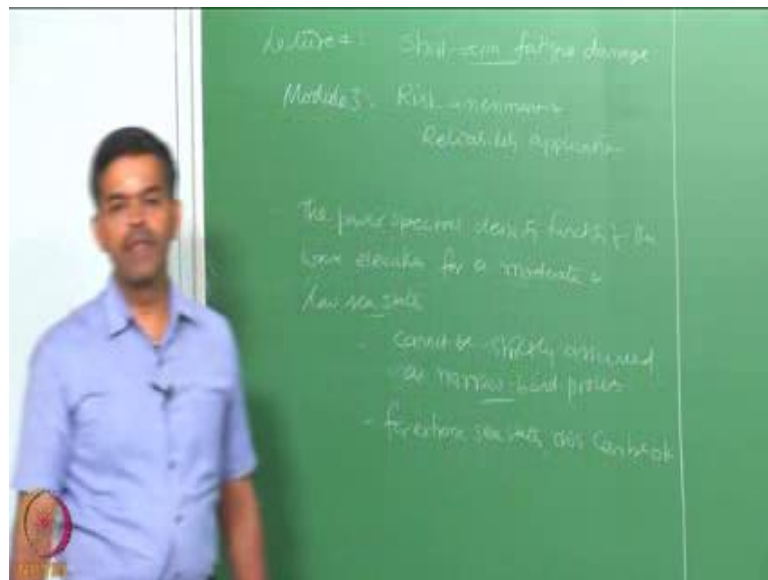


**Risk and Reliability of Offshore Structures**  
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**Module - 03**  
**Risk assessment and Reliability applications**  
**Lecture - 04**  
**Short-term fatigue damage**

Friends welcome to the 4th Lecture in Module 3.

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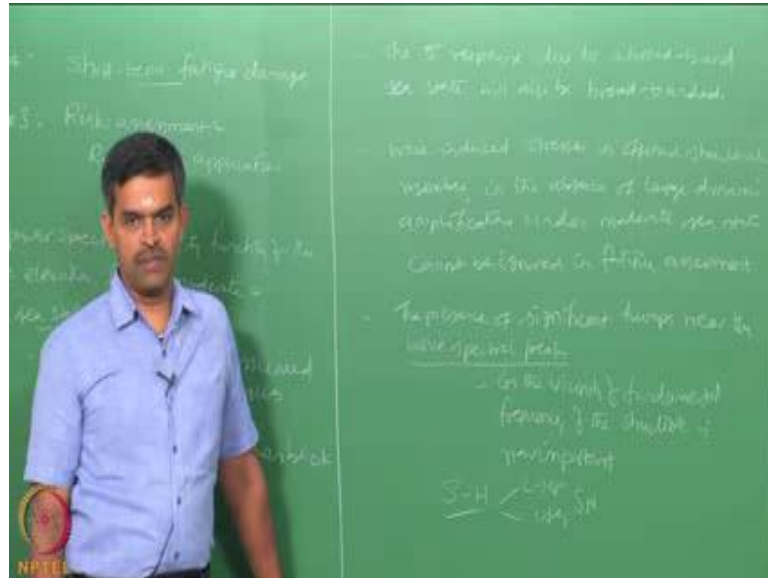


In this lecture we are going to talk about Short-term Fatigue Damage. This is a lecture in Module 3, where our focus is towards Risk Assessment and Reliability Applications. We all believe that fatigue reliability is one of the direct applications of finding probability of failure of a given system under cyclic loads where we have been discussing it continuously for about few lectures from now. So, I am talking about the short-term fatigue damage in this lecture which is online course on Risk and Reliability of Offshore Structures.

Now, the power spectral density function the wave elevation for low and moderate states cannot be actually strictly assumed to be a narrow band, moderate and low sea states cannot be strictly assumed to be a narrow band. You would realize from the last lecture that it is essentially important that the cyclic load defects cost with the high spectral

wave need to remain as relevant process that is why we model this from a relay distribution. Strictly speaking in reality the low and moderate sea states cannot be actually modeled as a narrow band process, whereas for extreme sea states this can work.

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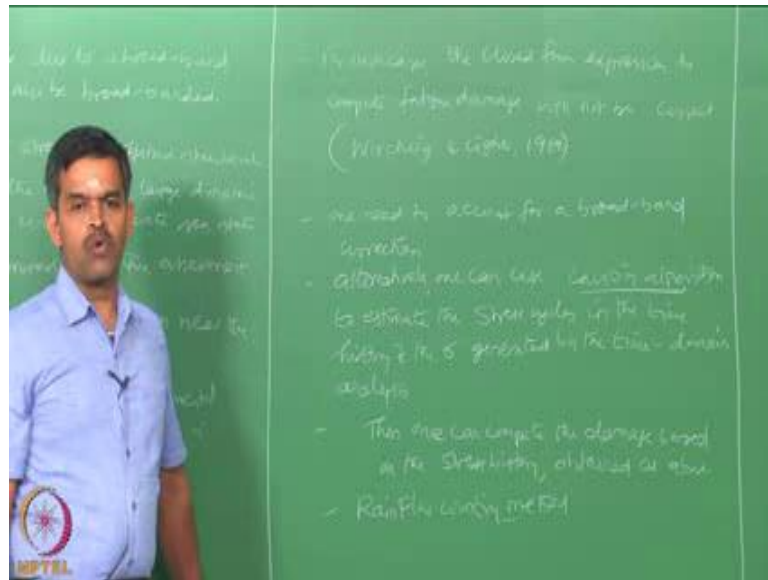


Therefore, the stress response due to broad-band sea states; will continue to remain as broad-band particularly when the dynamic response is not dominated by large amplification factors in any given natural modes of vibration. Therefore, the wave induced stresses in offshore structural members to be very specific in the absence of large dynamic amplifications which are exposed under moderate sea states. We will continue to remain as broad-banded which cannot be ignored in fatigue calculations. Someone cannot simply say I am going to idealize it as a narrow band process the stress ranges and tried to get the assessment which may not be correct, because in reality the moderate and low sea states are generally not narrow banded they are broad-banded.

So, therefore, they have significant humps near the wave spectral peaks. The presence of significant humps near the wave spectral peaks are really the concern one or may be vicinity of the fundamental frequency will become more severe, is now important because all through you understand that we are talking about the stress wave amplitude relationship for either here 2 segment S N curve or the 1 segment S N curve. That is how we assessing the fatigue damage so far in all our discussion what we had discussed in the earlier lectures.

So, here the peaks present near the wave spectral peaks and if that peaks happen to be closer to the vicinity of the fundamental frequency then the assessment of fatigue what we have been doing earlier without any modified methodology may give you a wrong assessment of fatigue damages. Therefore, what you do in such case.

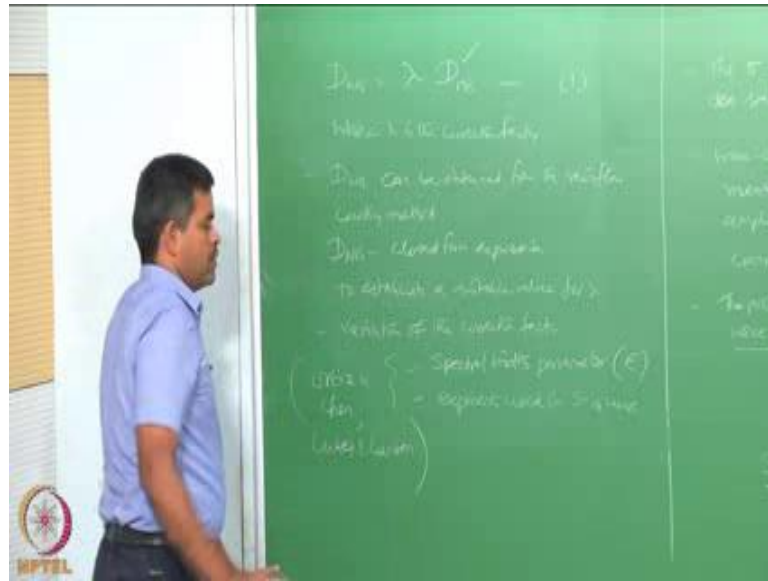
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For such cases, the close form expression for stress cycles and the fatigue damage derived for the spectral analysis will not be very accurate; the closed form expression to compute fatigue damage will not be correct as said by Wirching and Light in 1980. Therefore, a general approach to a broad-band correction is to assume a suitable counting algorithm. So, one need to actually account for a broad-band correction on the estimated values of the fatigue damage or alternatively one can go for suitable counting algorithm. We estimate the stress cycles in the time history of the stresses generated by the time domain analysis.

So then one can use this estimate of stress range history to obtain the damage, then one can compute the damage based on the stress history obtained as above from this process. Instead of assuming the stress history as a narrow band process you go for a typical counting algorithm. Counting means for a given stress cycle time history you try to find out how many number of times or what is the cycle by which the stress has exceeded a specific threshold value. So, rain flow counting method you seen as one of the alternative method is very popular in offshore fatigue assessments.

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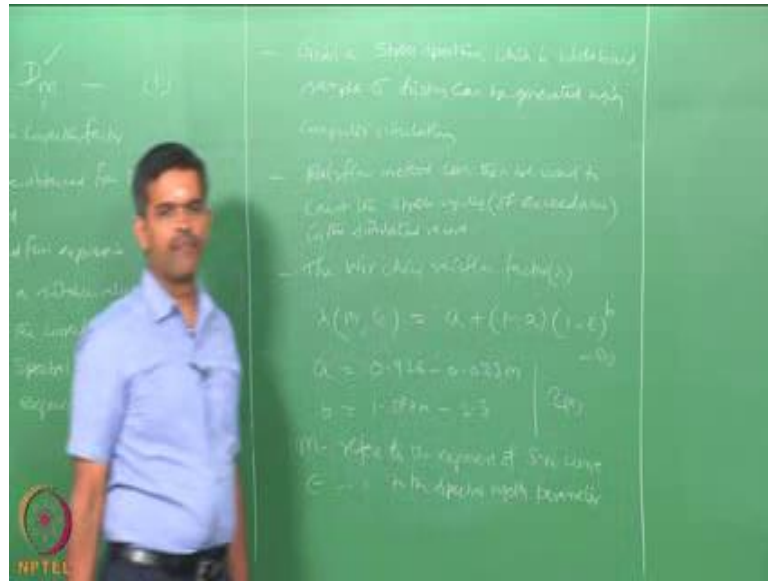
So, the narrow band and the wide band are connected by the following relationship if really wanted to adopt the correction factor to the fatigue estimates what you done from the narrow band you can connect this by a factor lambda, if you know the estimate of fatigue damages from a narrow band theory what you already assumed you can always adopt the correction factor alpha as we said here to account for the wide band corrections.

In this case  $d_w b$  denotes the wide band process obtained from the time domain analysis and  $d_n b$  denotes the narrow band process obtaining by making a narrow band assumption, whereas  $\alpha \lambda$  is what we call as a correction factor. The damage from the wide band can also be directly obtained from the rain flow counting method, whereas the damage from the narrow band has already have closed form expression which has been given and discussed at length in last set of lectures.

Now the issue is to establish a suitable value for the correction factor lambda. Now discussion is to establish a suitable value for lambda. Now there are very few spectral shapes which have been considered based on which the lambda can be computed. So, the variation of this correction factor depends on the spectral width parameter, let us say epsilon. And the second could be the exponents used in a sine curve approach.

Now, these are studied by various researches in detail by let us Ortiz and Chen, similarly Lutes and Larsen.

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Let us say given a stress spectrum which is essentially a wide band. The sample stress is history can be generated using computer simulation that should not be a big problem, because once we know the spectrum define can always find the stress history. Rain flow method can then be used to count the stress cycles in the simulated record. A stress cycles let say of exceedance or positive up crossing barriers in the simulated record. So, one can follow this approach therefore a narrow band and wide band can be connected by the relationship shown from equation 1.

Now the Wirching rain flow factor lambda is expressed by the following equation which depends on the slope or the exponents of S N curve, and the spectral width parameter which is a very important discussion or decision between the narrow band and the wide band, which is given by a plus 1 minus a 1 minus epsilon to the power b; equation number 2. Where, a is given by 0.926 minus 0.33 m and b is given as 1.587 m minus 2.3; equation 2.

So, we all know that m in this case refers to the exponent of S N curve and epsilon refers to the spectral width parameter.

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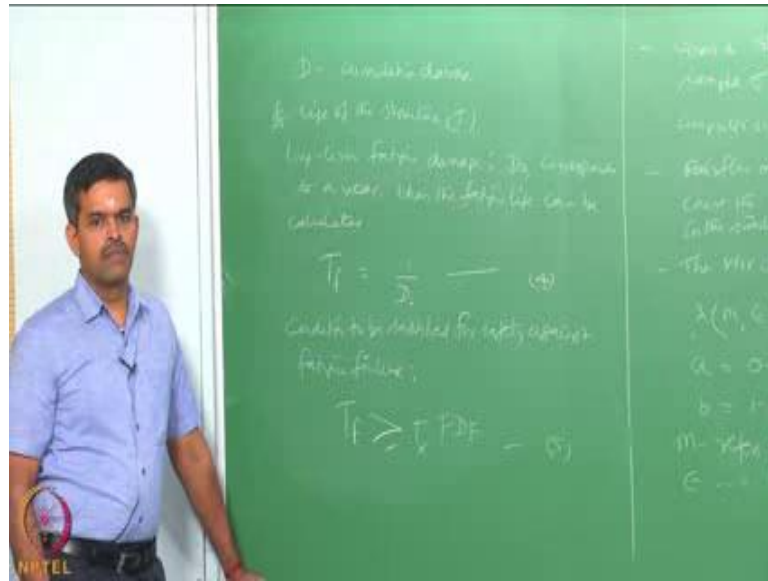


Of course,  $m$  refers to the S N curve corresponding to low cycle ranges only that is important. So, one thing either find out directly the up crossings of the stress cycle history from the wide band of the real sea state using the rain flow and using Wierching factors one can estimate and covert the data the damage obtained from the narrow band to that of the equivalent wide band. Why we are interested in the wide band because of a simple reason as we stated earlier, the sea states of medium and low cannot be actually processed or analyzed as a narrow band process.

However, in the whole context there are many uncertainties in fatigue analysis. Let us talk about the uncertainties in fatigue analysis may be arising from the wave loads essentially linearization of the wave loads that can be one major problem. Inaccurate stress concentration factors what we call S C F, inappropriate fatigue design curves that can be many uncertainties. Now the fatigue damage estimates or sensitive to these errors due to various factors or influenced cost by these factors in the overall estimate.

Therefore, to check whether the system is safe under the fatigue damage estimate one can use the relationship  $D$  should be less than or equal to 1 by FDF, where FDF is called fatigue design factor usually greater than 1. It is implied that the total damage  $D$  which is cumulative over the design life time  $T$  of the structure is what we are estimating. Alternatively if the calculator long term fatigue damage let say  $T_0$ .

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So equation D 3 we are trying to compare the total cumulative damage cost to the system that is fatigue damage if the life of the structure is T. And the long term fatigue damage estimate is D 0 which corresponds to a year, then the fatigue life can be calculated thus one is interested in knowing the safety check; fatigue life can be calculated, the fatigue life can be given by T f is equal to 1 by D 0; equation number 4.

Once we know this then the either the member or the joined is to be considered safe against fatigue failure if the following condition is satisfied. So the condition to be satisfied for safety against fatigue damage or fatigue failure, because that is what we are interest in reliability or application reliability is that T f should exceed or equals T x into FDF. Now various international courses give appropriate fatigue design factors as guidelines.

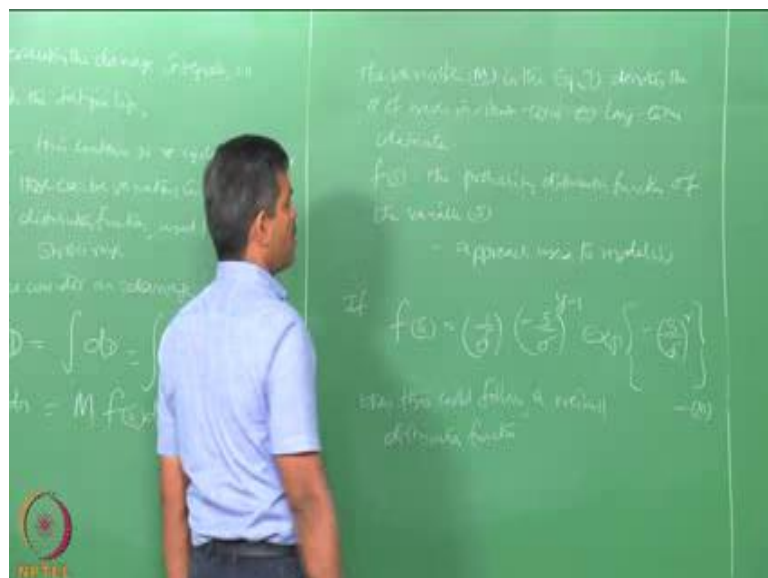


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Interestingly, while evaluating the damage integrals in estimating the fatigue life, let us say the fatigue damage integral contains basically n numbers of cycles to failure. The variants of the exponential distribution probably may be a different density function. So, there can be variations in the distribution function used for the stress cycles or stress ranges. Let us consider here damage integral say D, where D is given by integral of this which is going to be integral of d n by N, where d n is m into f of s ds.

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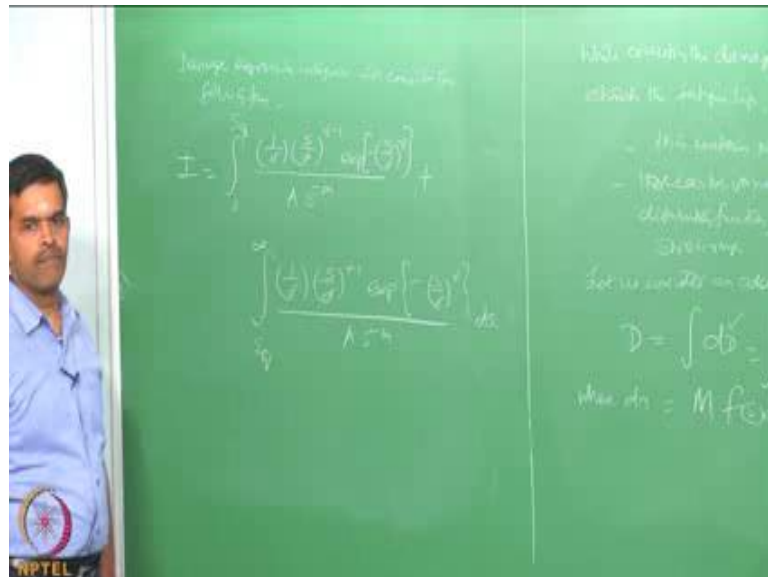




Now, the variable  $m$  in the equation 7 denotes the number of waves in a long-term or a short-term climate, and  $f$  of  $s$  would be the probability distribution function of the variable  $s$ . Of course it depends upon the approach used to model  $s$  that is important. If let us say  $f$  of  $s$  is given by  $\frac{1}{\lambda} \left(\frac{s}{\lambda}\right)^{\nu-1} e^{-\frac{s}{\lambda}}$ ; let us say equation a.

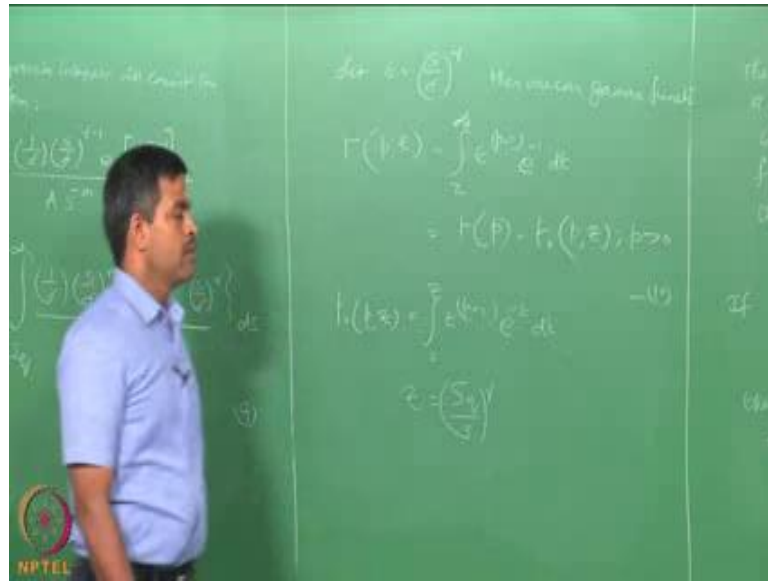
Then this could follow available distribution function, so that depends upon what is the particular pdf we are using for estimating the  $d_n$  value based on which the integral is evaluated.

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So, the damage expression integrals will consist the following form is integral I. Can be integral from 0 to stress value of  $q$  which can be  $\frac{1}{\lambda} \left(\frac{s}{\lambda}\right)^{\nu-1} e^{-\frac{s}{\lambda}}$ ; plus integral of  $s$   $q$  to infinity, we are splitting this integral into two parts is going to be  $\frac{1}{\lambda} \left(\frac{s}{\lambda}\right)^{\nu-1} e^{-\frac{s}{\lambda}}$  divided by  $\lambda^{\nu-1} ds$ ; equation number 9.

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While evaluating this integral let  $t$  be expressed as  $s$  by  $\delta$  to the power  $\nu$ . Then one can use gamma functions. Then the integral in the equation 9 can be replaced as let say that our gamma functions can be gamma function of  $p$  and  $z$  is set infinity  $t$  to the power of  $p$  minus 1  $e$  to the power of minus 1  $dt$ , which can be also said as gamma of  $p$  minus gamma 0  $b$   $z$  provided  $p$  is greater than 0; equation 10. Gamma 0  $p$   $z$  is also written as 0 to  $z$   $t$   $p$  minus 1  $e$  to the power of minus  $t$   $dt$  and  $z$  is actually  $s$   $q$  by  $s$  to the power of  $\nu$ . This gamma functions can be easily evaluated using a MATLAB program.

So, there are various uncertainties in fatigue estimates. We know that the input values of fatigue damage estimates which are important as slope of the  $S-N$  curve to be used for the design, the spectral width, the material characteristics, variation in input load effects, etcetera may cause errors due to many reasons. Therefore, the basic fatigue data which is used in developing  $S-N$  curve is actually the source of large scatter for these variations. So, fatigue damage models especially under random stresses can be only approximate it cannot be accurate.

There can be defects in discontinuity in welded joints which can complicate this process of estimate further, because a crack initiation and propagation depends upon the discontinuities at the point of origin of this fracture. This is also an important aspect that relates to the fatigue damage models. The statistical models which are generally used in ocean climate modeling can also have inherent errors. The force or the load model being

used for estimating the wave and current forces are only approximate and they can also often be used from empirical relationships. The hotspot stresses used in fatigue damage estimates contain also uncertainties because, a variety of stress analysis procedures each having different accuracy are adopted in practice.

Therefore, all these factors relate to accumulation of errors in structural modeling for obtaining the nominal stresses which is one of the important inputs for estimating the fatigue damage, therefore evaluation of the hotspot stresses using stress concentration factors etcetera can also lead to a lot of uncertainties in fatigue damage estimates.

Further in offshore structure influence of corrosion will also matter, dynamic amplification of local displacement structures like component structures can also be a problem, the cathodic protection which is being used as an inherent part in the design can also have influence on fatigue and estimates which are actually not modeled appropriately using the literature. Therefore, it is very interesting for us to actually apply these uncertainties to some extent in an experimental model. So, a tubular joint is investigated using experimental and analytical procedures the results are compared which we will discuss in the next lecture.

Friends, we are now going to discuss a very interesting problem where the tubular joints are actually loaded on an experimental setup and the stress concentration factors are derived based upon the loads estimated using the census on the failure joints then they are compared with low carrying capacity which are estimated based upon the empirical relationships advised by international codes to actually understand the reasons for these uncertainties.

As discussed just now there are many uncertainties which lead to a general statement making that fatigue estimates in offshore structures cannot be accurate they can be relatively better if I start using the experimental models for fatigue damage estimates as well, which we will discuss in the next lecture.

Thank you very much.