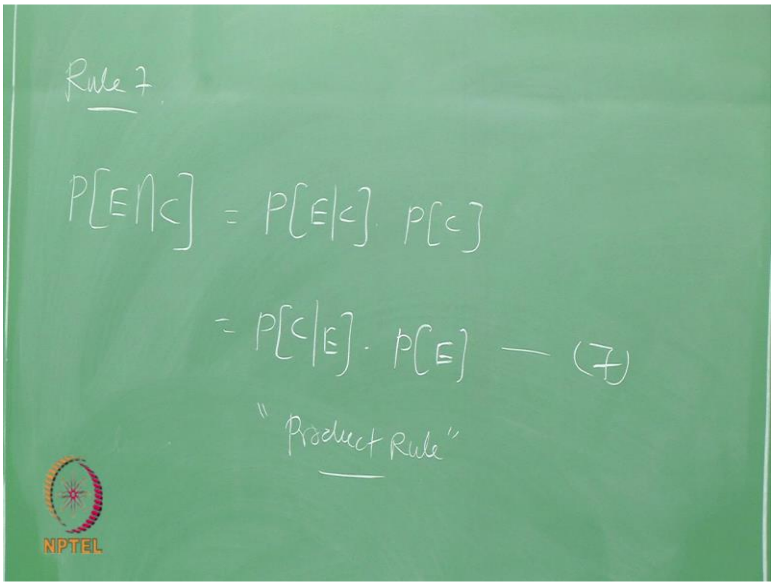

Let us talk further on rules of probability. Rule 7 states that, probability of occurrence of an event with that of the cloud, which you require the numerator can be also expressed as probability of occurrence of an event with that of cloud multiplied by probability of having cloud seen in the picture. This is also equal to probability of occurrence of cloud because of probability of occurrence of rain multiplied by probability of occurrence of rain. We call this as rule 7. This is also called as product rule.

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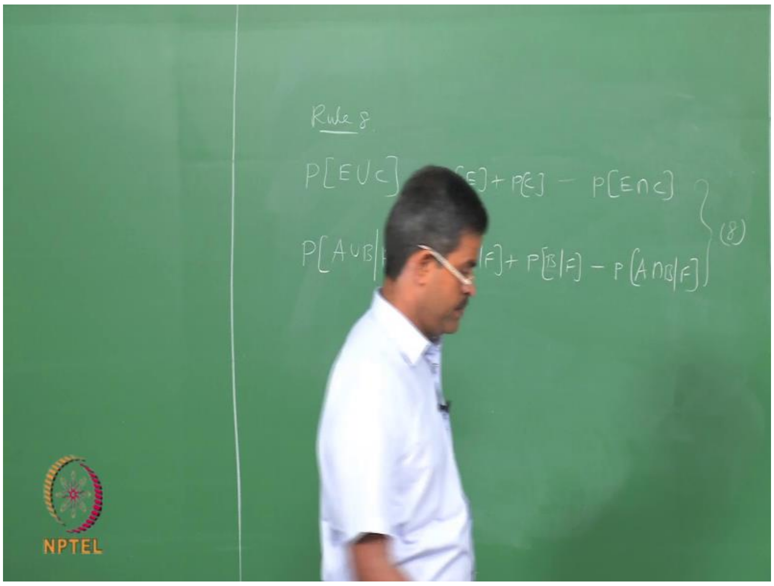

Let us expand the discussion further talking on rule 8. Rule 8 states the probability of E and C is equal to the probability of these events alone their sum minus the probability of their OR. If you really wanted to know the probability of any two events occurrence; given the knowledge F, can also extend the same way probability of that event given F plus probability of that event given F minus probability of (Refer Time: 27:07) We call this as equation 8, which is the rule 8.

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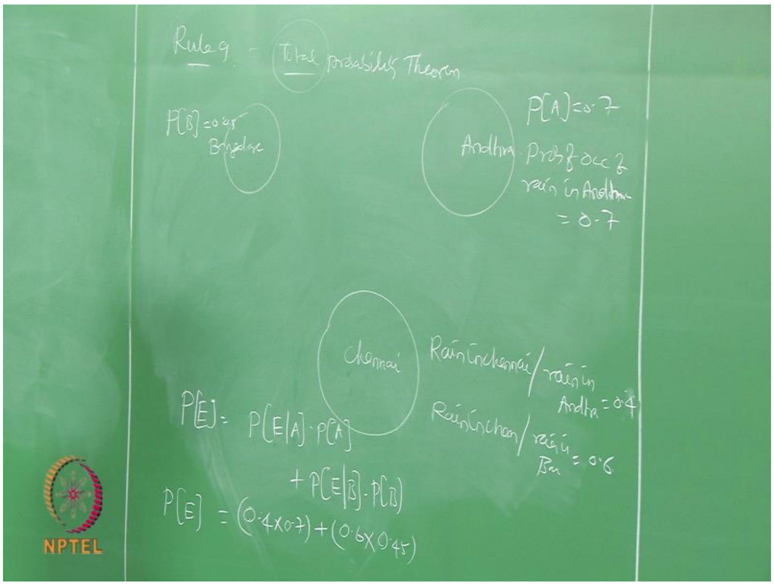

We extend the discussion further to write down the rule 9, which leads to the total probability theorem. You can expect some total effect in the whole equation. I have a very interesting question to ask here. Let us take an example. Let us say this is Chennai. (Refer Time: 28:03) coastal place let us say Andhra. There is interior slightly let us say Bangalore. We are talking about the event of rain in this case. The probability of occurrence of rain in Andhra let us say is 0.7, that is, P of A.

If there is rain in Andhra Pradesh, the previous historical data or the meteorological data shows there will be possible rain in Chennai and that knowledge status of occurrence of rain in Chennai, that is, rain in Chennai because there is rain in Andhra is point let us say 4. It can be easily evaluated from the meteorological data; you can also compare the historical curve and see if there is rain in Chennai; how many times there was rain in Chennai because there was a parallel rain in Andhra Pradesh. One can always find out in a given data, this number can be established.

Similarly, probability of rain in Bangalore is also known. Let us say 0.45. And, looking at the past data, one can always say that, whenever there was a rain in Bangalore, there was also a parallel rain in Chennai. So, the probability of rain in Chennai because rain in Bangalore is let us say - 0.6 higher, because it is closer let us say from the Andhra coast.

Now I am interested in finding out what is the probability of rain in Chennai given that the probability of the neighborhood are known to me and the influence of that event on my event is also known to me. So, I am trying to connect all the neighboring events or the knowledge of neighboring events. Therefore, we call this as total probability. Please try to compare this with the earlier cases space of events; the space of events earlier told, where only related to the local knowledge, that is, when there was a cloud in Chennai; when there was a fog in Chennai; when there was a sunshine in Chennai; we are trying to estimate the probability of occurrence of rain in Chennai. We are not considering the influence of neighborhood knowledge on Chennai.

However, in this case, we are not considering the space of events within Chennai; we are looking at the space of events may be extended to the neighborhood and trying to find out what is the probability of occurrence of rain in Chennai if there would have been rain in Andhra and there would have been rain in Bangalore. So, the equation is very easy to understand and express. So, probability of occurrence of event; so, that is rain in Chennai is essentially probability of occurrence of rain in Chennai because of rain in Andhra into probability of rain in Andhra plus probability of occurrence of rain in Chennai because of rain in Bangalore; probability of rain in Bangalore.

So, you can simply substitute the values; probability of occurrence of rain in Chennai because of the rain in Andhra is 0.4. So, 0.4 multiplied by the probability of occurrence of rain in Andhra is 0.7 plus probability of occurrence of rain in Chennai because that rain is happening Bangalore is 0.6 we can see from the previous historic data. So, 0.6 multiplied by the probability of occurrence of rain in Bangalore. So, what you see from the data now is 0.45. So, you can compute this; you can always find for the probability of rain Chennai.

If you know that, there is a probability indicated the meteorological data that, there is going to be rain in Andhra tomorrow or there is going to be rain in Bangalore tomorrow. One can easily find out this, the total probability confluences the knowledge status of the neighborhood to that of the interesting event of your choice. One can use this data interestingly in the other way. Let us see how. So, this is what I am saying as an equation 9. This is the 9th rule.

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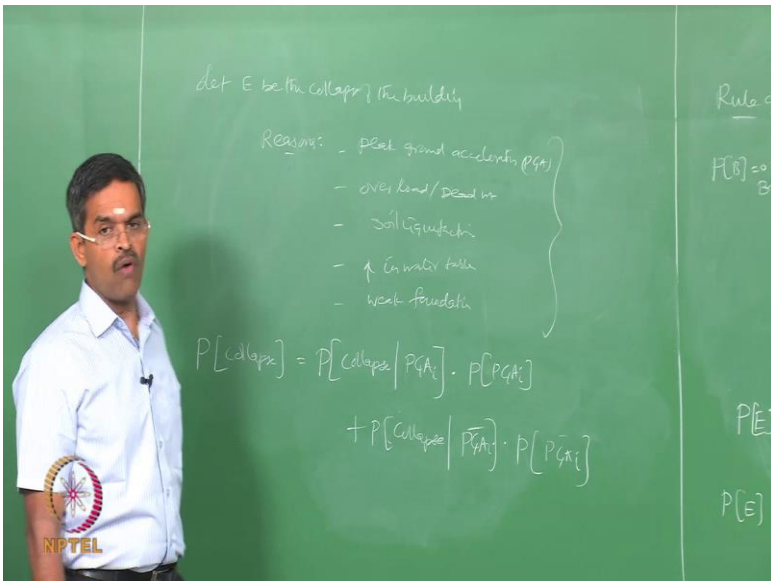

Now, I want to use this data slightly in a different way. Let us say I am looking for the collapse of the building. Let E be the collapse of the building. There are many reasons why this collapse could happen in a given building. It can be due to; what are the reasons? So, we are trying to ascertain the space of events for the occurrence of these events. That’s what we are saying reasons. It can be peak ground acceleration. Let us say PGA. It can be overload in terms of dead weight. There can be soil liquefaction. It can be raise in water table. It can be due to weak foundation. There can be many spaces of events, which can result in failure or collapse of a building. So, what we call these are all space of events; I am trying to use the knowledge of equation 9 to express this slightly in a different manner.

Now, I can say the probability of collapse of a given building can be written as probability of collapse given PGA of i buildings in the given locality multiplied by the probability of occurrence of that PGA of i plus probability of collapse of the building given because of negation of PGA i; I mean the collapse not due to the peak ground acceleration, but all other reasons multiplied by probability of PGA i negation. I am actually using the same algorithm, but slightly in a different understanding.

So, instead of counting for all the neighborhood data individually and getting influence on them, you can always focus on one specific event and try to get the influence of all other events on this. This is very useful. Especially we look at the failure of an offshore structure because of one specific predominant factor, which may not be peak ground acceleration, but can be corrosion.

So, the structural member can fail because of corrosion, because of improper welding, because of shock and impact loads; there can be many space of events, where we need not have to find out the influence of each one of them separately on the collapse of the structure; we can always say the deterioration happened predominantly due to the corrosion effect and what is their influence and what is the influence of all other events on the collapse. So, total probability theory helps us to connect the knowledge level of different status available in a given system overall to express the confidence of occurrence of a specific event of your choice as expressed by equation 9. Let us move this discussion further to take it to rule 10.

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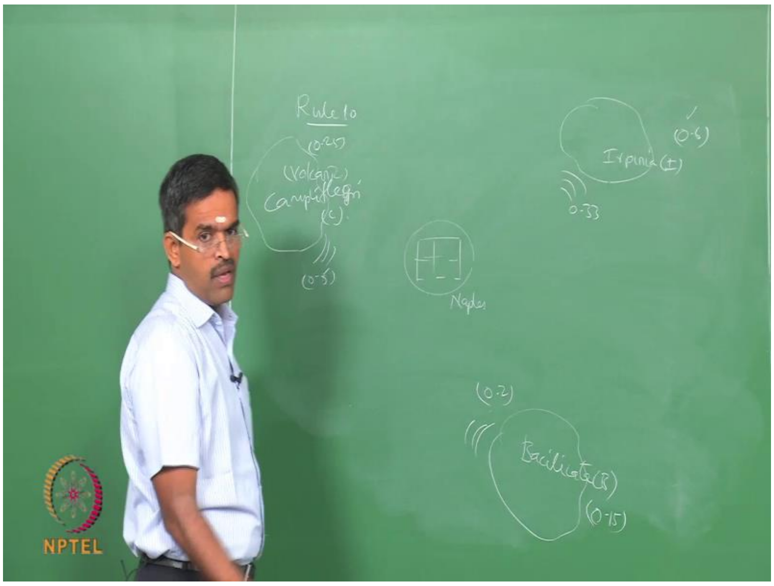

Now, I am going to look at an interesting example in this case. Let us say I have a city; I have a building let us say a platform located in a site called let us say Naples. I am giving an example of a different country Italy, so that it is easy for us to understand and interesting also. Naples is closely located to three other cities, which is one is called

Irpinia, which is indicated as I; other is called Bacilicata indicated as B; third one is called Camplifegri indicated as C. All these are earthquake prone zones.

In fact, Camplifegri is volcanic as well. So, obviously, any volcanic eruption happening in Camplifergi can cause the occurrence of earthquake or a failure of a building due to earthquake in Naples, because they are all closely located. It appears as if we are discussing the rule number 9 once again here; but, I am looking for a reverse problem here. I will come to that point exactly now.

Let us say occurrence of earthquake in Irpinia is 0.6, which is known. And, if earth quake occurs in Irpinia earthquake felt in Naples is 0.33. Similarly, occurrence of earthquake in Bacilicata is 0.15. If there was an earthquake in Bacilicata in the previous history, occurrences are felt – feeling of the earthquake happening in Naples is 0.2. Occurrence of volcanic eruption in Camplifegri is 0.325 let us say. And, if at all there would have been a volcanic eruption in Camplifegri, earthquake occurred in Naples; that influence was taken as 0.5. These are all data collected from the literature, which are essentially from the meteorological data of Europe. Of course, the numbers may not exactly equal to the second order or second digit, but they resemble the meaning what I am expressing here.

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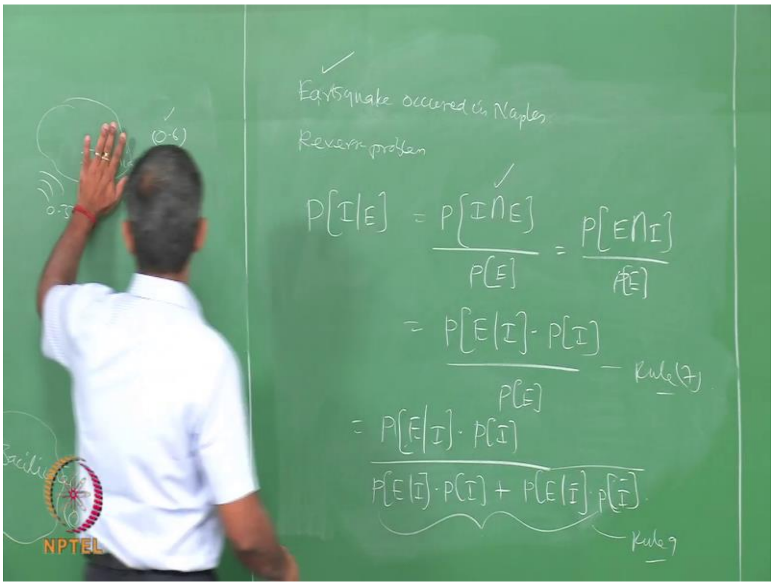

Now, my interest is what is the possibility of expressing the statement; that is, I have an earthquake in Naples. Let us say earthquake occurred in Naples. I want to know what is the probability that is earthquake is due to event occurred in Irpinia; that is a reverse problem. Earlier, in the total probability theory, we have used the knowledge of different space of events as knowledge level of the neighborhood to find out the probability of occurrence of an event at specific location maybe in Chennai from Bangalore and Andhra Pradesh.

But, in this case, I am looking for a reverse problem. The reverse problem in the sense, the event has occurred in Naples. I wanted to know what is the probability that this occurrence has happened because there was an event in Irpinia. So, I am looking for the probability of Irpinia, where the event occurred in Naples, which is expressed as probability of let us say I intersection E by probability of E, which is as same as probability of E intersection I by probability of E by rule 6. We already know this. I can also expand this as probability of E given I with probability of I by probability of E. This is as per rule 7. We already know this.

We can also express this as probability of E of occurrence of I probability of I divided by probability of E occurrence of knowledge of I with probability of I occurrence plus probability of event in E with I bar with probability of I bar. Of course, this can be derived from rule 9. Just now we saw. I gave an example; just now we saw. Now, the question is I do not have the probability of I bar, which is a negation of I mean except Irpinia, I do not have other things together. So, I can expand this further. Let me rub this here.

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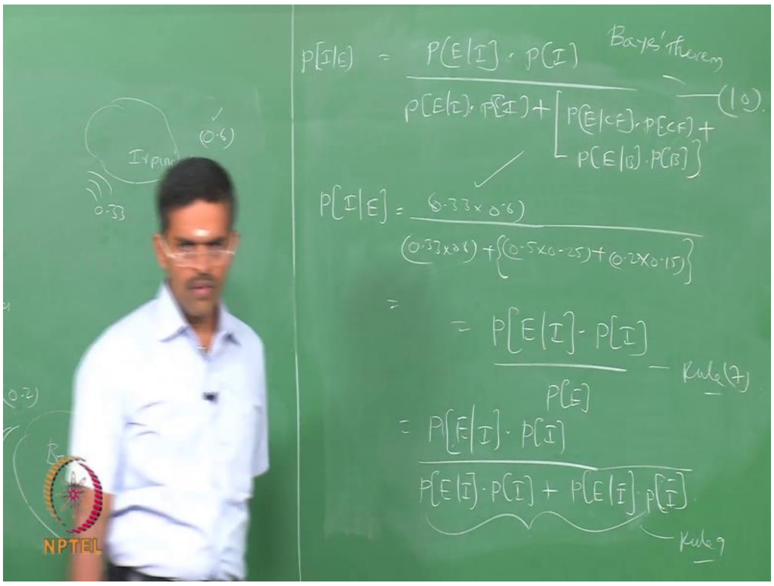

I will expand this part further which can be now; we are looking for probability of Irpinia on Naples, which can be probability of E given I probability of I divided by probability of E given I multiplied by probability of I plus probability of E Camplifegri – probability of Camplifegri; probability of E given Bacilicata – probability of Bacilicata. So, we have expanded I bar using the remaining - two knowledge on it. So, I can substitute these values very easily.

So, I can easily find what is the probability that the earthquake in Naples occurred because of that occurred in Irpinia can be simply probability of earthquake occurred in Naples because of knowledge in Irpinia, which is 0.33 multiplied by the occurrence of event divided by 0.33 and 0.6 plus – probability of event occurred in Naples because of Camplifegri – 0.5 and the probability of occurrence of event in Camplifegri – 0.25 plus – probability of occurrence of event of earthquake in Naples because of Bacilicata – 0.2 into probability of occurrence of event in Bacilicata – 0.15. So, I can easily evaluate this.

Similarly, I can also find what is the probability of earthquake occurred in Naples because there was a volcanic evolution in Camplifegri. So, I can always say probability of Camplifegri because of event in Naples; same way I can expand. So, very interestingly, the Bayes' theorem; this is actually an extension of Bayes' theorem is useful and this is actually the rule 10, is useful in expanding and understanding the possibilities of estimating occurrence of any specific event if you know the influence of the other similar events contributing this event and the knowledge level can be varied.

So, we have discussed in this lecture ten rules of probability, which are very useful, basically understandable and applicable with simple examples. We will extend this further discussion to physical meaning of these rules using possible reasoning in the next lecture. Should you have any doubts, please do not hesitate to write to me for any clarification, so that we will try to help you out. Always I will advise you to go through the parallel reading material advocated by me as references in this website of NPTEL.

Have a good day, bye.