**Risk and Reliability of Offshore Structures**
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**Module – 02**
**Reliability theory and Structural Reliability**
**Lecture – 13**
**Failure Domains – II**

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Friends, welcome to the lecture 13 on module 2 on the course on Risk and Reliability of Offshore Structures. In module 2, we are talking about structural reliability. In this lecture, which is the 13th lecture in module 2, I will continue to discuss the failure domain which we started in the last lecture so I put failure domains 2.

In the last lecture, we already said that behaviour of general system can be modelled either as a parallel system composed of path sets. So, general system can be modelled either as a parallel system composed of path sets with each path set acting in the sub system of the component in series. So, each path set will actually become a subset in series; alternately, one can also look for a series system composed of subsets or cut sets each cut set will become a sub set in parallel as we saw in the last example.

Now the as a system in series composed of cut sets each cut set will of course become a sub set in parallel. So, the first order approximation of general system reliability is based on the cut set formulation, which I will discuss now which already discussed partially in the last lecture. A similar approach can also be developed using a path set formulation in order to estimate the probability of failure of a given general system using cut set formulation. Now let us talk about the cut set formulation.

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So, we know that probability of failure as expressed in the last lecture is simple given by probability of U m then intersecting with i element of C m that is a subset of g i of x less than or equal to 0 equation number 1. Now, we will like to know that E m i going to the corresponding event which is section of i element C m g i of x less than or equal to 0, so E m is the event that the parallel sub system represented the cut set C m so E m is actually the is actually the event that parallel subsystem being represented by the cut set C m fails or cut set failure. So, therefore, the probability of failure as we said is probability of U m E m, where E m is the now the event. So, now, we also have something inclusion exclusion theory of the set theory then one can write the following statements very easily which we express the last lecture.

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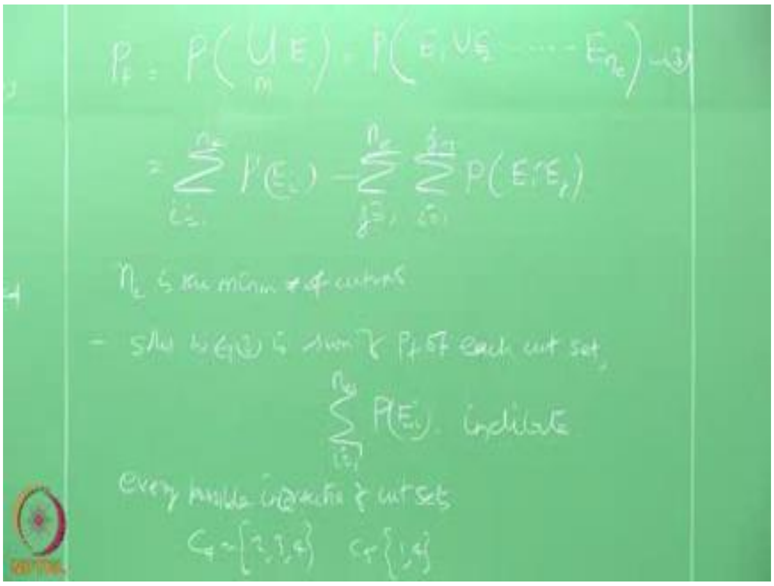

And probability of using inclusion exclusion theory of the set theory one can say probability of failure can be simple given by probability of U E 1 which can be said as probability of E 1 U E 2 as it goes up to U E n e which can be also simply i only P of E i which is minus i j of j minus 1 which is probability of E i E j. Now, in this case, N c is a minimum number of cut sets compressing the system or identified for the system. Therefore, the above equation can be simple solved by summing the probability of failure of each cut set, so solution to equation 3 is sum of probability of failure of each cut set. So, now, which is nothing but sum of probability of each cut set i equals 1 to let us say n e P of E i.

I mean one can say n c, please see let us say n c here n c, n c, where n c actually a minimum of cut sets so which will indicate every possible intersection of cut sets is not it. So, we said that after identifying a minimum cut set C 4 which is 2, 3, 4 for the classed coated example. And C 5 is 1 and 4 being the minimum cut sets which are adapted for the given example we can substitute appropriate sign and then apply the summation series to find the probabilities of failure of the whole general system. So, in this case, E 4 and E 5 will be represented.

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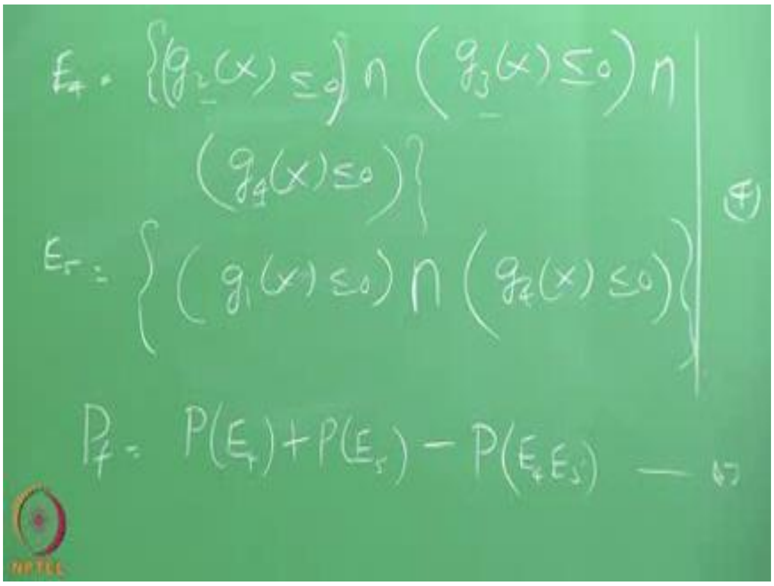


And we already said E 4 and E 5 will be actually the events associated with the parallel subsystem with cut sets C 4 and C 5 fail. So, E 4 and E 5 can be simply said as g 2 of x less than or equal to 0 intersection of g 3 of x less than or equal to 0 section of g 4 of x less than or equal to 0. And obviously, E 5 will be because E 4 corresponds to C 4, C 4 is having elements 2, 3, 4. Therefore, we have written 2, 3 and 4; similarly, E 5 is having elements one and 4 so I should write g 1 x less than 0 intersection 1 and 4, so g 4 x so equation number 4, which of course expressed in a last class as well. So, probability of failure is now given by probability of E 4 event plus probability of E 5 event using the theory of probability what we studied in the last module probability of E 4, E 5. So, we have already expressed E 4 probability of E 4 probability of E 5 then E 4, E 5 etcetera with beta 4 beta 5 etcetera.

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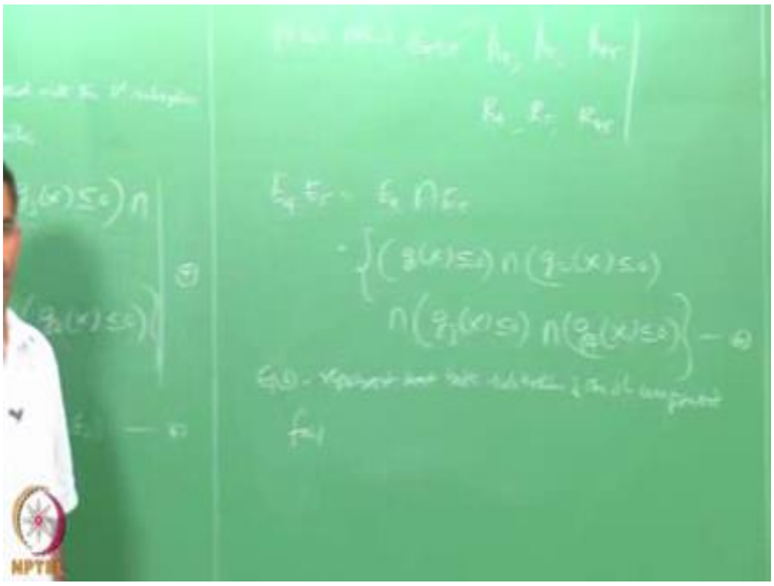

Now, interestingly we know probability of E 4 in the last lecture probability of E 5, E 4 E 5 we expressed all of them. We also know beta 2, beta 5 we already know beta 4 then we already expressed the equation for beta 5, we already expressed equation for beta 4, 5, we also know equation for R 5 and R 4 5, which you have given in the last lectures is not it.

And we wrote the matrix for matrix and vector for R'S and B's respectively for this particular problem. So, now, E 4 E 5 is E 4 intersection E 5, which can be simply g 1 of x less than equal to 0 domain intersection of g 2 of x less than equal to domain intersection of g 3 of x less than equal to 0 domain intersection of g 4 of x less than equal to 0 domain call this as equation number 6. So, equation 6 represent the event that both sub systems are parallel components fail; equation 6 represent that both sub systems of the parallel component fail. So, none of looks at the failure domain of this which shows the first order approximation which will look like this.

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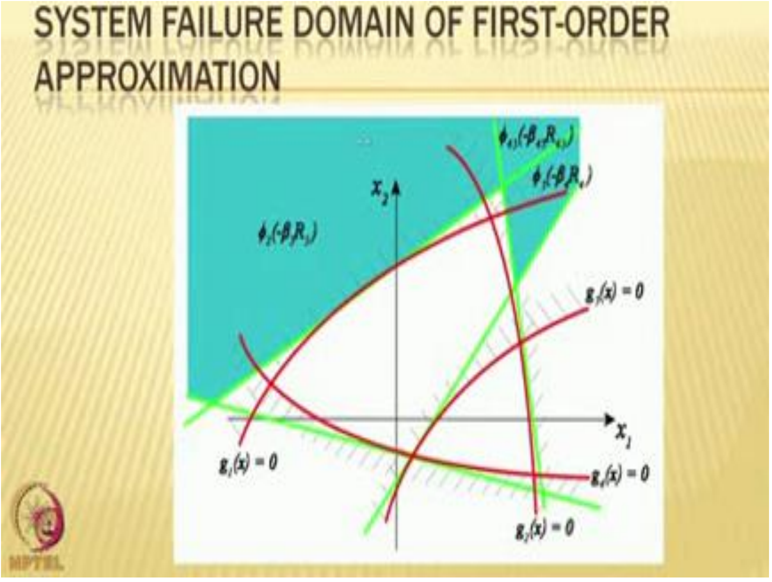


So, please pay attention to the system failure domain or first order approximation shown in the screen now. One can see here the failure probabilities using beta 4 5, R 4 5, beta 4 R 4 beta 5 R 5 which we just now wrote the equations on the blackboard. So, the red lines indicates the failure domain let say g 1 of x equal 0 and this of course indicates g 2 of x equal to 0, and this indicates the performance function g 4 of x 0, and this indicates g 3 of x 0.

We are looking for the intersection of this the shared portion will show the system failure domain which is the intersection of all these as seen in the expression or equation number 6. So, g 1, g 2, g 3, g 4 are all shown in the screen which you can see here the failure domains. Then of course the probability of failure 5, 4, 5, 5, 2 etcetera which you have computed based upon beta and R’s which you are given the equations. So, one can find the tangents and one can see the intersection when the shaded portion shows the failure domain the system failure domain using first order approximation.

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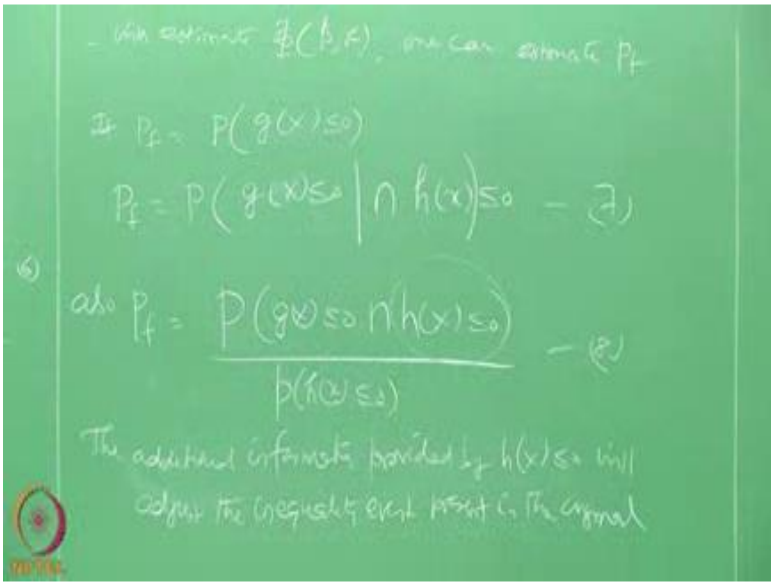

The figure showed the failure domain of the first order approximation of the system using first order approximation. For most of the system problems, one can see here that the probability of failure can be estimated even when there is a difficulty in estimating phi's of beta. So, the crush of the problem is to estimate phi's function of beta R you where know there are difficulties but still one can estimate with the estimate of phi beta R, one can estimate probability of failure of the given system which has been either replaced by equivalent parallel systems with minimum path sets or equivalent series system with minimum subsets.

Let say if the probability of failure is given by probability of let us say g less than 0 function for a true reliability analysis then the inequality observed is there in the given component now modify the above equation now can be modified as probability of failure which can be P of g of x less than 0 less than less than 0 given intersection of h of x less than 0. The above equation let us call this as equation number 7. Now, one can also expand this equation slightly in a better manner also probability of failure can be said as probability of g of x less than 0 intersection h of x less than 0 by probability of h of x. So, the addition information provided by this function will be able to adjust the inequality event will adjust the inequality events present in the original.

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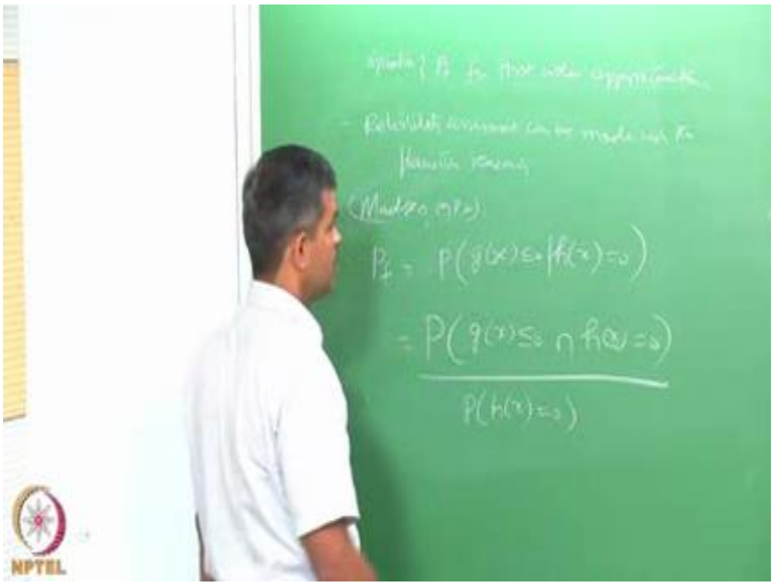

Equation of probability of failure for first order approximation, one can also use this addition information based upon the reliability assessment can be made using the plausible reasoning which you already studied in the earlier modules. So, Madsen, 1987 as given further information on the specific application we say probability of failure can be redefine slightly from equation 8 further that probability of g of x less than 0 given h of x equals 0. Which can be said as probability of g of x less than 0 intersections h of x equals 0 divided by probability of h of x equals 0 - equation number 9.

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Which can be further extended as probability of g of x less than 0 intersection h of x equals 0 divided by epsilon of P of h of x equals 0 h of x let us say minus epsilon equation number 10. Which can be further extended as epsilon tends to 0 dou by dou epsilon probability of h of x equals 0 h of x minus epsilon less than or equal to 0 can be said as probability of h of x probability of h of x minus epsilon minus del epsilon less than equal 0 minus probability of x h of x minus epsilon less than 0 divided by d epsilon which can probability of epsilon less than h of x less than equals epsilon plus d epsilon by d epsilon - equation number 11.

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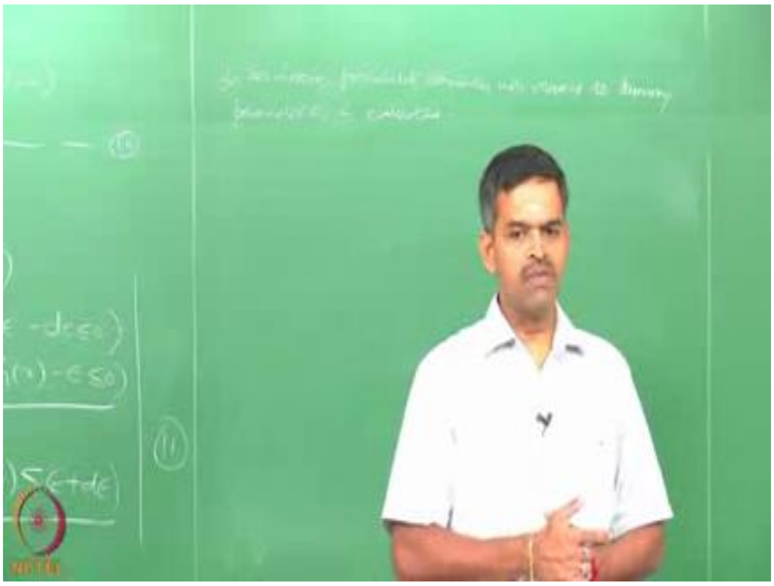

One can see here in equation 1, in this method the probability sensitivity with respect to dummy parameter epsilon is required to be evaluated. Probability estimates with respect to dummy parameter epsilon is evaluated. So, friends in this lecture we discussed about the failure domains how it is extended further. And how one can actually find out the failure probability of a general system which comprises of either minimum path sets parallel system or minimum cut sets series system, where essentially they converse to failure of the minimum parallel system with minimum path sets leaving to failure of sub system of system in series. Whereas minimum cut sets lead to sub systems fail in parallel.

So, both of them will give; obviously, the same answer as we discussed in the last lecture as well. So, here we have already told you how to compute the first order approximation and we also showed you the failure domain of the general system comprising of cut sets minimum cut sets in the presentation. So, the equation can be further modified to find probability of failure as given by Madsen, 1987. So, one can find out the probability of failure by identifying intersection of different failures of course, with respect to here dummy parameter epsilon as you see from this equation 9, 10 and 11.

I hope the module what we discuss so far to understand the first order second moment onwards different system reliability and component level reliability. We have also discussed about the system reliability derived from the component level failure as subsets of the main system using path sets or subsets, which we have already discussed with cut sets and path sets. One can easily find out the probability of failure of assembly of this as intersection or union of the failure domains given by different surfaces domains of different variables.

So, we will also look into some application problem of this very quickly in the next lecture before you move on to further extension of this for higher order reliability theories.

Thank you very much.