Risk and Reliability of Offshore Structures
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Module – 02
Reliability theory and Structural Reliability
Lecture – 26
Stochastic process II

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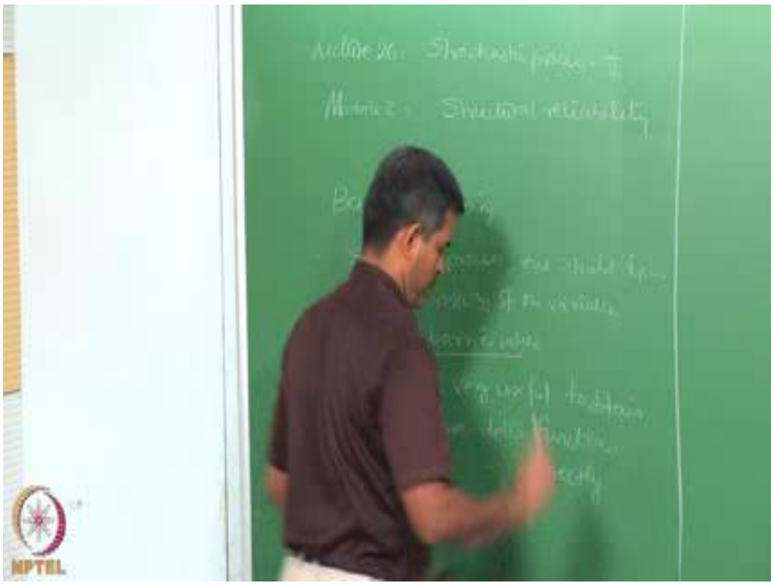

Welcome friends, we will take about the 26th lecture, where we will address the stochastic process and then move onto fatigue reliability. This is 26th lecture in module 2, where we are focusing on structural reliability. This is an online course titled risk and reliability of offshore structures under NPTEL, IIT Madras. In the last lecture, we discussed that the stochastic process can be qualify as a random process or stochastic variables can be qualified to be called as a random variable provided they are selected in the interval t 1, t 2 etcetera where t is the space of index set. As an example we applied this indication as a realization of this process on a Gaussian process and we also showed that Gaussian process will remain strictly stationary.

One important issue related to stochastic process is what we call as barrier crossing. Now what is the barrier crossing issue? For a stochastic process, it is very important to know the number of crossings of the given threshold value what we call barrier. So, in a stochastic process, one will be interested to know the number of crossings of the variable over the barrier value. The barrier value can be any value what the designers or analysts once this value is fixed as a barrier or an upper limit, one would like to know how many number of times a variable has crossed this threshold value. Now, interestingly the extension of this is essentially applied to what we call fatigue reliability. So, this is very useful to obtain the direct delta function which can be now done indirectly.

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Let us try to understand the realization of the stochastic process again. Let us say we draw a stochastic process variable, the space of event or the index is t. Let us say a signal typically looks like this. Let me draw a threshold value which I am going to indicate in my analysis, let us say this is my x of t, where threshold value is marked in this color. I call this let say x of t equal to some value which is z. Now I want to mark the threshold crossing values let us say 1, similarly 2, 3, 4, 5 and so on. Let us say we start the signal varying from the limits t 1 and the signal ends at the limit t 2. Now to solve the problem of estimating the expected number of positive passages why it is called positive passage, I am looking only the upward crossing, this is actually not touching the value. So, it is not crossing the barrier I am only trying to mark the upward crossings in the given system.

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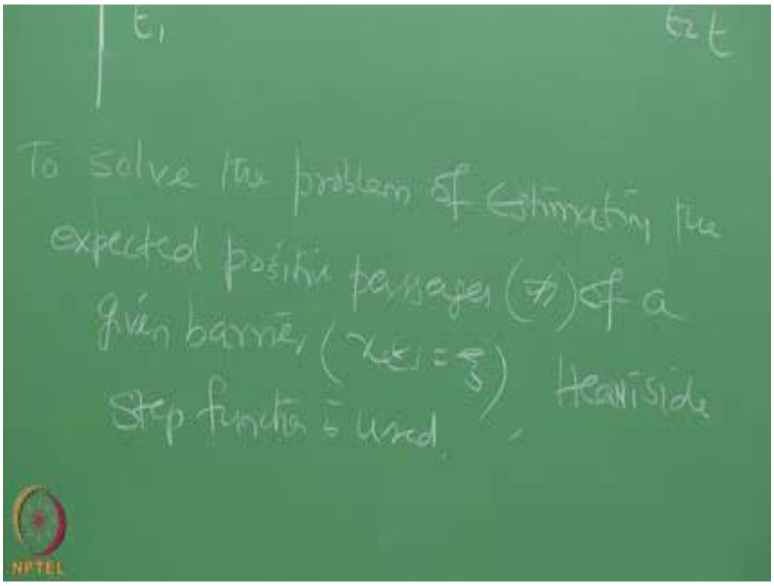

Now, to solve the problem of estimating the expected positive passage in terms of its numbers of a given barrier in this case the given barrier is a fixed value zeta. We look for Heaviside step function.

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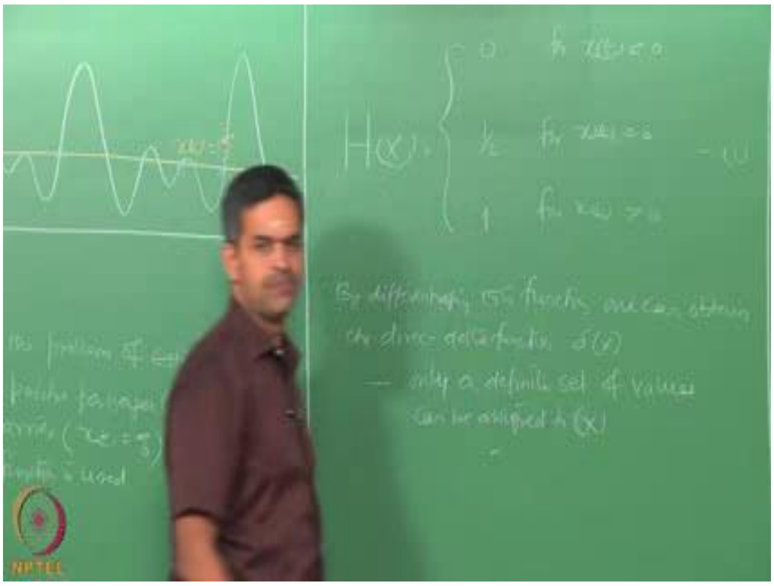

The Heaviside step function H of x is said to be 0 half and 1 for different conditions for x of t less than 0, for x of t equals 0, for x of t infinity times, I will call the equation number 1. By a formal differentiation of this function, one can easily get the direct delta function, which is indicated as delta of x.

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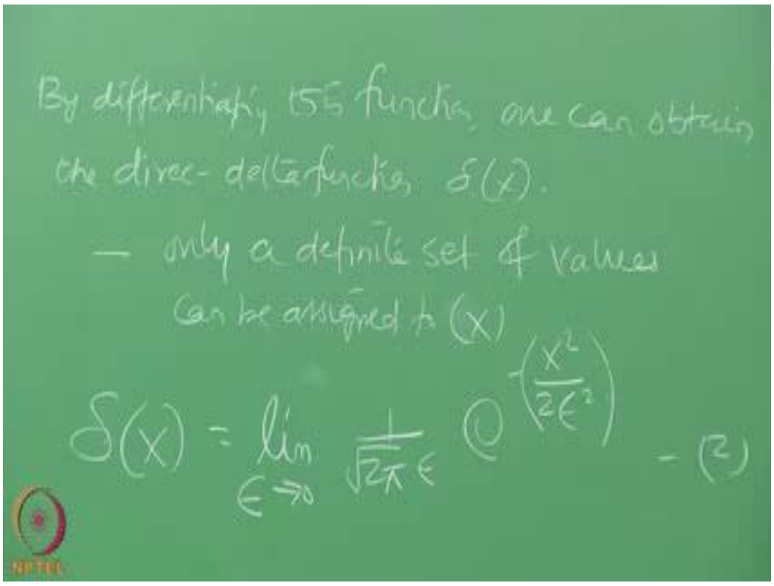

Then the advantage of that method of obtaining direct delta function is only a definite set of values can be assigned to x which is given by the direct delta of function of x limit 0 1 by 2 phi epsilon e to the power of minus x square by 2-epsilon square equation number 2. So, one can easily find the direct delta function which has an advantage of having the qualified definite set of values which can be assigned to the variable x in a stochastic process provided we will be able to find out the number of positive passages using the Heaviside function as we just presented in equation 1.

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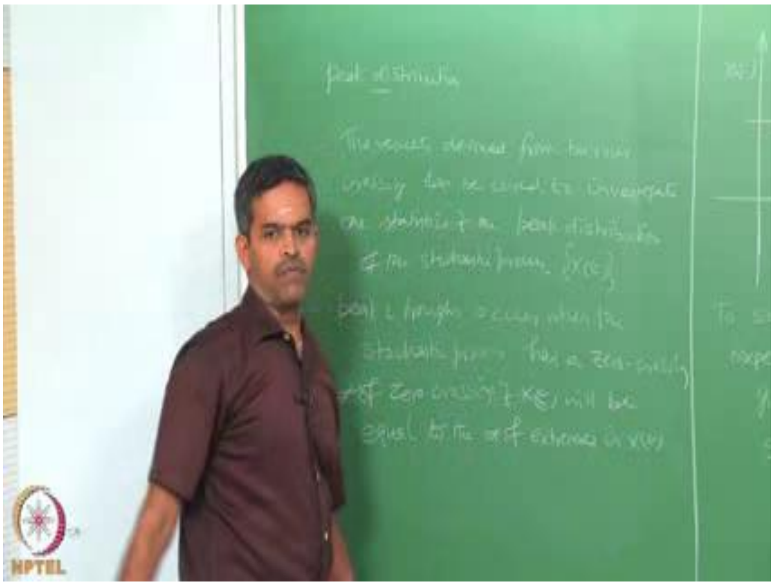

Now, the next concern about the stochastic process is the peak distribution because this is another concern otherwise in general in probabilistic estimates using random variables. Now the results derived from barrier crossing can be used to investigate the statistics of the peak distribution of the stochastic process. In our case the stochastic process, what we are talking about is x of t where t is a subset of capital T is called the index set. Interestingly the peaks and troughs occur when the stochastic process has a zero crossing the number of zero crossings of this process is equal to the number of extremes in the given process in x of t.

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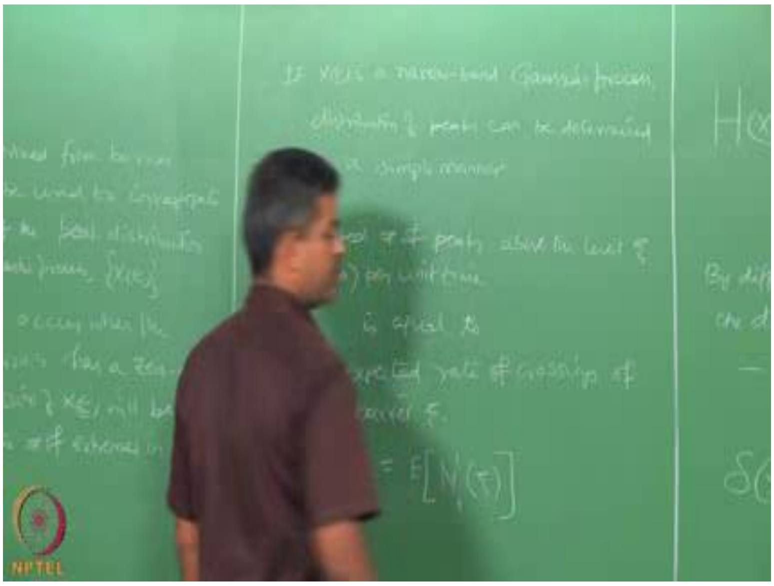

If x of t is a narrow band Gaussian process then distribution of peaks can be determined in a very simple manner the expected number of peaks above the level in our example of discussion we say the level is zeta and we want to say zeta is more than 0 we are looking for the positive passage. So, the expected number of peaks above the level zeta per unit time is actually equal to the expected rate of crossings of the barrier zeta which is actually equal to the expected value of n prime plus of zeta.

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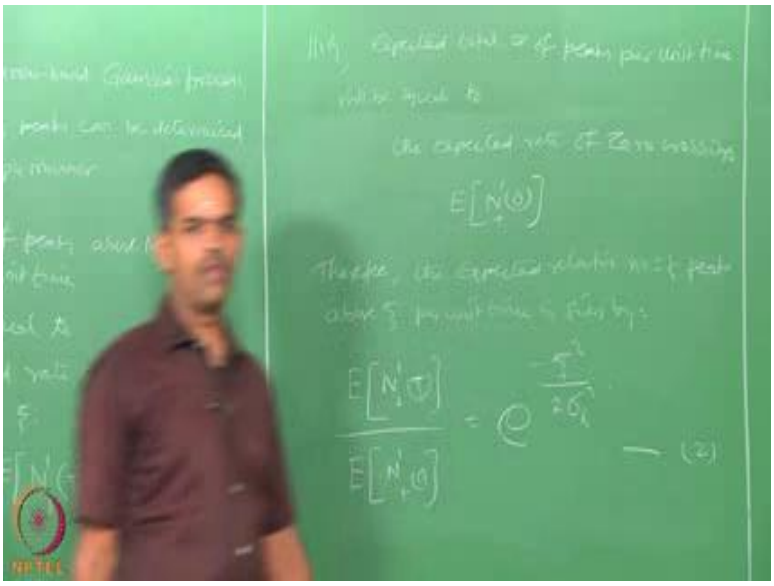

Similarly, the expected total number of peaks per unit time will be equal to the expected rate of 0 crossings which will be simply E N prime plus 0 because we are looking for the positive passage. Therefore, the expected relative number of peaks above zeta per unit time is given by expected number of peaks as given by this value divided by expected number of peaks as given above 0, which is same as minus zeta square by 2 sigma x square, let us call this equation number 2.

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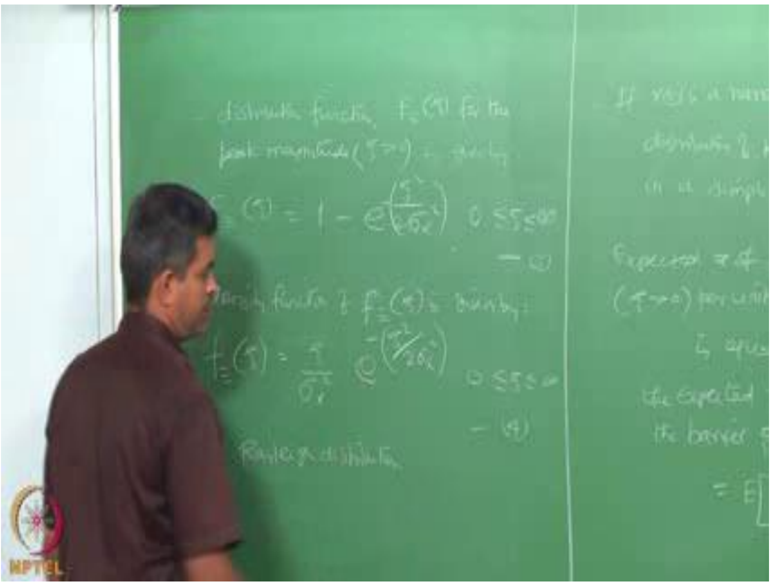

The distribution function which is F equals of zeta for the peak magnitude for the condition zeta is more than 0 is given by f function of zeta is 1 minus e zeta square by 2 sigma x square there is a minus sign here for 0 less than zeta less than infinity which we call as equation number 3. The corresponding density function which is of zeta is given by we will talk about small f density function distribution function of capital f zeta is given by zeta by sigma x square e to the power of minus zeta square by 2 sigma square for zeta sorry 0 less than zeta less than infinity - equation number 4. Interestingly equation 4 will expected to follow a Rayleigh distribution.

So, friends instead of handling a random variable, we alternatively pick up the stochastic variable which can be expressed in a space set capital T, what we call as an index set. When the realization of this process happens we can always check the positive passages of this variable with respect to any predefined threshold value what we call as a barrier. We can always find the barrier-crossing rate as expressed in the equations given to you on the blackboard just now, this can be applied directly for fatigue reliability. So, the concept what we discussed so far as the barrier crossing can be usefully applied directly in fatigue reliability analysis which we will discuss further. I will also take up a case study where we have estimated the tubular joints capacity, which can be one of the important inputs for reliability analysis of offshore structural joints.

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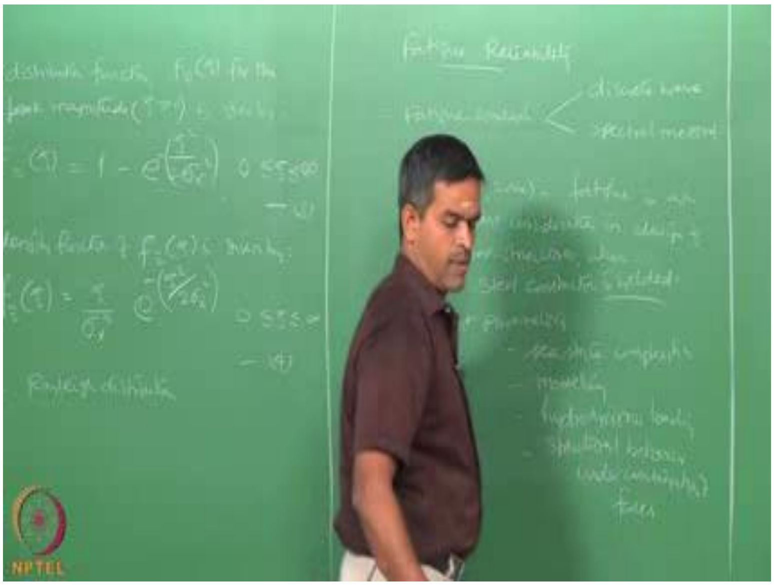

So, let us now expand our discussions further with a background of stochastic process onto fatigue reliability. Discrete wave and spectral methods are used generally used for fatigue analysis. We can have two waves by which can be done; one is what is called discrete wave approach, another is spectral method. We all know fatigue is a very important consideration in the design of offshore structures especially when the connections are welded in steel construction.

So, as indicated by ABS - American Bureau of Shipping 2003, 2004 fatigue is an important consideration in design of offshore structures where the members are essentially steel where steel construction is welded. So, we are talking about the welded joints and its capacity to resist the loads in terms of reversal of forces. Different parameters will affect this, different parameters for example, sea state complexities, complexities involved in modeling, hydro dynamic loading let say structural behavior under combination of forces will govern the fatigue failure.

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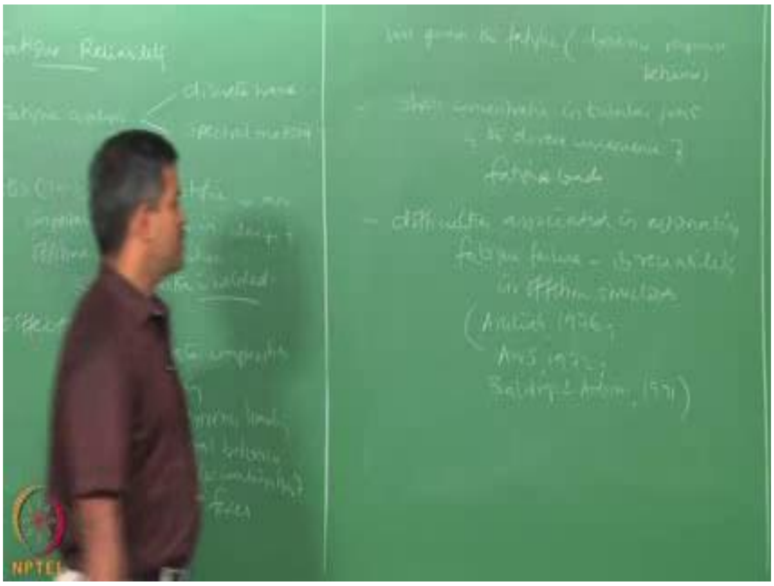

Especially, when you look for the dynamic response behavior, so friends one can easily understand here, fatigue reliability can be handled with input variables as a stochastic variable talking time as an index set, because we are looking for the dynamic response behavior which is predominantly affected by the factors which can result in a fatigue failure. So, if you ask me a question what is the direct influence of fatigue on the joints of steel construction, it results in what we call stress concentration in tubular is the direct consequence of fatigue loads.

Now, there are various difficulties associated in estimating the fatigue failure in offshore structures in terms of its reliability in offshore structures as indicated by Aruliab 1976 sorry 1976 American welding society 1972. You can also look at Baldrop and Adams in 1991. So, researchers have indicated several set of difficulties which are associated in estimating the fatigue reliability especially or in particular the offshore structures where the joints are essentially tubular. Now, one is interested to know what is the fatigue damage caused by the set of loads on the given system, and what could be the probability of failure if the fatigue loads exceeds the number of cycles of sustainability of this kind of reversal of loads by the material or with a joint configuration.

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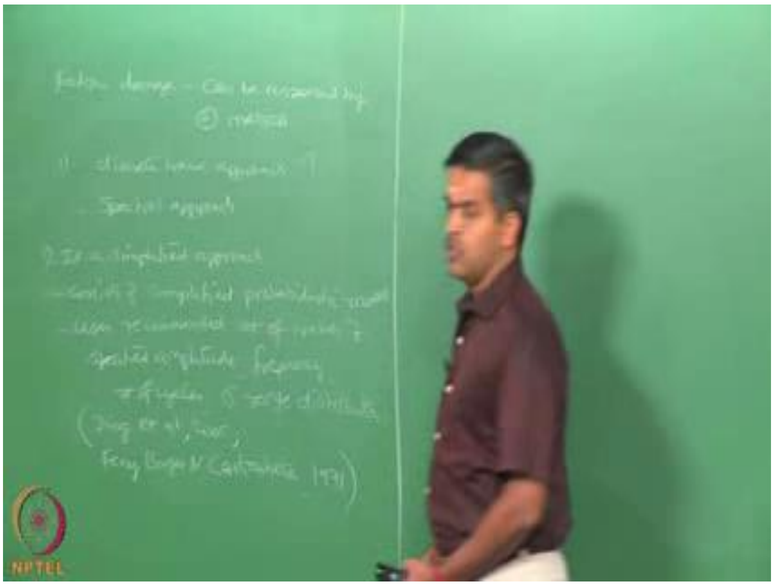

So, one is interested in understanding or identifying what we call fatigue damage this can be assessed by two methods one is the discrete wave of approach as we said in the beginning other could be the spectral approach discrete wave approach consists of simplified probabilistic models for the wave environment. It is a simplified approach consists of simplified probabilistic models. It uses the recommended set of waves for specified amplitude their frequency the number of cycles of occurrences or the stress range distribution as suggested by the literature the references can be ding et al 2005 ferry Borges and Castarheta 1971. So, discrete wave approach uses specific amplitude of waves the frequency content the number of cycles or the stress range distribution for estimating a fatigue failure which is more or less a simplified approach suggested by the researchers.

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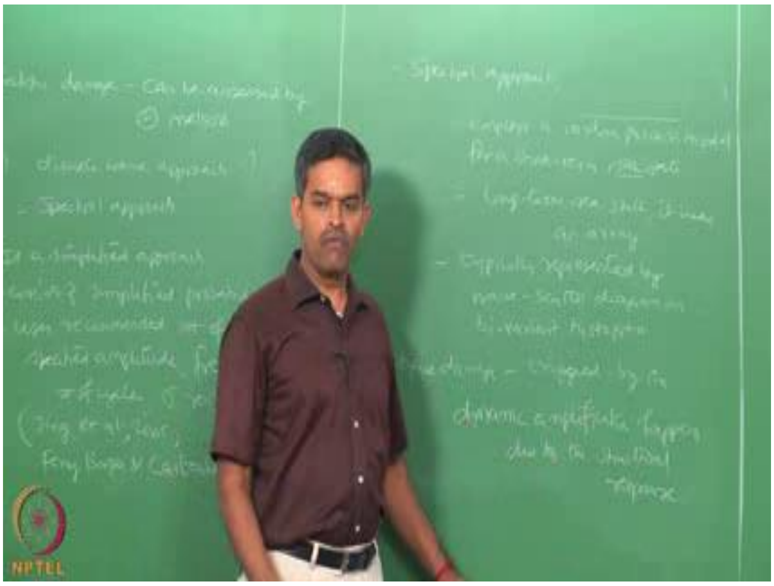

Alternatively, look at the spectral approach it employs a random process model for a short term sea state it looks for an array which are constituted for a long term wave environment. So, looks for a random process model or a short term sea state in case of long term sea state it uses an array on the random vectors now the random process or the sea state is typically represented by what we call wave scatter diagram or bi variant histogram.

We all understand and agree that the predominant load acting on offshore structures is wave loads cyclic stresses caused by this kind of loads induce fatigue damage in offshore structural members especially at the tubular joints of steel jackets and towers. Since, they are very high stress concentration zones the fatigue damage is generally aggravated by the possible dynamic amplifications caused on the structural response as well. So, here the fatigue damage is triggered due to or triggered by the dynamic amplification happens due to the structural response.

Now, interestingly we have formed dominated structural systems in the recent generation the structural responses are going to be very high and the dynamic amplification if not designed properly at certain frequency bands will be phenomenally or significantly high. So, the damage caused by the stress concentration factors at the tubular joints because of reversal of forces or reversal of amplitudes or directions of wave load acting on this members will cascade the effect of failure in the presence of larger responses which amplifies the failure because of the dynamic amplification factor presence in the given system.

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So, therefore, fatigue analysis or failure due to fatigue loads or in general fatigue assessment in offshore structures consists of establishing a fatigue demand of that element. Then compare it with this fatigue strength. If you look at this whole statement closely, this is nothing but the basic concept of reliability. You know what the strength of the member of the material is; you know what is the demand or the action applied by the forces on the member.

So, when you are able to estimate the fatigue demand of the set of structural elements in a structural form of an offshore structure, and also able to compare it with fatigue strength of that members or the structure. Then one can always assess whether there is any probability of failure caused by the fatigue loads what in general we call fatigue reliability as suggested by Goodwin et al 1999, Hasan et al 2009. So, this nothing but a broader base or what we call fatigue reliability.

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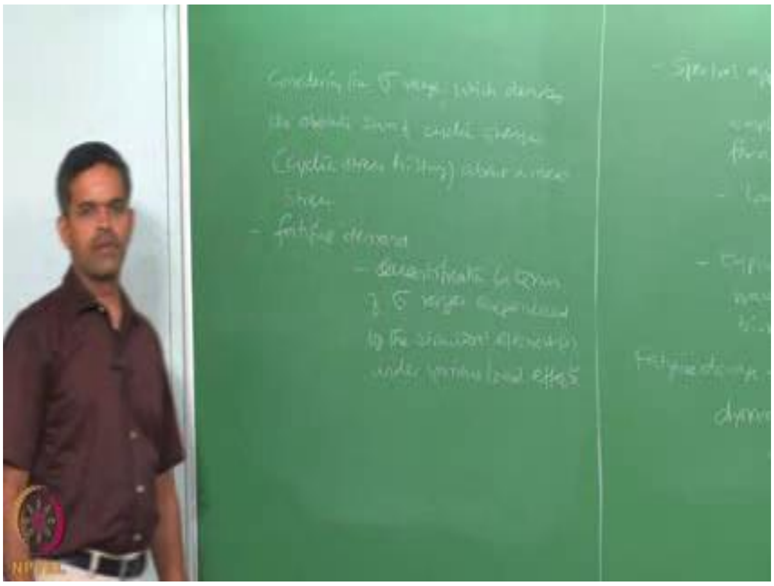

Considering the stress ranges, the material or the members can withstand which is denoted or which denotes let us say the absolute sum of cyclic stresses or one can say even cyclic stress history about a mean stress. Fatigue demand is nothing but it is quantification terms of stress ranges experienced by the member due to various load combinations.

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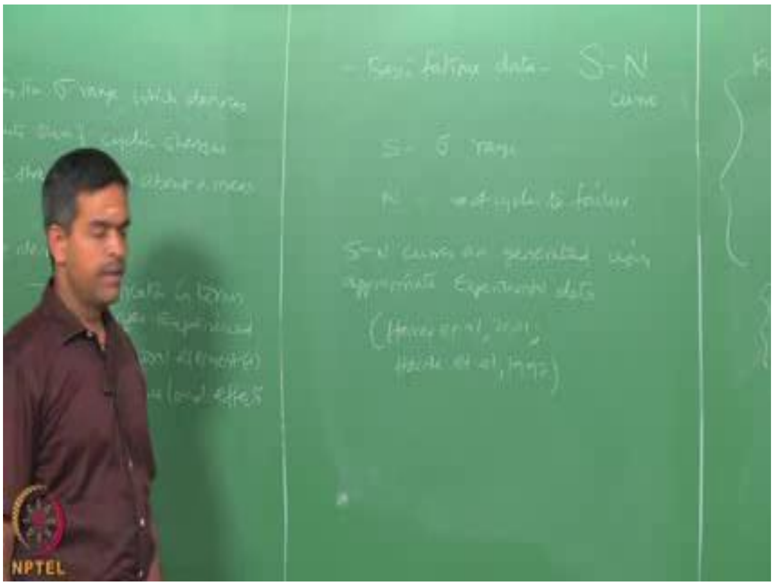

Basic fatigue data is actually the relevance to the S-N curve. Basic fatigue data has a very strong relevance to the S-N curve. S-N curve actually represents the stress range and the cycles of failure. So, S is the stress range and N is the number of cycles to failure. S-N curves are generally generated using appropriate experimental data. Of course, this is verified by many literatures proof. If you look at Haver et al 2001, Horde et al 1997, they have commented upon the limitations of the S-N curve obtained from the experimental verification. If you look the whole discussion in the context of offshore structures, we understand the primary source of loading in offshore structures arise essentially from motion waves which is cyclic in nature. So, now the description of the input load which is going to cause a cyclic effect on the offshore member can be described in two ways.

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So, the cyclic load which is causing fatigue damage essentially comes from wave loads predominantly in offshore structures. Of course, various researchers have experimentally analytically examined and verified the statement. As suggested by these researchers, the cyclic load effect caused by the waves on offshore structures can be described in two ways. One again using a discrete wave approach; alternatively, using a stochastic model. The discrete wave approach deals with nondeterministic nature of wave heights, whereas the stochastic model approach represents sea state in spectral character which is more realistic. Therefore, the second method is most probably and commonly used for fatigue assessment in offshore structures as suggested by Madavan Pillai and Veena in 2006.

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Let us say we want to represent the cyclic load caused by the wave effects on the structure by any one of these methods, now what is the major task as far as we are concerned in fatigue assessment. In the major task in fatigue assessment is to compute the stress concentration at critical points what we call as hot spots. So, at these points, fatigue damage will be estimated; in particular, we have to establish a relationship between the wave height and the stress concentration. So, the whole focus is to establish a relationship between the wave amplitude and the stress what we call S-H relationship at critical points essentially the hotspots. So, the whole exercise of fatigue assessment is converged to estimating or computing a stress concentration of hotspots.

So, we need to establish a relationship between the stress and the wave amplitude. To do this, various steps are involved; we will discuss this in detail in the next lecture. We will further elaborate the fatigue assessment in terms of fatigue reliability applied to offshore structures. Then we will take up very specific example of estimating the stress concentration factor for tubular joints of different structural forms of geometric forms which has been done experimentally and then we will compare them with the empirical issues given by international codes for our better understanding. Therefore, one can then explicitly understand the limitations of estimating the stress concentration factor which is one of the vital requirement of the fatigue assessment in a given tubular joints of offshore structures.

Thank you very much.