Risk and Reliability of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module - 02
Reliability theory and Structural Reliability
Lecture – 14
Application problem-I

After understanding the Structural Reliability Theory from the previous lectures, we are now continuing with Lecture-14 on Module-2, where we will take up an Application Problem. This problem and solution will be discussed in two lectures, I call them as application problem number 1 and of course next lecture will be application problem (Refer Time: 00:48) on the same problem.

So, welcome back to the 14th Lecture on Module-2 on online course Risk and Reliability of Offshore Structures.

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As I said in general offshore structures have uniqueness because they are form dominant design. Recent developments offshore platforms are shown very high dependence on different kinds of structural forms, I can quote some examples. Starting from new generation platforms like tension leg platform where buoyancy exceeds the weight by a large amount, triceratops where the hull or the topside hull and the buoyant legs or isolated using injured connections or let us say ball joints etcetera. There are classical examples in the literature which shows that the recent development of offshore platforms are more form dominant than they are function dominant as used to be earlier.

So, now let us take up one example of application of reliability problem, we will consider a tensional platform for our discussions now.

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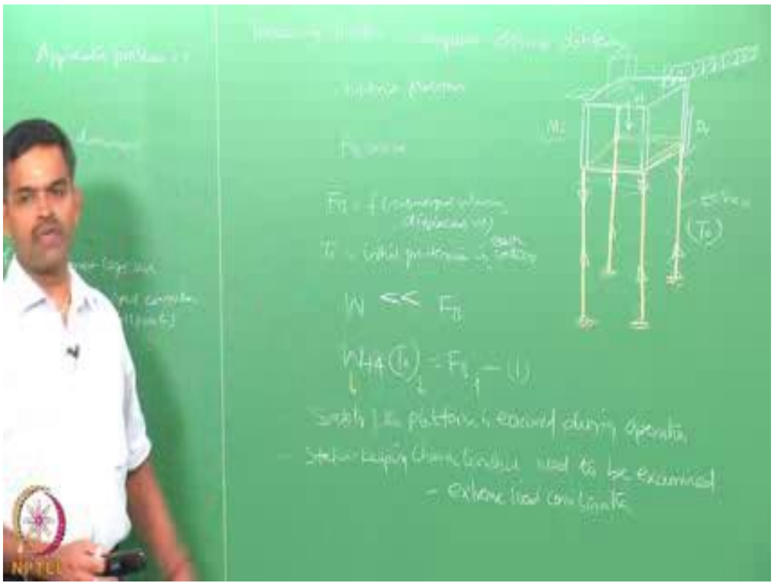

We already know tension leg platform is a compliance system is an compliant offshore platform which is also called as an hybrid platform, because of one essential reason. I will tell the reason very quickly which you remember from the first module lecture but still. So, let us say I have a platform consist of pontoons and column members, let say these are all column members, let us assume the rectangular platform in shape. I have 4 column members as I showed here and I have pontoons members and the bottom as well as on the top. And the top side will now consist of all details as required may be a frame derrick, may be helipad and so on, it is a multi tier system let us say the machine room etcetera all complications.

So, the water level approximately is here that is the main sea level. Now, the system has a mass let us say which is acting downward therefore the weight which will be now balanced by of course the buoyancy force. Now, we already said in the design buoyancy force exceeds the weight of the platform by a large amount, this is advantage of this. When the buoyancy force, we all know that buoyancy force actually is a function submerged volume or you can say displaced volume. So, displaced volume depends upon the surface area of the circular member and the depth of immersion what we call as draft or design draft.

So, the volume of the system the submerged volume depends upon the size, the size is very large and the diameters again very large, so it has got a very large displaced volume. Therefore, buoyancy is a very excess compared to the weight the advantages, when buoyancy exceeds the weight the platform can set up flowed very easily therefore transporting erecting commissioning becomes easy.

However when you want to install the platform in position for oil exploration then the station keeping is very important, so we hold down this platform using tethers which will be support at the bottom an anchor to the sea bed firmly. When the platform is brought in position these tethers will be all slackened, all will be loose, there is no initial tension in them, so there is a design draft which I can mark here, this may design draft us say d r. I add more weight to the top side so the platform will start immersing further. So the cables in all the tethers, these are all tethers will be all slackened, we be loose, so the tethers will be connected to these points firmly. Once the excess weight is removed the platform will be pushed up by the buoyancy force and these tethers will not allow the platform to move up, they will pulled the platform that is why they are called Tension Legs.

So, the initial tension given in the legs is what we call as T 0, so T 0 is the initial I should say pre tension in each tether let us say. Now, if you write the equation of static equilibrium for this particular platform I should say W and F B are not matching, as W is far lesser than F B but I want to match this I want to equate this so I had T 0 and say this is my F B. Since there are 4 legs I should multiply this with 4 T 0. So, W is acting downward, T 0 is acting downward, F B is acting outward; this is my equation of static equilibrium. Interestingly, I am looking at the safety at the platform during its functional intended use.

As I said reliability is assessing or guaranteeing or ensuring performance of a system in terms of its fractional functional operation when it is demanded to perform at a specific time under specific conditions. The specific conditions in our discussion is nothing but the sea state where this being commissioned.

So, we would like to check whether the safety of the platform is ensured during operation. To ensure this I must check whether the platform remains safe and well conditioned and the installed commissioning stage. Therefore, station keeping characteristic essentially come from the measurement of T naught or T tethers or compliance of tethers for the given loading imbalance. So, station keeping characteristics of the platform need to be examined. We try to examine this under extreme load combination, because under extreme load combination this can cause imbalance to the T tethers or tension in tethers which in turn will affect the stability of the platform because that is the equation of static equilibrium of the given platform.

Therefore, reliability sometimes also checks the safety of the platform under intended operation. So, this is may be the reliability index may not be computed from this problem, but safety of the problem at extreme load combination will be verified which is also of course essentially part of the reliability study.

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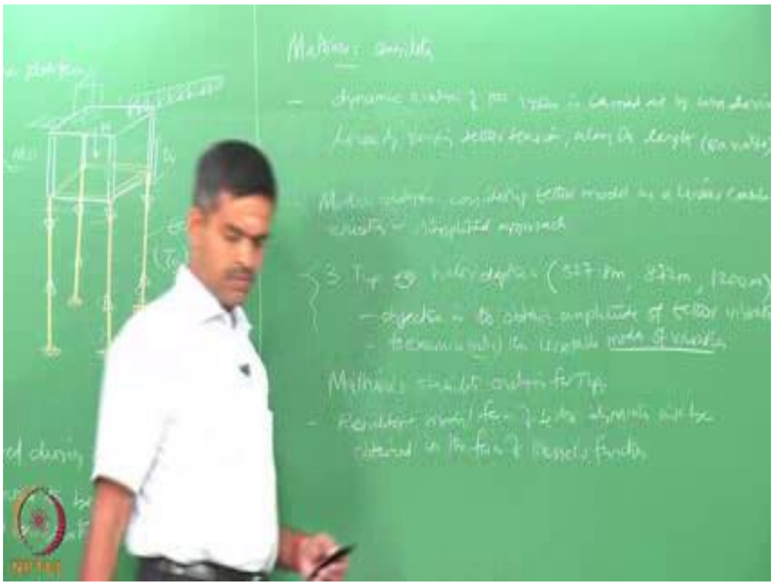

To do this we assess the stability of the platform using Mathieu's stability. One can study this using Mathieu's stability. So, the example what will now discuss will assess a tension leg platform of a given installed platform of a known sea state. We will assess the stability of the platform in terms of its functional safety. Therefore, this is going to be a dynamic analysis of tethers and T lp as a system considering a linearly varying tension along the tether length.

We are going to check the stability using dynamic analysis of the system is carried out by considering, so to challenge the stability of system there should be a variable is or not that variable is going to be by considering linearly varying tether tension along its length so that is the variable here, T 0 is the variable. To ensure this we perform model analysis we need to perform model analysis considering the linear cable equation for the tether model which may be a simplified approach does not matter, but let us look into this as an example study.

We will pick up 3 tension leg platform installed at various water depths, because we would like to know the parameters on which this variable is dependent then only we will able to establish the cross correlation effects of each of this variable under the action of these parameters. So, we will consider 3 T lp's installed at different water depths. For example, the water depths are 527.8 meters which are real T lp’s, I will give you the examples later 872 meters and 1200 meters. What we are looking at is the objective of this, to obtain the amplitude of tether vibrations and also to examine the unstable modes of vibration. So, you perform Mathieu’s stability analysis for T lp’s I should say.

The resulted model form of tether dynamic model will be then obtained in the form a Bessel’s function, because it is then it is easy to check the stability. As we all know in engineering practice wherever we start any problem numerical, analytical, experimental we always have set of assumptions or idealization.

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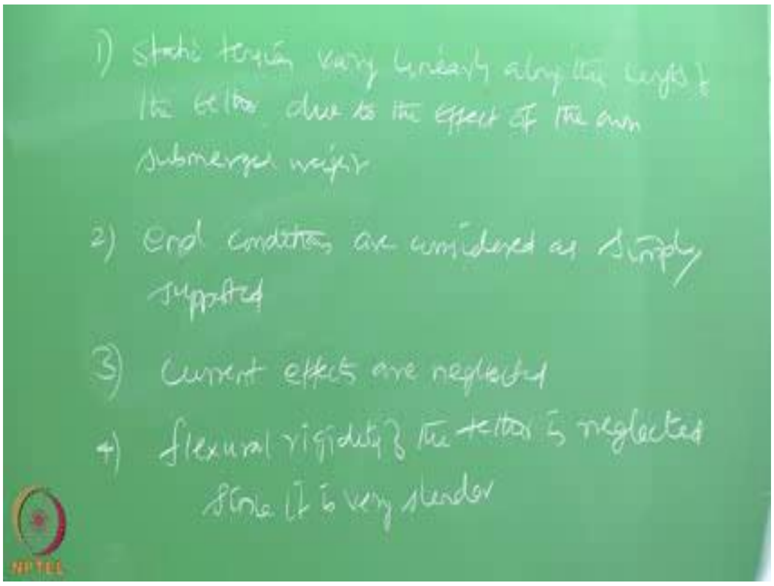

So, the following assumptions are made let us see. Static tension vary linearly along the length of the tether due to the effect of own submerged weight. We are looking at the tension variation due to two cases; one is because of the submerged weight on its own, one is because of the variation in response of the system itself because it is a compliance system. Two, the n conditions are considered as simply supported. Of course, we consider only the hydro dynamic load, current effects are neglected, and only the wave load is considered. We also neglect the flexural rigidity of the tether, because it is very slender. Once that pressure rigidity tether is neglected is lateral movement is then given by a specific equation, let us do this.

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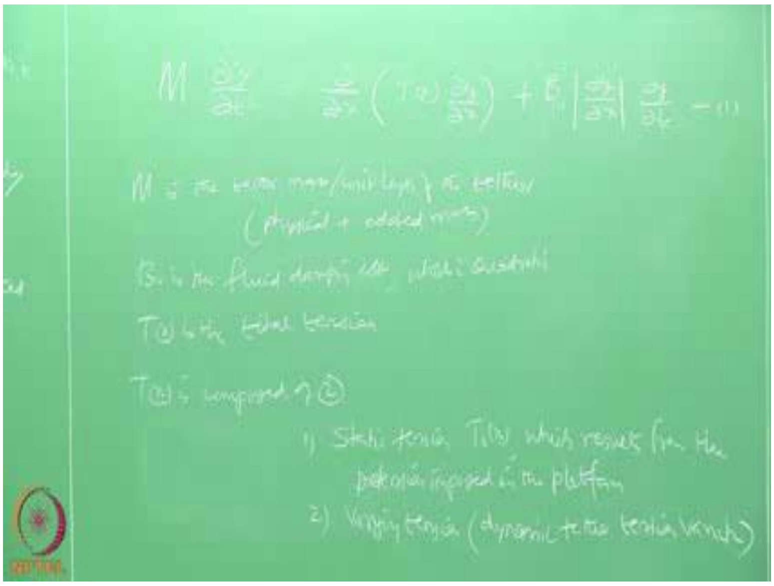

So, under the following assumption let us say, why assumption 4 is valid that is flexural rigidity is neglected. The lateral movement of tether under lateral loads is given by the following governing equation I can say M dou square y by dou t square minus dou by dou x of t x dou y by dou x plus b v dou y by dou x mod dou y by dou t, let us say equation number 1. In this case M is a tether mass per unit length of the tether. This includes physical plus added mass if any, added mass will come due to the submergence effect; b v is the fluid coefficient which is quadratic. T x is the total tension.

Now the total tension acting along the tether is composed of two things. One it will be due to the static tension which I call as T 0 x, T 0 stands for initial pretension or static tension which results from the pretension imposed on the platform. The second could be the varying tension along the tether length, which we call dynamic tether tension variation.

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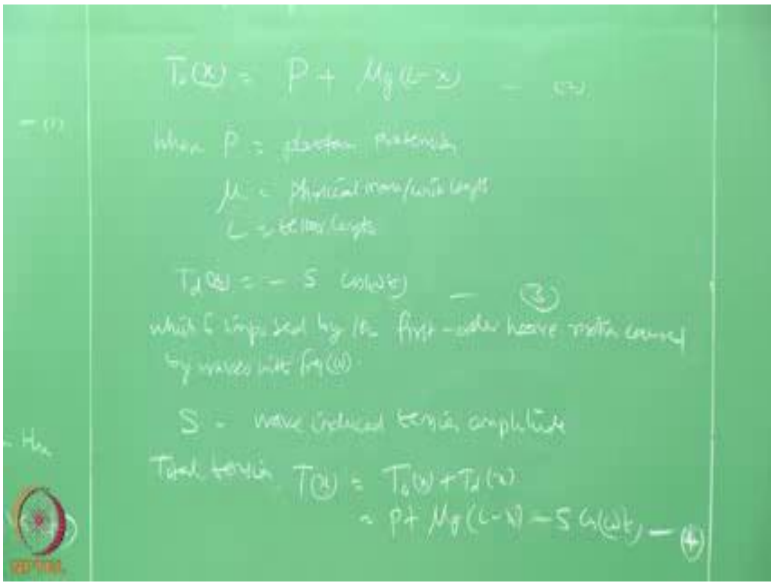

So the varying tension along the length of tether due to the action of submerged weight. One can now say T 0 x is p plus mu g l minus x, where P; let us say equation 2, is platform pretension, mu is a physical mass per unit length, L is a tether length. The dynamic tension, the second component is given by T d x which is minus S cos omega t, which is imposed by the first order heave motion caused by waves with frequency omega. In this case S is wave induced tension amplitude Therefore total tension which I need to substitute in equation 1 that is T x is P that is T 0 x plus T d x which is P plus mu g L minus x plus minus S cos omega 2.

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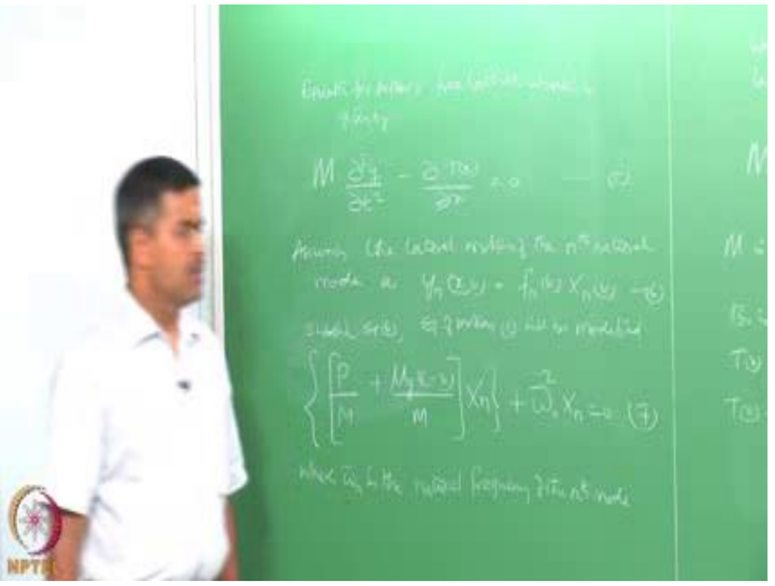

Now, the equation for tethers free lateral vibration is given by M dou square y by dou t square minus dou t x by dou x set to 0, which equation number 5. Now assuming the lateral motion of nth natural mode has y n of x of t can be written as f n of t, X n of t. Now let us substitute 6 in 5 above. Rather, let us modify substituting equation 6, the equation of motion what initially we proposed in equation 1 will be now modified as P by M plus mu g L minus x by M of X n plus omega square X n is 7; let us say equation number 7; where omega bar n is the natural frequency of the nth mode.

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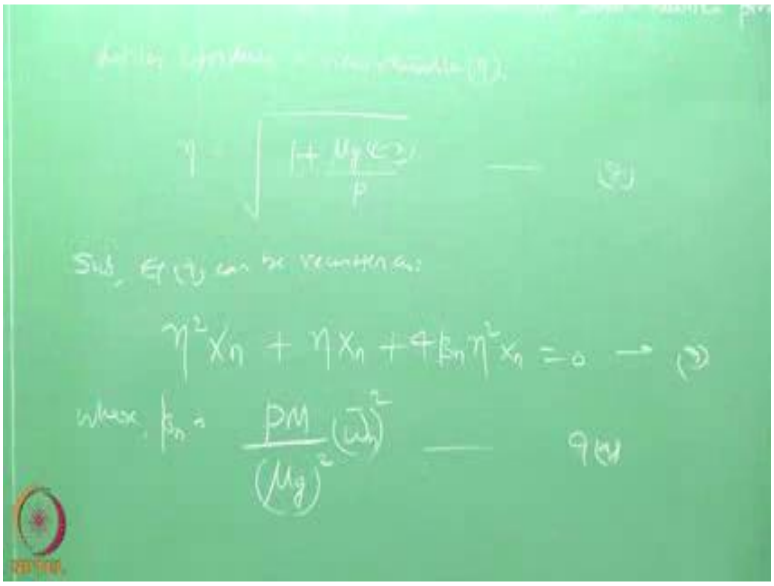

Now, this dynamic model shown in equation 7, leads actually to a classical Sturm Liouville problem. Let us introduce a new variable eta, where eta is 1 plus u g l minus x by P; equation eight. Now substituting equation 7 can be rewritten as eta square X n plus eta n x m plus 4 beta n eta square X n is 0, where beta n can be said as P M by mu g square omega n square; equation let us say 9. You look at the equation 9 closely.

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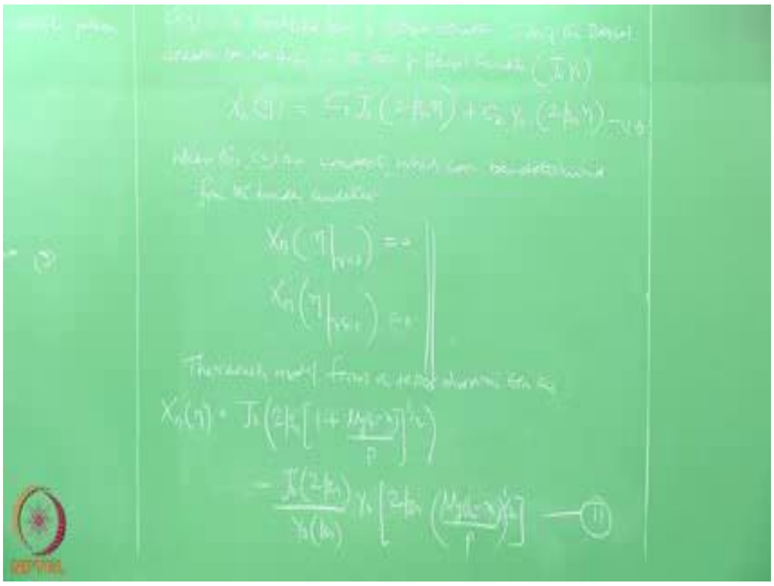

The equation 9 is actually a modified form of Bessel equation. The solution of Bessel equation can be given in form of Bessel function which in this case is J 0, Y 0. So, X n n is C 1, J 0 2 beta n n is going to be eta plus C 2 Y 0 twice; so equation number 19. Where, C 1 and C 2 are constants which can be determined from the boundary conditions. So, what are boundary conditions? X n eta r equals 0 is 0, and X n eta r equals 1 is also 0; both n's are simply supported.

Therefore, the resulting model the tether dynamic equation as given by J naught 2 beta n 1 plus mu g l minus x by P to the power half minus J 0 twice beta n by Y 0 half beta n half Y 0, I will write down the term here; minus J 0 twice beta n by Y 0 of beta n of Y 0 2 beta n u g l minus x by p half, equation number 11.

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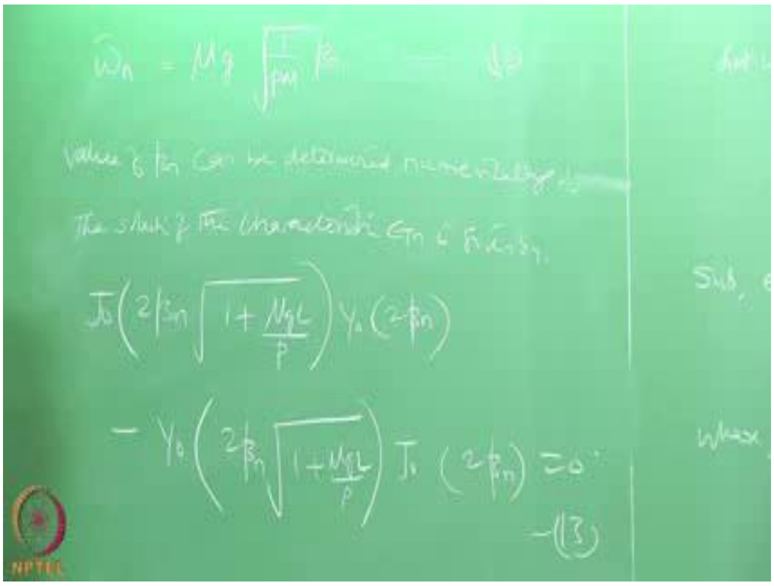

The frequency omega bar n will be given by u g 1 by P M of beta n; equation number 12. Now the value of beta n can be determined numerically, because the solution of the characteristic equation is given right, that is the reason why we can get beta n numerically J 0 2 beta n 1 plus u g l by P Y 0 2 beta n minus Y 0 2 beta n root of 1 plus mu g n by p J 0 2 beta n said to 0; equation 13. Substituting the value of the above equation in the original equation of motion what we had.

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Now, by substituting these values equation 1 will be modified using Galerkin's variation method. So, is nothing but multiplying it with X n of x and then integrating it along the tether length, so now we need coefficients to do this? Let us have, let the following coefficient be determined I 1 which is along the length you have to integrate; along the length 0 to l, so X n square d x. Let us say I 2 is again 0 to l d x n by d x of X n d x. Let us say I 3; 0 to l d square X n by d x square X n d x. Let I 4, be 0 to l d square X n by d x n square of X n x d x. Now let us say I 5 is 0 to l x cube x d x equation 14.

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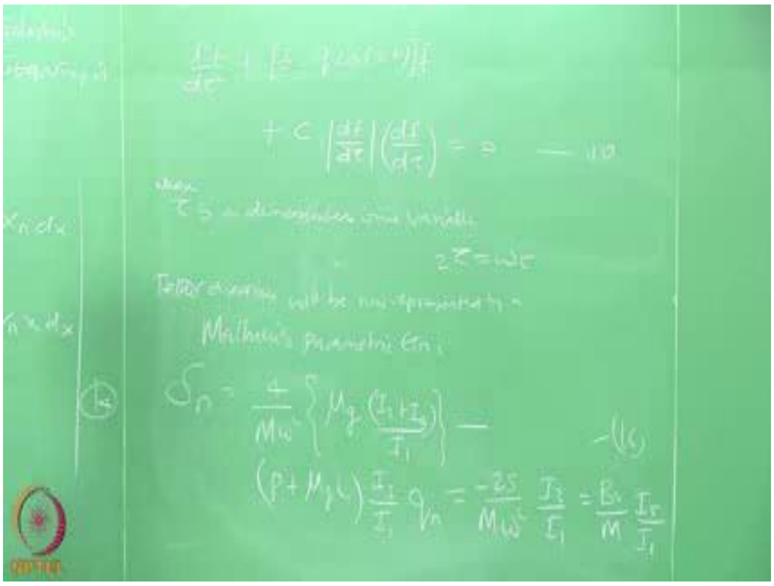

Now, with these coefficients in place equation 1 it is modified to a new form, so which is del minus q cos 2 tau of f plus C d F By d tau is 0; let us say call the equation number 15 with the modified form of the equation 1. In this case in the above equation where tau is the dimensional time variable which is know 2 tau this omega t. Now the tether dynamics will be now expressed represented by a Mathieu's parametric equation, there other 2 parameters, let us say what are these parameters? Delta n which is given by 4 by m omega square of u g I 2 plus I 4 by I minus P plus mu g l I 3 by I 1 q n which is equal to minus 2 S by M omega square I 3 by I 1. Which is further equal to b v by m I 5 by I 1; equation number 16.

Now the dynamic analysis will be performed assuming induced dynamic tension amplitude in the given problem which is as per the real case analysis what we are going to do.

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To assess the safety of the system, stability of the given system under the normal loading compression and operating conditions the dynamic analysis will be now performed under induced dynamic tether tension. As an extreme case, because we are looking for a failure case as an extreme case the dynamic tension amplitude equal to 60 percent of the nominal static tension will be imposed. Why it is done? As an extreme case to induce failure is or not, so we are going to check it. Therefore, it is a safety study.

So, we will apply this to a set of T lp examples and we will plot the Mathieu stability curve and show whether the induced tension as an extreme case results in failure or safety domain of the T lp, which will discuss as continued problem in the next lecture.

Thank you very much.