Risk and Reliability of Offshore Structures
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Module - 02
Reliability theory and Structural Reliability
Lecture -17
Application Problem-II continued

Welcome friends to the 17 lecture, on module 2, on the NPTEL course on Risk and reliability of offshore structures.

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We are talking on lectures in module 2, which is focusing on reliability theory and structural reliability. Today, we will discuss lecture 17, in module 2, where we will continue with the application problem what we discussed in the last lecture, application 2 as I continue. So, we are talking about the effect of the influence of the response of compliance system under, highly non-linear waves in the presence of earth quakes. So, the original p-m spectrum, which is a function of wind velocity, is a modified. So, the original Pierson Markowitz spectrum, which is a function of wind velocity, is modified as a function of modal frequency.

Subsequently, it is further modified to accommodate as a function of significant wave height and the modal frequency. This was suggested by Michel 1999 which we discussed

also in the last lecture I will write down the equation for continuity s eta eta omega 8.110 power minus 3 by omega 5 exponential minus 1.25 omega M by omega to the power 4 equation number 1. In this case g g square here, in this case g is acceleration due to gravity which is 9.0 meter per second square omega M is called the modal frequency. In this study, in general as advised by Mitchel; we have taken modal frequency as 0.46 radius per second which amounts to 0.07 hertz. S eta is the power spectral density function of the wave height and wave frequency the wave elevation eta t is realized, as a discrete some of many sinusoidal functions.

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Well doing so, vary the angular frequencies and random phase angles. So, eta t is given by, which we have seen in the last lecture as well this is going to be a summation discrete sum of various sinusoidal functions. So, we know that k i is a wave number, omega I is the discrete sampling frequencies, delta omega I is omega I minus omega I minus 1 which will give me the difference n is a number of data points and phi I is a random phase angles. Now, interestingly the generated waved profile is designed have a peak and specific time t naught which is going to be distinctly high in comparison to other wave heights and therefore, it can be classified as distinctly high sea waves. We look at the figure which is shown on the screen.

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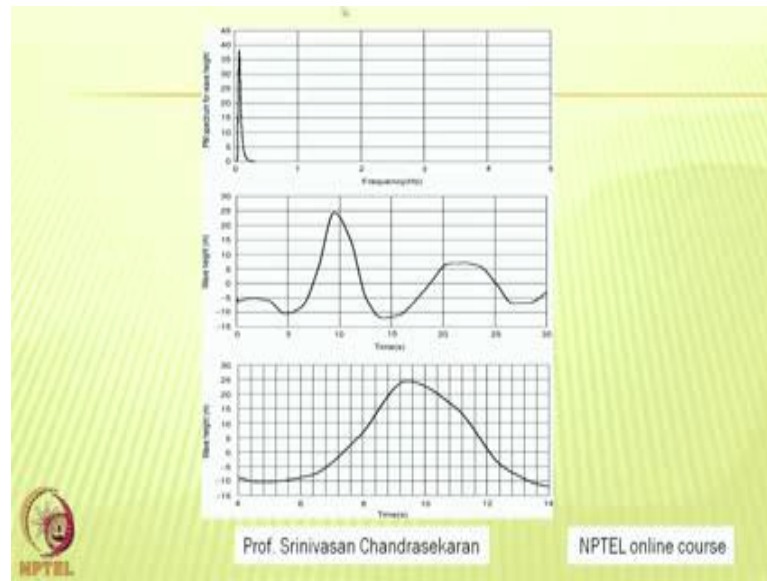
One can see here, the top one shows the generated Pierson Markowitz spectrum, using the modified values given by Mitchel the corresponding wave height in terms of wave height time history is shown here if you take a blow up any specific wave height or sea surface elevation which you see from here you can explain the characteristics of this very easily by looking at this figure which is classified as distinctly high sea waves because one can see here the wave height and the specific value is phenomenally and comparatively very high with respect to the preceding and succeeding wave heights. So, one can simply say this is a typical distinctly high sea waves. So, the figure showed the Pierson Moskowitz spectrum and the sea surface elevation sample; at means x is equal to 0. So, the location is at x is equal to 0 where the wave first interface with the structure.
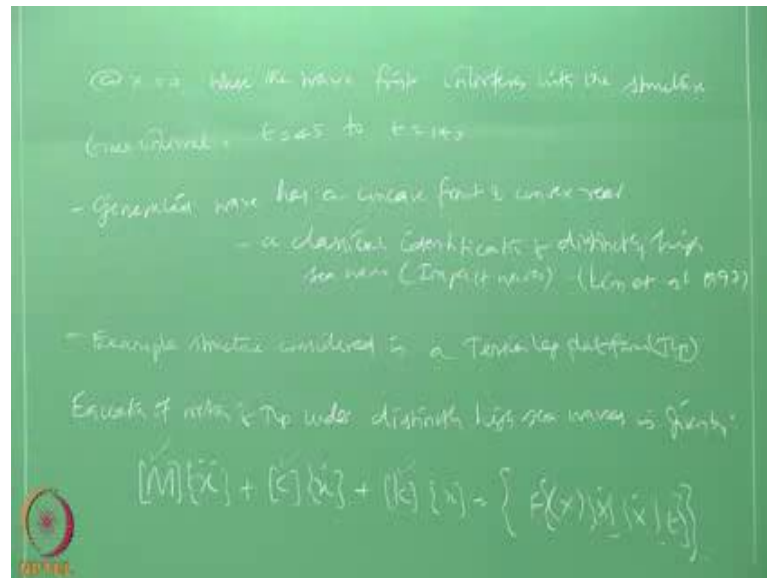
In this case, it is going to be a here. One can see a magnified wave history sea surface elevation see on the screen here a magnified wave history one can see here by looking at this the wave generated can be easily seen as a continuous function of time near the chosen time interval the time interval chosen; in this case is t is equal to four seconds to t is equal to 14 seconds. In this time interval, in the specific time interval one can easily see it is become a continuous function of time it is also seen the generated wave has a concave front.

Generated wave and a convex rear, which is a very classical identification of distinctly high sea waves, which can cause impact on the structure as given by Kamet in 1997, one can also see the specially varying high sea wave profile is the wave seen in the first mode point of TLP that is x is equal to 0 therefore, from that point onwards different phases of the same wave profile reaches different positions along the TLP as a wave passes away; that is how the discrete some of sinusoidal function as become a continuous function in terms of a wave which is causing a required impact on the structure which is distinctly high sea waves. Now, in the study example problem was the example structure taken considered is a tension leg platform. Kindly, look at the figure shown in the screen now.
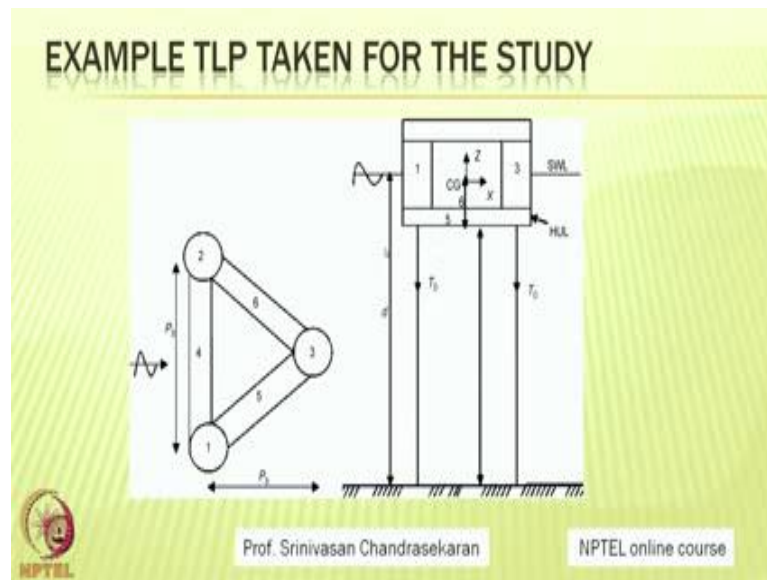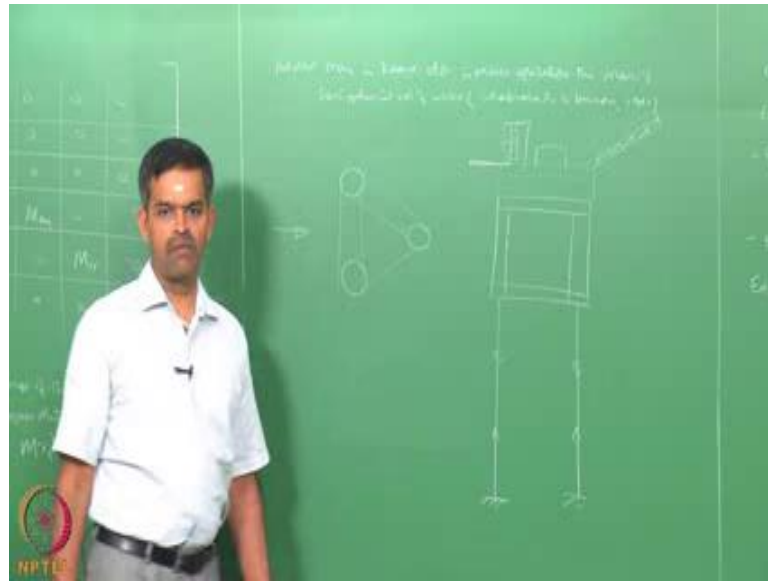
This is an example TLP is considered for the study presently the TLP has got three column members designated as 1 2 and 3 and 3 pointing members designated as 3 4 5 and 6 where which are located at the bottom and of course, the top is having the trust system where the top deck is assembled to the column members as seen the elevation, the water depth is marked as d the TLP in triangle as the dimensions in plan p b and p l plan along the breath and the plan along the length and thus the means level sea level of sea c g is taken as the mass center where the degree of freedom are marked.

So, one can see here the initial pretension is about t naught and the water is about c g as marked here. Now, in the equation of motion of TLP under distinctly high sea waves is given by M x double dot plus c x dot plus k x is going to be f of x x dot x double dot because I am going to include the acceleration as well and of course, the time of t. M is the mass matrix of given system c is the damping matrix k is stiffness matrix and of course, f is the hydrodynamic force vector. The structural mass is assumed to be lump at specific degree of freedom points and therefore, it is going to be diagonal in nature because we all know when the structural mass lumped at the points where the degrees of freedom are marked or measured then the mass matrix will be diagonal. So, the mass matrix in this specific case is going to be like this.

These are essentially the lumped in mass at respective degrees of freedom where M 1 1 M 2 2 and M 3 3 is the total mass of the flat form whereas M 4 4 M 5 5 and M 6 6 are actually total mass of movement of inertia about x y and z axis respectively. On the other hand, M 4 4 is going to be simply M r x square where as M 1 1 M 2 2 and M 3 3 can be said as M similarly M 5 5 will be M r y square and M 6 6 will be M r z square respectively. Interestingly the added mass term will also get now included in the mass matrix. So, we say plus M 1 1 that is going to be added mass which I am doing it here and M a 5 1 the added mass similarly because of the heat the displaced volume M a 3 3 and M a 5 3 remaining all terms will just not been indicated here will be marked as 0.

So, one can see here the additional mass matrix terms essentially arrays because of the variables (Refer Time: 16:00) effect on the given problem r x x or r x r y and r z are radius of gyration about the respective axis x y and z respectively M a 1 1 and M a 3 3 are the added mass terms in the surge and heave degrees of freedom whereas M a 5 1 and M a 5 3 are the added mass movement of inertia due to the additional mass is surge and heave degrees of freedom respectively. The heave added mass on the TLP (Refer Time: 16:36) is they can equivalent to the mass of the hemispherical volume of water.

So, the added mass a (Refer Time: 16:50) of freedom is actually taken equal to the mass of hemispherical volume of water one can equivalent to mass of hemispherical volume of water as suggested by Chakravarthi Hanna 1991 can see here from the mass matrix that the presence of half diagonal terms, indicates contribution of added mass due to high dynamic loading.

So, now we are talking about the example problem where, the TLP is encountering the wave with front two columns in plain. So, triangular configuration in plain of course, it is got the top deck supported by the members and bottom contour, we solve now push and the stand using (Refer Time: 18:15) of course, the top deck has the same facilities as that of conventional hull where all systems will represent as expected in the given system of t l p. Now, interestingly in this particular equation of the study contribution added mass is taken up to the M s l which included along the force vector the stiffness matrix now which is indicated as k depends on the tether tension and its responds dependence.

As well dependence on tether tension and structural response, for a triangular TLP the stiffness matrix 6 6 and k 4 4 reaming all members are 0 k 5 1 remaining all members are 0, in this case all members are 0. So, one can see here that heave degrees couple is almost all degrees of freedom there was three 1 1 to 3 6 all are present. So, heave is got a very strong coupling with all most all degrees of freedom it is very simple to understand that is concept in case if we have a TLP when you given a another motion to TLP the TLP actually sets an half sets on waves when it surgeons wave of course, because of the heave movement it is going to be half set.

So, heave surgeon's wave or any way coupled, because the variable some other effect there will be going to additional mass movement of inertia cause and that is going to cause a restoring movement in the pitch roll in all degrees of freedom therefore, heave is coupled almost with all degree of freedom here. So, it is why we very strong coupling with all degrees of freedom in a given system one can also see looking at the coefficients of a stiffness matrix one can easily see they are going to be tether tension dependent and response dependent one can see this details from my papers published and indicated in the NPTEL website. So, coefficients of stiffness matrix can be derived from the first principles k i j can be derived from the first principles.
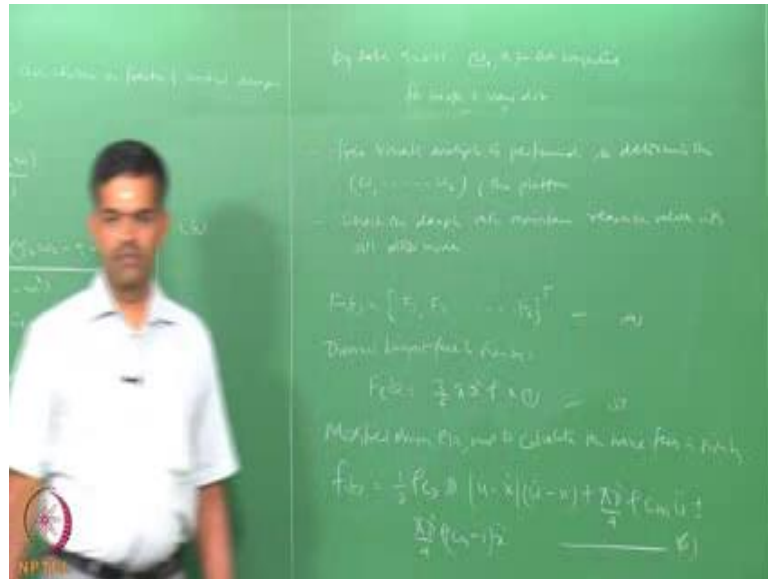
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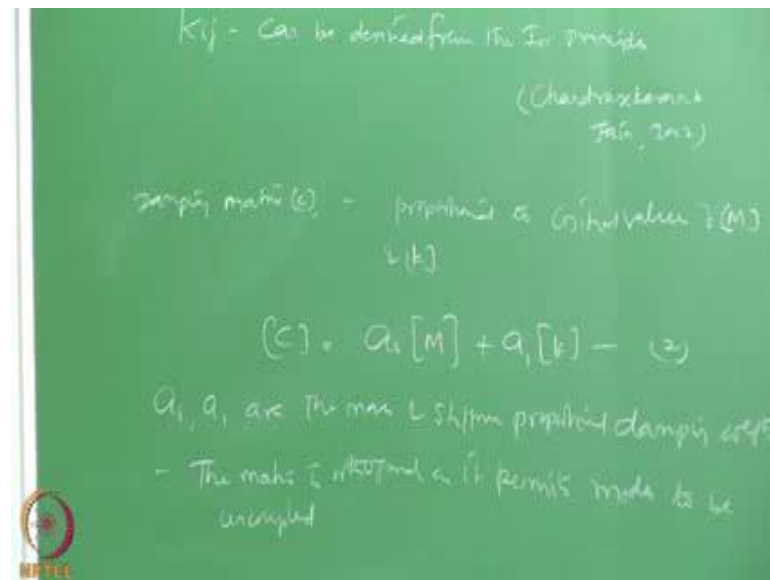
One can easily see from the literature Chandrashekaran and Jain 2002 damping matrix c in this case is assumed to proportional to initial values of mass matrix and c plus matrix. So, c matrix is given by a rally modal combination of this which is a 0 M plus a 1 k i call this equation number two where a 0 and a one are essentially the stiffness the mass are the mass and stiffness proportional damping coefficients the damping matrix given here is interestingly orthogonal, because it permits as it permits modes to be uncoupled by the Eigen vectors undone Eigen value problems. The damping constants a 0 and a 1 interestingly are chosen as fractions of critical damping at frequencies which are omega 1 and omega 2.

So, one can solve the simultaneous equation write now a 0 and a one can be written like this twice zeta 2 omega 2 minus zeta 1 omega 1 by omega 2 square minus omega 2 square where as a one is twice. Now omega 1 omega 2 zeta two omega 2 minus zeta 1 omega 1 by omega 2 square minus omega. So, I call this equation number three. So, the damping coefficients zeta 1 and zeta 2 are taken a certain percentage of critical damping the corresponding frequencies omega 1 omega 2 are chosen such a manner. So, the two different frequencies are chosen out of 6.

So, that the suitable mass and stiffness proportional damping can be used in the certain system. So, the damping attributed to a 0 k increases the increasing frequency whereas the damping attributed to a 0 k increases with increasing the frequency whereas, a 0 a 1 M will decrease with increasing frequency. So, accordingly on has calculated the values of a zero and a 1. So, that when you substitute here as a 0 M and a one k they are compromised. So, this was of course, refer from Chopra 2003.

So, by taking damping a critical 0.005 here 5 percent of critical damping the coefficient a 0 and a 1 are worked out or computed by solving this two simultaneous equations for surge and (Refer Time: 27:21) degrees of freedom that is omega 1 is related to surge and omega 2 is related to yaw degree of freedom.

Then subsequently a free vibration analysis performed to find out the natural frequency of the plat form, which can be used here for our calculations therefore, the damping ratios can be obtained. It has been seen that the damping ratios maintained a reasonable value with all other modes. So, one was to check the damping ratios maintain reasonable values with all other modes the force vector f of t is given as f 1 f 6, and suppose equation number 4 where f will be the force in any expected degree of freedom the force degree of freedom is of course, surge and 6 0 freedom is of course, the yaw motion.

The dynamic buoyant force is given by, which is also going to be the function of time it is for the equivalent t l ps of three by four rho x of t equation number 5. Wave forces are modal using modified Morison equation which is given by f of t is force for unit length pie d square by 4 rho c M u double dot plus or minus pie d square by 4 rho c M minus one of x double dot equation number 6 where x indicates the structural displacement velocity and acceleration of the structure u indicates the water particle kinematics.

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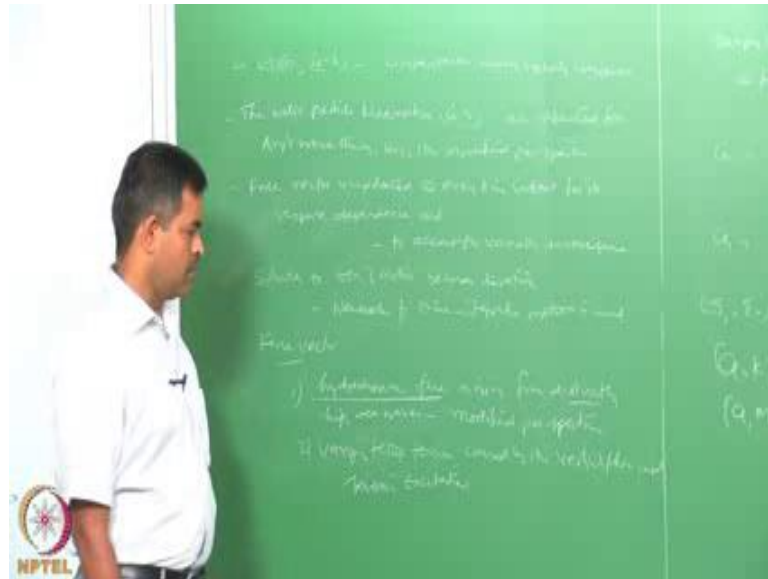
So, one can see here in equation 6 u dot minus x dot in equation 6 u dot minus x dot is the relative velocity component is the instantaneous relative velocity component between the structure and the water particle, of course x dot and x double dot are the structural velocity acceleration respectively these are the diameter of the member of the cylinder, rho is a density of sea water c d c M or the diagonal initial coefficient which are used in the equation.

The water particle kinematic u dot and v dot using the modified (Refer Time: 31:52) the force vector updated, at every time instead, for its response dependence and also to account for variable submerge. Now, as you can see from this equation it also depends on the response this of course, not known we got solving using iterative scheme of numerical procedure. So, therefore, solution to equation of motion becomes iterative becomes iterative and Neumann's beta time integration method is used in the present step.

Now interestingly the force vector has got two components compress of one. The hydrodynamic forces arising from distinctly high sea waves, as obtained from the modified p-m spectrum which I showed you earlier, in addition to that it, as also called what the varying tether tension which is caused by the vertical and horizontal (Refer Time: 34:19) excitation. When the t 0 or the tethers are actually anchored to sea bed when the sea bed is got vertical and horizontal assessment consideration the tension in

the tether continuously varies which affects the stiffness coefficients therefore, the equation of motion is updated at every time instance because 2 force components stop 1 is the hydrodynamic force arising from the distinctly high sea waves which we obtained from the modified p-m spectrum the second is of course, t 0 variations which causes in direct force on the system. Now, the tether tension variation which affects the stiffness coefficients is very simple.

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The tether tension coefficients which is affecting or influencing the stiffness matrix coefficients is dynamic in nature, it varies with time, it needs to be updated, and is given by let us say the variation delta T is given by of course, the ratio of x of t minus x g t the ground displacement. Let us call this as equation number 7 where x of t is the instantaneous response vector of the TLP and x g of t is actually x 1 g of t 0 I only looking the horizontal and vertical (Refer Time: 36:35) excitation we assume that horizontal excitation happens only in such degree in else where it is not there remaining all other degrees remains 0.

X 1 of g is of course, the horizontal ground displacement in surge degree of freedom and of course, x three of g is the vertical ground displacement in heave degree respectively. So, now, we are going to add both the forces that are arising from the distinctly high sea waves the hydrodynamic force and the indirect force caused by the change in tether tension given by equation 7 as delta T because change in tether tension affects the

stiffness coefficients which of course, is a part of the equation of motion, which we will see and discuss in the next lecture.

Thank you very much.