# Risk and Reliability of Offshore Structures

**Prof. Srinivasan Chandrasekaran**
**Department of Ocean Engineering**
**Indian Institute of Technology, Madras**

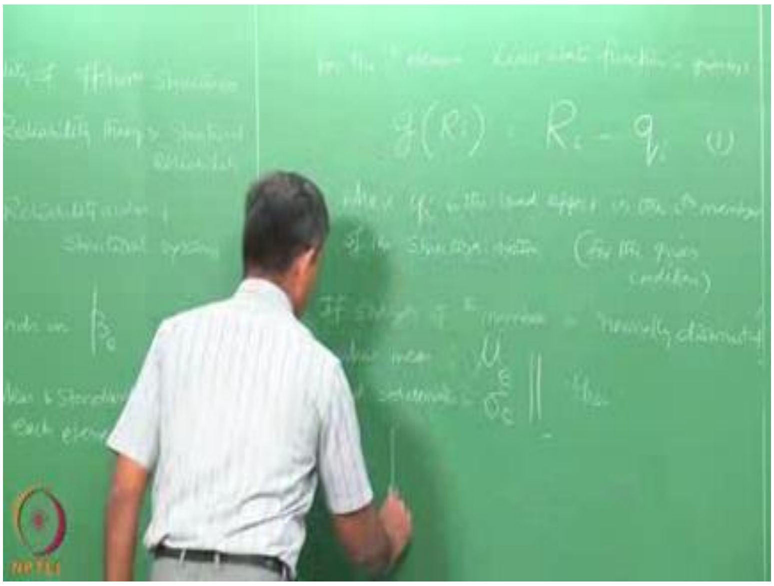
## Module – 02
## Reliability theory and Structural Reliability
## Lecture – 20
## Reliability analysis of structural systems

(Refer Slide Time: 00:21)

Welcome friends to the online course on Risk and Reliability of offshore structures. We are looking into lectures on module 2; module 2 is focusing on reliability theory and structural reliability.

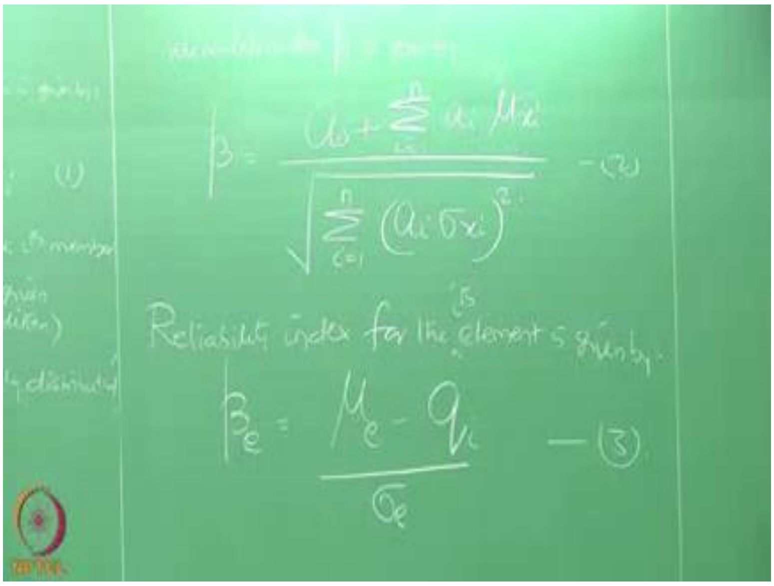
In this lecture, which is lecture-20 in module 2, we are going to talk about structural systems and its reliability analysis of structural systems. In the last lecture, we discussed that if I know the reliability index of an element, I can always find the reliability index of the entire system, we know that the reliability index of the entire system is dependent on reliability index of an element, which in turn depends on the mean and standard deviation of each element.

(Refer Slide Time: 02:16)



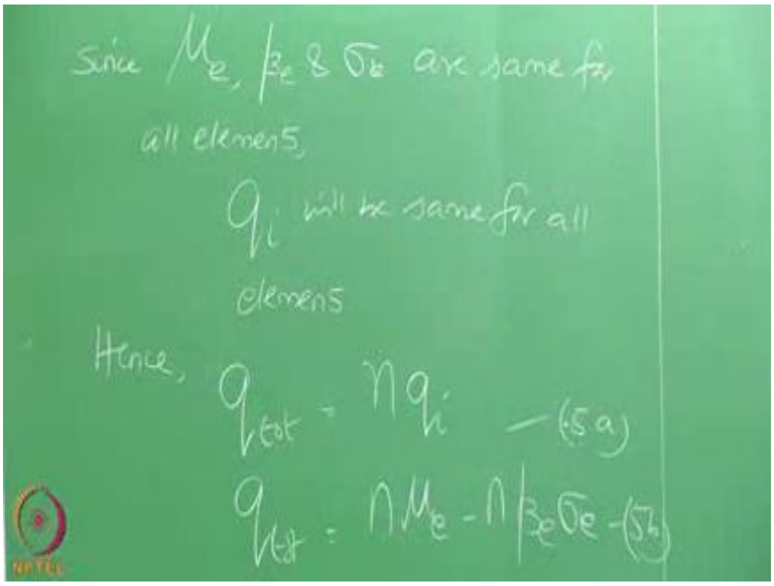
Therefore, for the ith element limit state function can be given by where q i is a load effect in the ith number of the system. Of course, without seeing for the given conditions, reliability study cannot be applied on infinite domain given conditions, and given state and given period, given combination, you can do reliability analysis; all these are to be prescribed before we do a reliability analysis. Therefore, in the given conditions one can write this. If the strength of the ith number is normally distributed whose mean is mu e and standard deviation is sigma e, then reliability index of the element is simply given by or let us say reliability index this is the equation (Refer Time: 05:08).

(Refer Slide Time: 05:15)



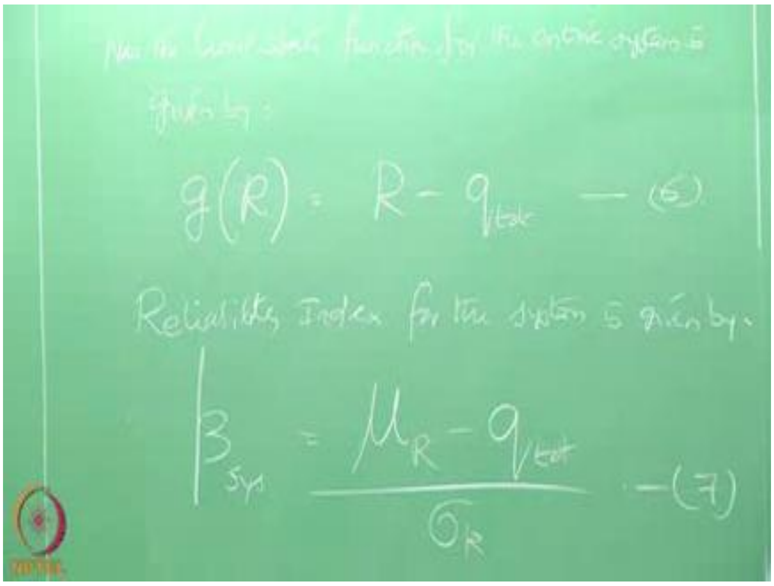

Equation number; so, the reliability index for the element is given by beta e which is mu e minus q i by sigma e of the ith element for the let us say ith element.

(Refer Slide Time: 06:24)



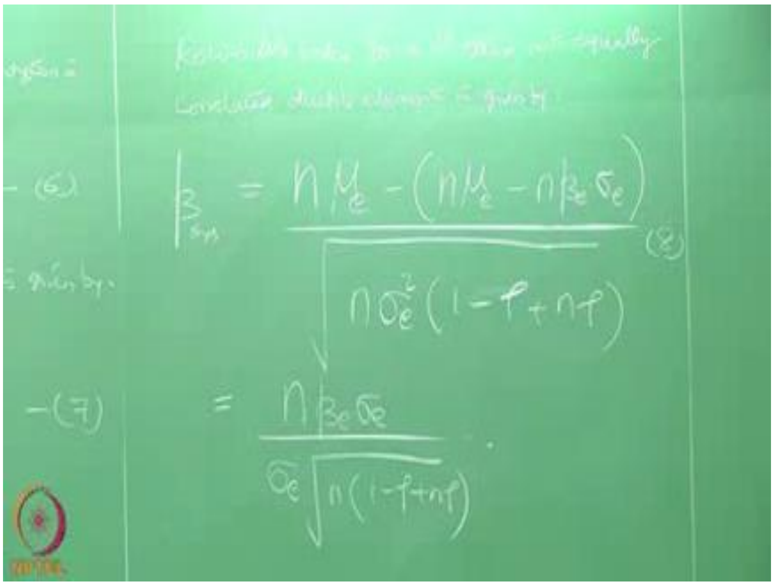

Now, q i which is going to be the load effect which is required for the reliability analysis is simply given by the mean of the element minus reliability index of standard deviation of the element. Now, since the main reliability index and standard deviation are same for all elements one can expect that q i that is the load effect will also be same for all elements, hence the total load effect can be simply said as n number of elements of q i. On the other hand, q total can also be said as n u e minus n beta e sigma e S q i is given by this equation from and this 5 a, 5.

(Refer Slide Time: 08:12)



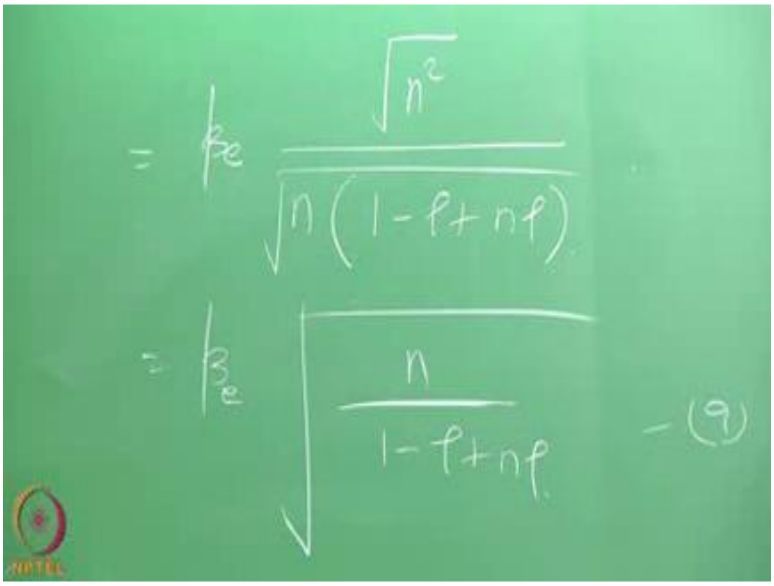

Now, the limit state function for the entire system is given by because earlier it was for the element now it is for the entire system. And the reliability index for the entire system is given by.

(Refer Slide Time: 09:42)



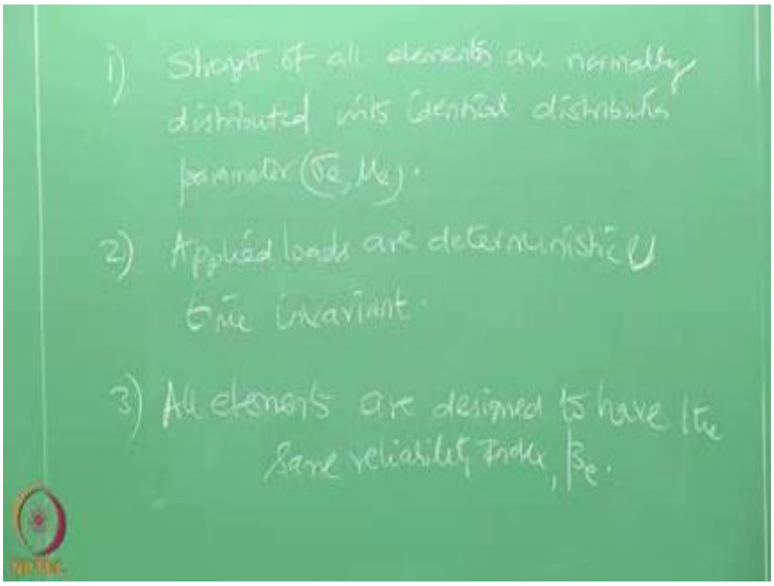
Now, reliability index for a parallel system, which contains with equally correlated ductile elements is given by, where rho is the correlation coefficient calls equation number 8, which can be further simplified as.

(Refer Slide Time: 11:31)



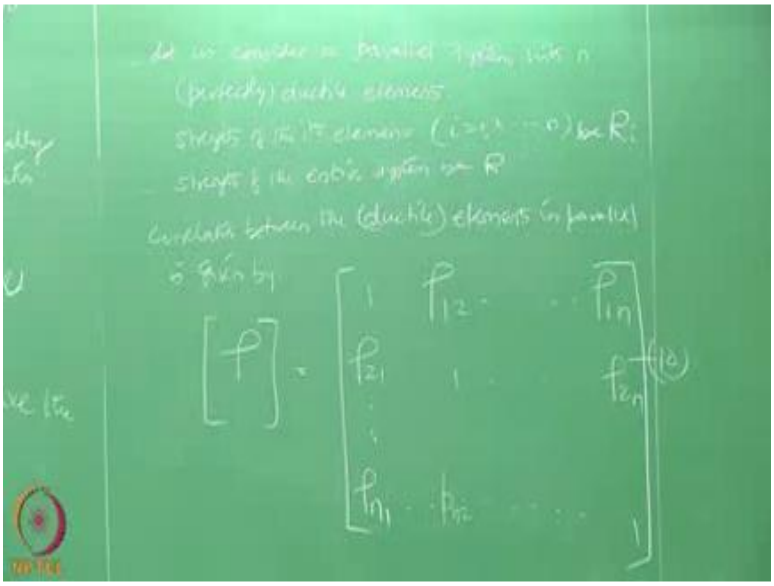
Which can be further said as which can be simply with element from (Refer Time: 12:04) equation number. This is square root here as well.

(Refer Slide Time: 12:31)



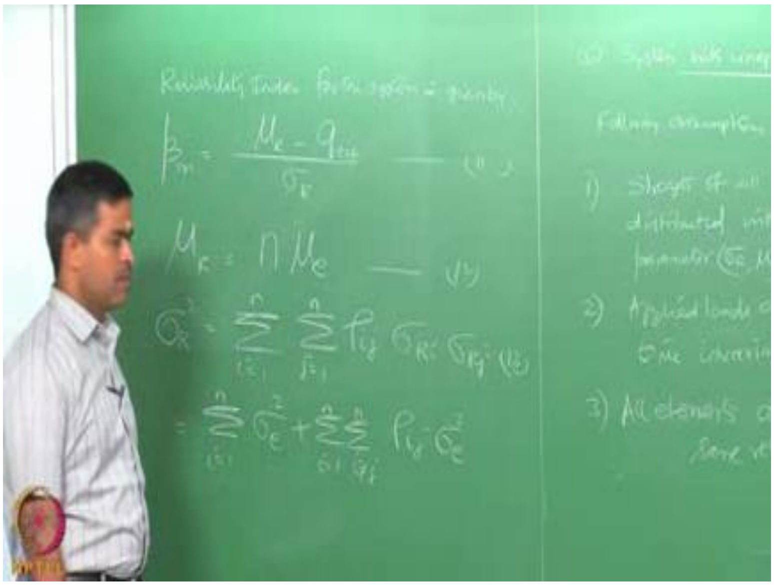
So, now let us talk about a system with unequal correlated elements. In real structure system has n number of elements which are generally unequally correlated. So, following assumptions are need to be made in such analysis; one, strength of all elements are normally distributed with identical distribution parameter sigma e and mu e. Secondly, applied load are deterministic and time invariant; thirdly all elements are designed to have the same reliability index beta e.

(Refer Slide Time: 14:52)



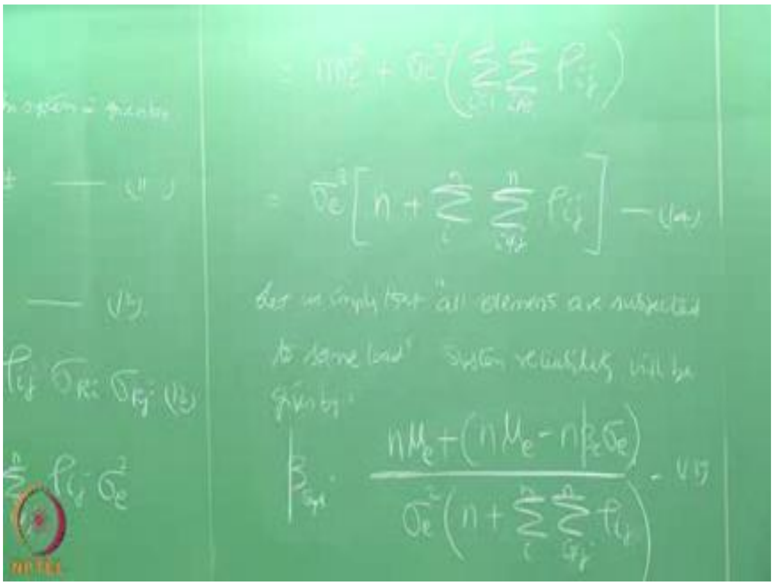
Let us talk about parallel system in ductile elements. Now let us consider a parallel system with n perfectly ductile elements, the strength of the ith element, where i is actually the number count of the element be R i and let the strength of the entire system be simply R. Now since we are talking about elements with unequal correlation, we need to also establish the correlation matrix. So, the correlation between the elements parallel is given by the rho matrix which is going to be unity rho 1 n, rho 2 1 unity rho 2 n rho n 1 1, rho n 2 let us say rho n 2 etcetera that is going to be the matrix which you call as equation number 10.

(Refer Slide Time: 17:38)



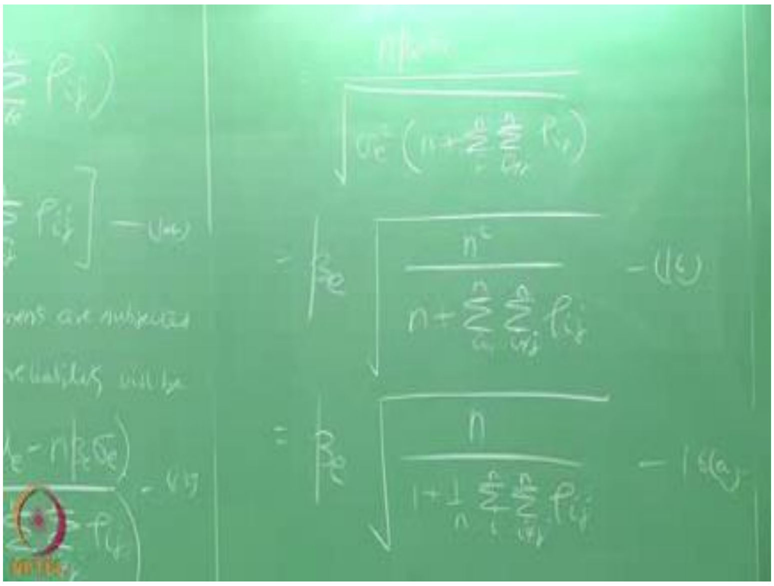
Now, the reliability index of the system is given by mu R minus q total by sigma R - 11. Now mu R is of course, n mu e; variance is submission of rho i j sigma R i sigma R j which can be said as.

(Refer Slide Time: 19:20)



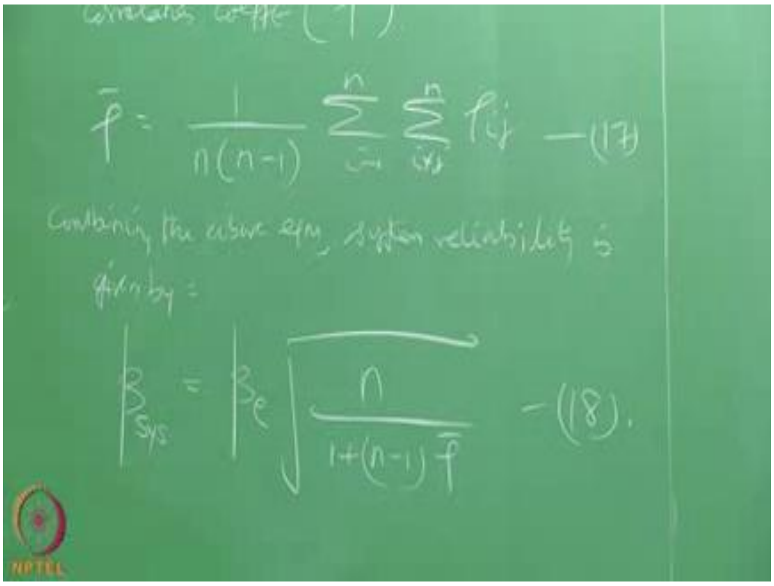
Which can be further said as. Therefore, we can say sigma e square n plus summation to n summation to n i not equal to j i rho equation number 14. Now, let us implies that all elements are subjected to same load in that case system reliability will be given by provided equation 15.

(Refer Slide Time: 21:42)



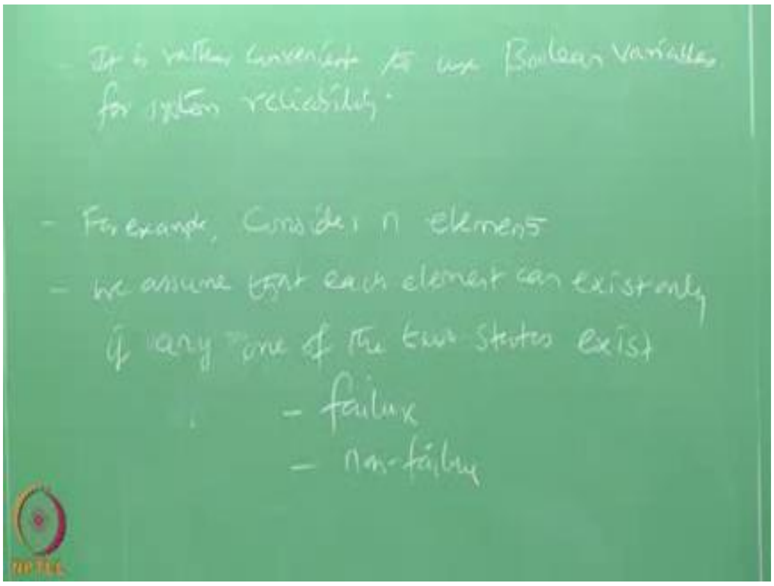
Which can be n sigma e by root of sigma e square n which can be simply beta e equation number 16, which further simplifies to beta e root of n by 1 plus n by n of summation of i naught equals j i n n rho j.

(Refer Slide Time: 23:10)



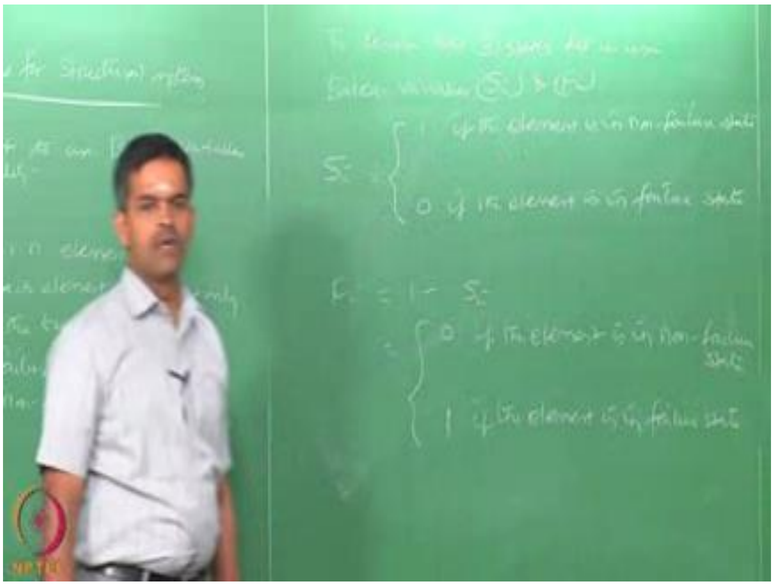

Now, as we understand that all numbers are unequally correlated, we can use an average correlation coefficient, which I call as rho bar. So, rho bar which is the average correlation coefficient of all the members is given by 1 by n of n minus 1 double summation i 1 to n i not equal to j n rho i j equation number 17. Combining the above equations, one can write the system reliability as let say combining the above equations system reliability is given by (Refer Time: 24:42) reliability by 1 plus n minus 1 equation number 18. So, we discussed about the reliability estimate of a structural system comprising ductile numbers in parallel with unequal correlation coefficient.

(Refer Slide Time: 25:18)



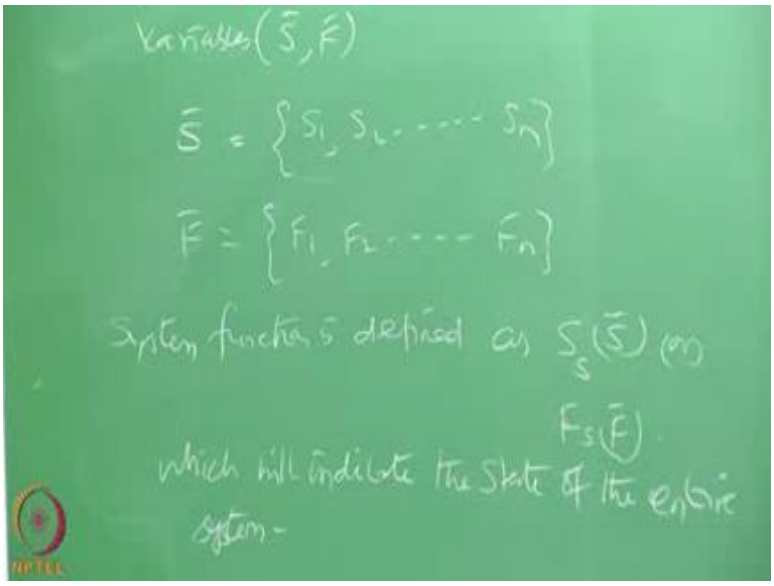

Now, let us talk about the reliability bounds for structural systems. In system reliability, it is often convenient to work with Boolean variables. So, it is rather convenient to use Boolean variables for system reliability. You may ask me a question why it is so, system reliability which is consisting of elements maybe in series or in parallel is amounting to failure of an element amounting to failure of a system or failure of all elements amounting then to failure of a system. So, it is talking about only either success or failure either performing or non-performing. So, it is always easy and convenient to use a Boolean variable for system reliability. Let say for example, consider n elements. We also assume that each element can exist only if one of the two states are existing namely failure or non failure only two states.

(Refer Slide Time: 27:55)



Now to describe these two states in Boolean variable let us try to use certain variables. Describe these two states let us use Boolean variables S i and F i which are defined as follows. S i will be actually equal to 1, if the element is a non zero failure state; it is 0, if the element is in failure state. F i which is also a Boolean variable is 1 minus S i, which again says 0 if the element is a non failure state because it is 1 minus S i where S i 1 F i become 0 for a non-failure state; and obviously, it is 1 if the element is in failure state.

(Refer Slide Time: 29:50)

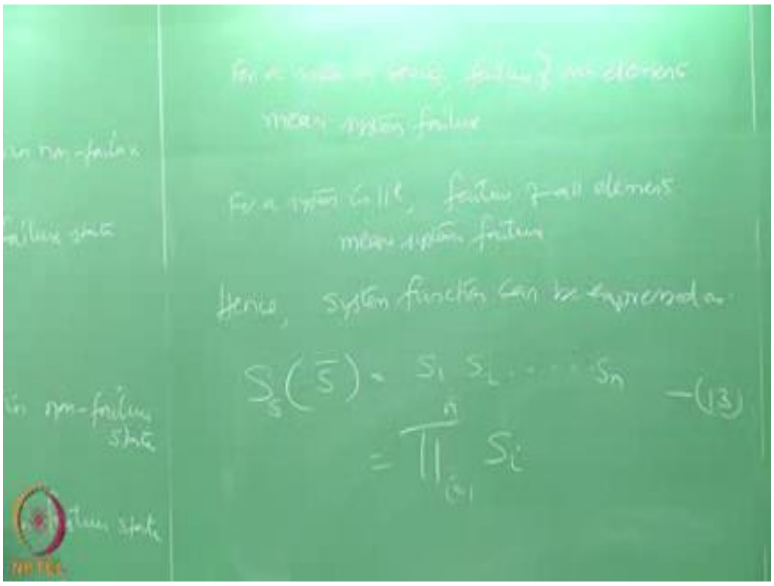



So, based on the states for elements, one can define the state of the system. State of the system using Boolean variables let us say using S bar and F bar. S bar is the set of S 1, S 2, S n; and F bar is a set of F 1, F 2, F n. Now the system function is defined as S s S bar or F S F bar this will now indicate the state of the entire system. We can write it here, which will indicate the state of the entire system.

(Refer Slide Time: 31:36)

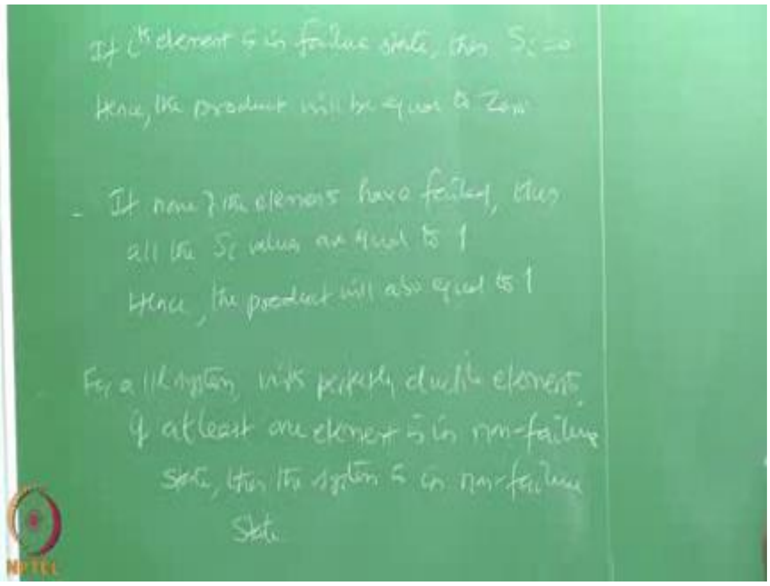
Now, the function is defined as S s S bar is 1 if the element is in non-failure state; it is 0, if the element is in failure state. Similarly, F S F bar is 1 minus S s S bar which is now 0 if the element is in non-failure state because the non-failure state S s S bar is 1; since F S F bar is 1 minus this so for a non-failure state, this becomes 0. And of course, it becomes 1, if the element is in failure state is it.

(Refer Slide Time: 33:13)



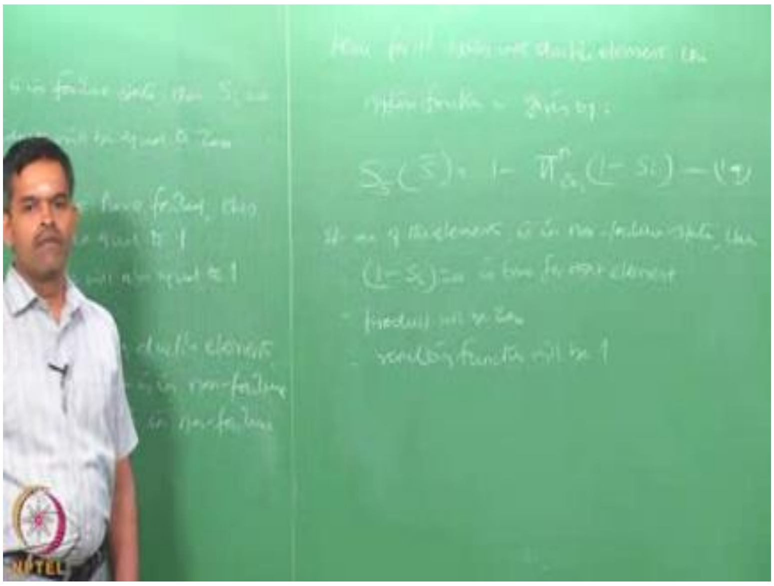

Now, for a system in series failure of one element means the system failure. Now for a structural system in series failure of one element means system failure. For the system in parallel failure of 1 element or failure of all elements means system failure. Now hence the system function can be expressed as S s S bar is S 1, S 2, S n which is nothing but phi i equals 1 to n S i. If the ith element is in a failure state, I will call this equation number 13.

(Refer Slide Time: 34:57)



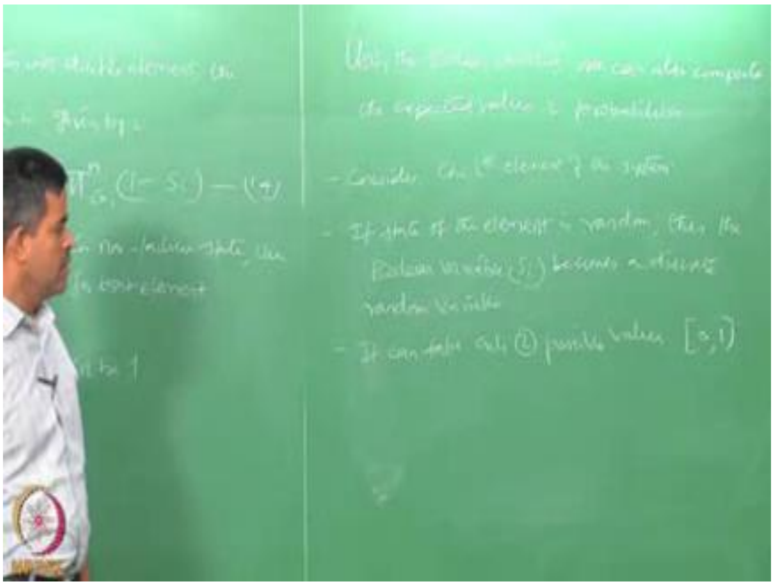
If ith element is in failure state, then we know that S i is 0. And hence the product has shown in equation 13 will also be 0. If none of the elements have failed then all the S i values are equal to 1, it is a non-failure state, therefore the product will also be equal to 1 which indicates non-failure state. Now, for a parallel system, this is for the system in series for a parallel system with perfectly ductile elements, if at least one element is in non-failure state then the system is a non-failure state.

(Refer Slide Time: 37:12)



Therefore, the corresponding system function is given by this S s bar if 1 minus phi i equals 1 to n of 1 minus S i equation 14. Therefore, looking at the equation 14, one can say that if one of the elements is a non-failure state then 1 minus S i will be 0, which is true for that element product will be 0. And the resulting function will be one which implies that system is a non-failure state. By using the Boolean variables, one can also compute the expected values and probabilities.

(Refer Slide Time: 39:09)



Let us say consider the ith element of the system. If the status element is random S i then become a discreet random variable. Therefore, there are only two possible ways or values, so it can take it can assume only two possible values 0 and 1; and the probability associated with each value can be calculated, which we will discuss in the next lecture.

Friends, in this lecture, we are able to understand how to compute the system reliability for known element reliability, if the elements are in series and parallel. If the elements have equal correlation coefficient, if the elements have unequal correlation coefficient where we replace the correlation coefficient by an average value rho bar from a matrix what we have computed and shown in the lecture.

Interestingly for system reliability, it is more convenient to express the reliability of the elements in Boolean variable; therefore system reliability can associate only with two problems of possible values 0 and 1. Then in that case the probability associated with each of this value can be interestingly studied and can converge to system reliability very easily with a better understanding.

Thank you very much.