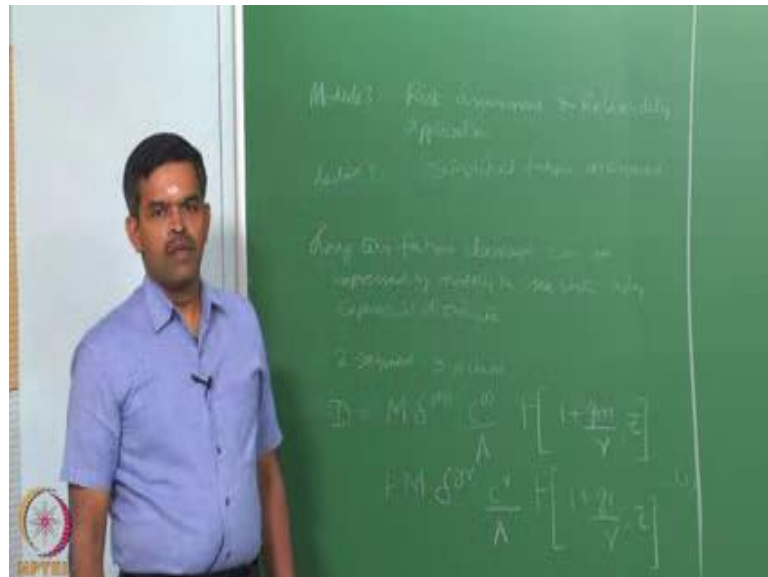


Risk and Reliability of Offshore Structures
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Module – 03
Risk assessment and Reliability applications
Lecture – 03
Simplified fatigue assessment

Dear friends, welcome to the third lecture on module-3 where we are focusing on Risk assessment and Reliability applications.

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This is going to be your third lecture in module-3, where we are going to discuss Simplified fatigue assessment. The long term fatigue damage which is essentially caused due to the sea state model can be expressed by using an exponential distribution as we saw in the last lecture.

Essentially we are looking for what is called the S-H relationship or on the other hand one can also select the parameters in the H n plot. So, now, the 2 segment S-N curve which can be used for estimating the long term fatigue damage by exponentially distributing the sea state can be given by a simple expression as we saw in the last

lecture, call equation number one. Alternatively one is looking for a single segment S-N curve.

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$$D = M S^m \frac{C^m}{A} F\left[1 + \frac{\gamma}{\delta} z\right]^{-1}$$

Simplified fatigue assessment

For an assumed long term S distribution, as a two parameter Weibull distribution.

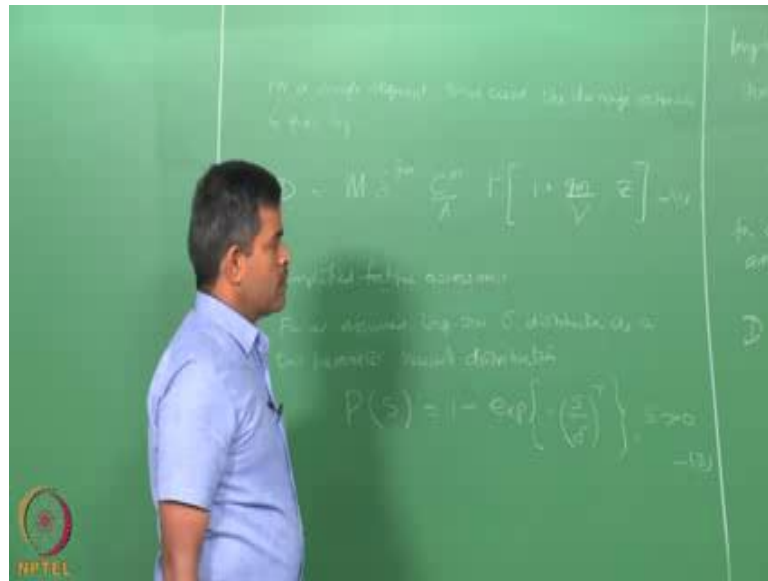
$$P(S) = 1 - \exp\left[-\left(\frac{S}{\delta}\right)^\gamma\right], S > 0$$

-(3)

Then the damage estimate can be computed by where the second segmental path or the second slope is removed.

Now, let us extend this concept for a simplified assessment. Now we are looking for let us say a simplified fatigue assessment if we assume a long term stress range distribution. So, for an assume long term stress distribution as 2 parameter Weibull distribution which can be said as the stress distribution probabilities P of S 1 minus exponential of minus of S by δ raise to the power γ where S is greater than 0. Based on the Weibull distribution assumes for a long term stress distribution the long term damage.

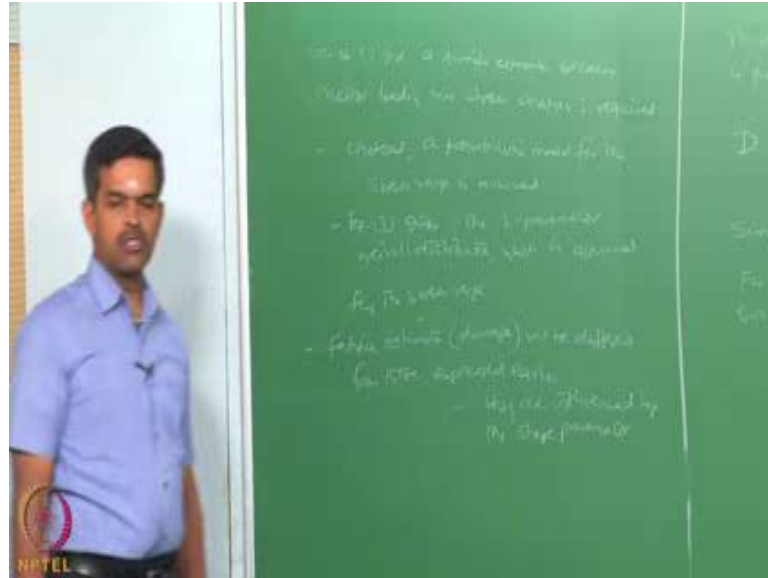
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For a single segment S-N curve is given by D as we saw earlier a gamma 1 plus. For your 2 segment S-N curve the long term assessment is given by M by A 1 plus m by mu z plus N T delta r by c gamma naught function 1 plus r by nu z is going to be equation 5.

Equation 4 for a single segment or equation 5 for a 2 segment which can give me a long term damage assessment is actually a method of giving a simple estimate of fatigue damage. How one can say that equation 4 and 5 gives me a simple estimate. Equation 4 5 give a simple estimate because neither loading nor stress analysis is required to obtain the information.

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Then what have we done in this case, in this case instead of that a probabilistic model for a stress range is assumed. So, in this case equation 3 gives the 2 parameter Weibull distribution which is assumed for the stress range. Now if I take life estimates calculated from the above discussion or from the above equation they actually vary from that of those expressions what we gave in the earlier lectures because they will be influenced by what is called as a shape parameter.

So, fatigue estimates in fact, the damage - long term or short term both will be different from those expressed earlier due to the basic reason because they are influenced by the shape parameter considered for the probability distribution of the stress range. Now this causes really an ambiguity in the estimate because the equations where you assume here probabilistic model for the stress range gives me one type of estimates of long term fatigue damage. If we do not assume a probabilistic model and actually incorporate the loading and the stress analysis in total then it gives me a different kind of fatigue estimate this gives me an ambiguity, reduce this ambiguity - what one can do? One actually can calibrate the results.

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Calibrate the results by comparing it with that of detail analysis for specific case preferably offshore structures. That is important. We also said there is one more factor which plays a very important role in the fatigue estimates that is what we call the effect of dynamic amplification. In case of offshore structures, where essentially they are form dominant there is always a possibility that the dynamic response make it amplify at the sequence certain frequency bands because systems are actually designed to remain (Refer Time: 13:53).

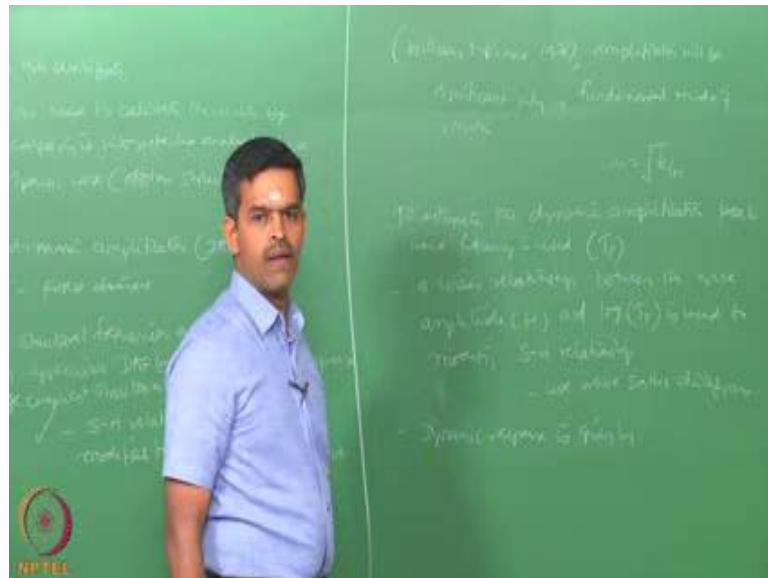
The buoyancy force on the system is exceeding the weight of the system itself. So, system remains a float. So, the transportation, commissioning, (Refer Time: 14:06), everything becomes simple. When it is (Refer Time: 14:11) commissioned they can be either ballasted and then deballasted or they can be anchored to the specific side using DPS dynamic portioning systems and the work can be carried out for exploration or production etcetera.

So, when we have a system which is undergoing larger displacements because of the wave action or because of the wind action then there is a possibility on certain frequency bands for which the system is not tuned it may undergo a large dynamic amplification. So, a system which has got the very high dynamic amplificationn may initiate tag

propagation and the failure caused by the fatigue damage can be even more serious. So, if the structural frequencies are capable of causing appreciable dynamic amplification factor to the response, I am talking about the structural response this is very, very true, in case of large compliance structures this is very, very important this will happen.

Then what we are going to do is the S-H relationship, what we discussed need to be modified to account for this effect.

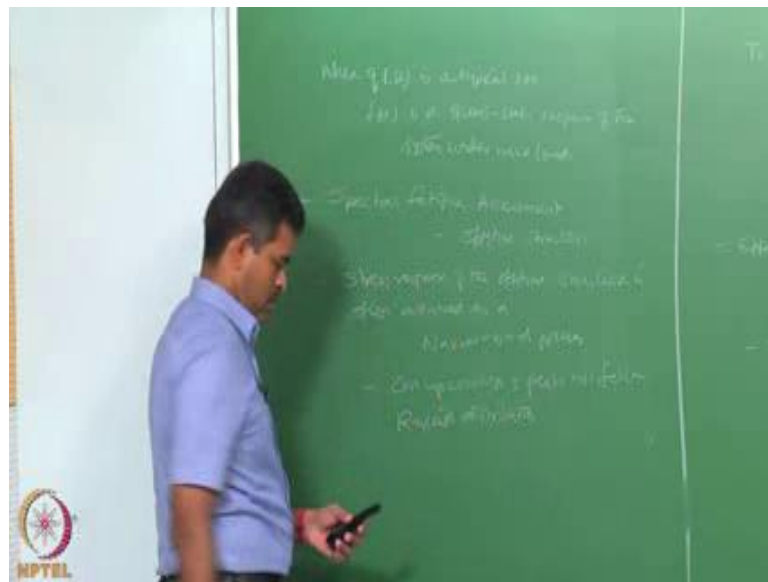
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But interestingly researches show that the amplification is significant, will be significant only in fundamental. So, as a designer one need to be careful in fixing the fundamental mode of the system because we all know that the fundamental mode of vibration is a structural characteristics where mass and stiffness of the system can be adjusted accordingly. So, that the fundamental mode of vibration depending upon the modal participation of the first mode can be selected in such a manner, can be designed rather in such a manner that they will not land up in large amplification factors because if the design in due a very large amplification factor then that may add up to the fatigue damage which will be caused by the cyclic load effects on the system.

Now, to estimate the dynamic amplification peak wave frequency is actually used. Then a linear relationship between the wave amplitude which is H and \log of this let us say, I will say T_P , peak period \log of this is used to modify the so called S-H relationship. Because we already said that the S-H relationship needs to be modified, if you have to account for the dynamic amplification caused by large compliance structures. So, you need to modify this S-H relationship with H and \log of T_P , one can use wave scattered diagram to do this, to pick up the peak wave frequency of the input loading.

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In that case the dynamic response is given by σ equals let us say f of H and q of H equation number 4, I mean this will be equation number 6 - already we have equation 4 and 5. So, equation 6 where q of H is a typical dynamic amplification factor and f of H is the quasi static response of the system of the system under wave loads.

Now, there is another discussion available in the literature to estimate the fatigue damage what we call as a spectral fatigue assessment, which is exclusively useful for offshore structures please spend some time in discussing this. Friends it is a well known fact that the response of a linear system subject to random loads with Gaussian distribution will also remain Gaussian. Although the Gaussian c surface forms the basis of a spectral analysis, the load is the response, the load acting on the system and the response which is

the effect of the load on the system can become also non Gaussian unless a consistent linear session procedure is adopted in the through and through analysis.


So, we are not bothered about the response of the system or the fatigue assessment, but we are focused on the stress response of the offshore system. Let us say the stress response of the offshore structure is often assumed as a narrow band process, for which both the 0 up crossings and the peaks will follow Rayleigh distribution.

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the cumulative fatigue damage, on the basis of Miner rule is given by,

$$D = \frac{T^*}{T_0} \left[\int_0^{S_q} \frac{P(S)}{N(S)} dS + \int_{S_q}^{\infty} \frac{P(S)}{N(S)} dS \right] \quad (7)$$

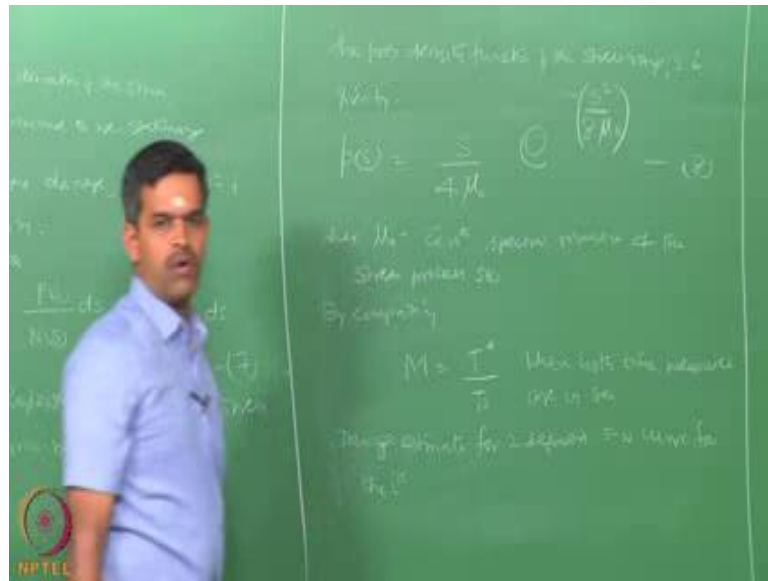
where, $P(S)$ is the Rayleigh distribution for the stress range in the narrow-band process.



Let us say, if T^* denotes the duration of the storm which is assumed to be stationary then the cumulative fatigue damage of the system on the base of Miner's rule is given by D equals T^* by T_0 integration 0 the stress range q P of S by n of S which are the distributions broadly distributions of the stress range $d S$ plus S q to infinity P of S by n of S , for the equation number 7 – where, P of S is the Rayleigh distribution for the stress range in the narrow band process.

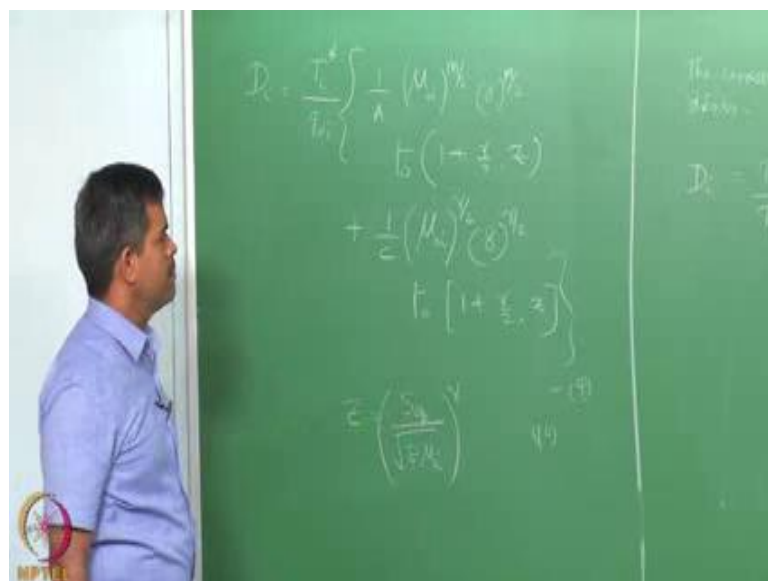
The probability function of stress range S is given by S by 4 μ_0 e to the power of minus S square by 8 μ_0 , equation number 8 – where, μ_0 is the 0th spectral moment of the stress process S of t .

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Let us try to compare this equation with the earlier expression what we had. Now by comparing we can understand that M will be equal to T^* by T_0 where both times are measured in seconds.

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Now subsequently the damage estimate for a 2 segment S-N curve for the i th short term sea state is given by; for the i th short term sea state is given by D_i is equal to T_i star by T_0 i by a μ_0 the 0th moment to the power M by 2, 8 to the power r by 2 gamma function, $1 + \gamma_0$, $1 + r$ by 2 z plus. 1 by c , I am talking about 2 segment therefore, there are 2 terms - one is with the slope a at the constant a , other is the slope c at the constant c as we discussed earlier also; μ_0 i raise to the power r by 2, 8 raise to the power r by 2, gamma naught $1 + r$ by 2 z - equation number 9. Where z can be the stress range S_q by root of 8 μ_0 i to the power.

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$$D_i = \frac{T_i^k}{T_0^k} A (\mu_0)^{m/2} (8)^{m/2} \Gamma\left(1 + \frac{m}{2}\right)$$

- (11)

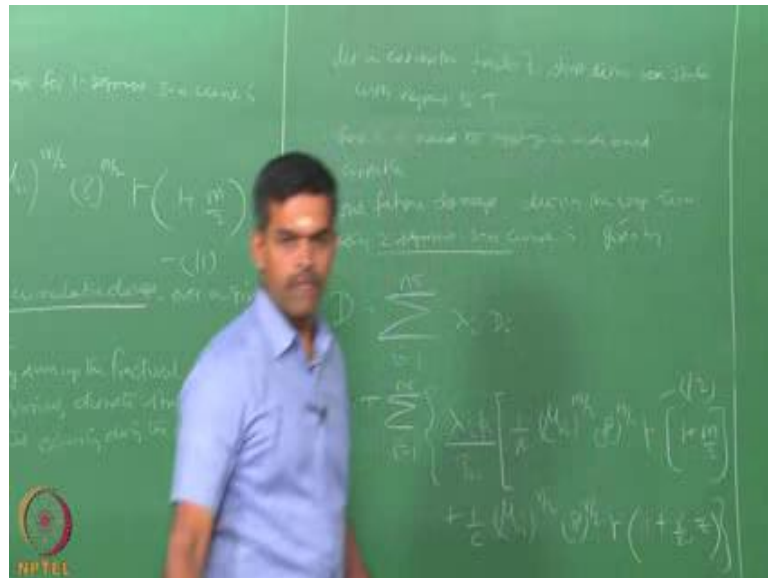
To obtain the total cumulative damage over a given period of time (T), one may sum up the fractional damages due to various discrete short-term sea states occurring during the period, T .

The corresponding one segment curve corresponding damage for one segment S-N curve is given by D_i that is for the i th short term sea state, T_i star by T_0 i a μ_0 i to the power M by 2, 8 to the power M by 2. In fact, in this equation also this should be M by 2. Please make a change in equation 9, it should be M by 2, this slope is M this slope is r here, M by 2 gamma function $1 + m$ by 2. So, equation number 11.

Now, I am interested in (Refer Time: 31:38) the total cumulative damage over a total period of time. To obtain the total cumulative damage over a given period of time T . One may sum up the fractional damages, due to various discrete short term states occurred or occurring during the period, one can sum up. So, let us say for this purpose let us denote

the fraction of time of the i th short term sea state over a long duration T . So, we need express. So, let us express, let us try to express the fraction of short term sea state with respect to the capital T what you are looking for the overall period.

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So, that the above estimates of damage or based on a narrow band model. So, there is a need to apply a wide band correction to this. So, there is a need to apply a wide band correction. So, in that case the total fatigue damage during the long term using 2 segment S-N curve is given by D is going to be - I am talking about the cumulative one, I am talking about the total damage over a period of time T . Therefore, I should say D is going to be the sum of i equals 1 to n $\lambda_i D_i$ that is the factor which is going to make the correction for a wide band because the process assumed is a narrow band which I say is going to be T that is period over which I am looking at a damage.

Summation of i equals 1 to n $\lambda_i t_i$ by $T O_i$ of, I am talking about the 2 segment S-N curve. So, there are 2 different segments. So, I will use the same equation 1 by a 0th moment raised M by 2, $\Gamma\left(1 + \frac{M}{2}\right)$ plus the second slope which is $1 + c$, μO_i raise to the power of r by 2, $\Gamma\left(1 + \frac{r}{2}\right)$ raise to the power of r by 2, gamma function of $1 + \frac{r}{2}$ which I can call equation number 12.

So, friends in this lecture we have tried to understand how the fatigue assessments can be easily model using spectral estimates. We have also seen how a simplified fatigue estimate can be replaced, instead of going for a full sea state model where the stress range and the load amplitudes or model. We have also employed the discussions of equations explaining, the 2 segment curve and the 1 segment curve appropriately with different slopes a and c or constant a and c and the slopes m and r respectively.

We have understood that how one can easily model and estimate the fatigue damage from the i th sea state which is modeled as a Weibull distribution, with the 2 parameter where sea states or the response stress ranges becomes the Rayleigh distribution. Then once you know the damage cause with the i th sea state for a specific case, you want to find the cumulative damage can sum them up over a period of time t where we have shown the discussions for understanding the cumulative damage D for the sea state of i th short sea states which we have estimated using a 2 segment as well as one segment S-N curves.

We will extend this discussion for short term fatigue damage in the next lecture. Then we will also discuss a specific case study experimental investigations where we estimated the stress concentration factors and the fatigue damage on tubular joints and the different kinds of combination of loads, and compare that with estimates given by the literature and highlighted a difference of understanding between the actual behavior and that of recommended for the design curves.

Thank you very much.