Risk and Reliability of Offshore Structures
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Module – 02
Reliability theory and Structural Reliability
Lecture – 12
Failure Domains

Friends, let us continue with the lecture 12, where we are going to talk about the Failure Domains in system reliability.

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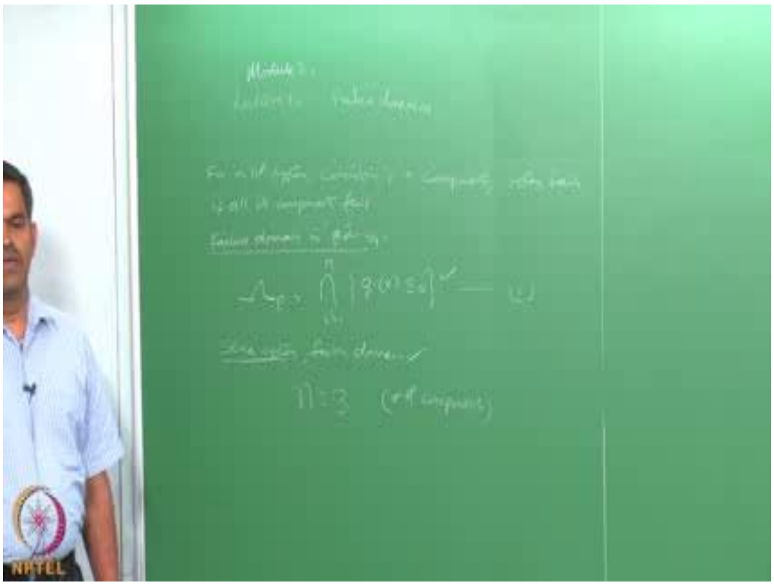

This is lecture in module 2 on system reliability or reliability theory, on the online course risk and reliability of offshore structures. We already said for a series system and a parallel system. 1 can easy estimate the failure domains. So, let us revise for a parallel system which consists of n component. n components the system fails if all its component fail. Therefore, the failure domain is given by as we saw in the last lecture, for a parallel system is actually intersection of the component failure domains i equals 1 to n, n is the number of components, g i x less than or equal to 0 that is a equation.

Now, let us try to understand the failure domains of series system and parallel system graphically. So, we know that series system failure functions, we know the failure domain of the parallel system. Let us try to understand this graphically 1 is of course, intersection of the failure domains of different sub components other is the union. Because you know the failure condition for series systems is, if any fails the system fails where as for the parallel system see if all of them fail the system fails. Therefore, accordingly we have formulated the failure domain we are given the governing equation, for g f x as we saw in this lecture and as we understood in the last lecture.

Now, let us try to understand how the limit state function, looks like for a failure domain of series system and that of a parallel system consisting of let us say 3 components let us say n the number of components is 3. So, in the figure the hatching along the limit surface will be done. So, that it indicates the region in which the corresponding limit state function is lesser than 0. So, kindly pay attention to the figure shown in the screen now.

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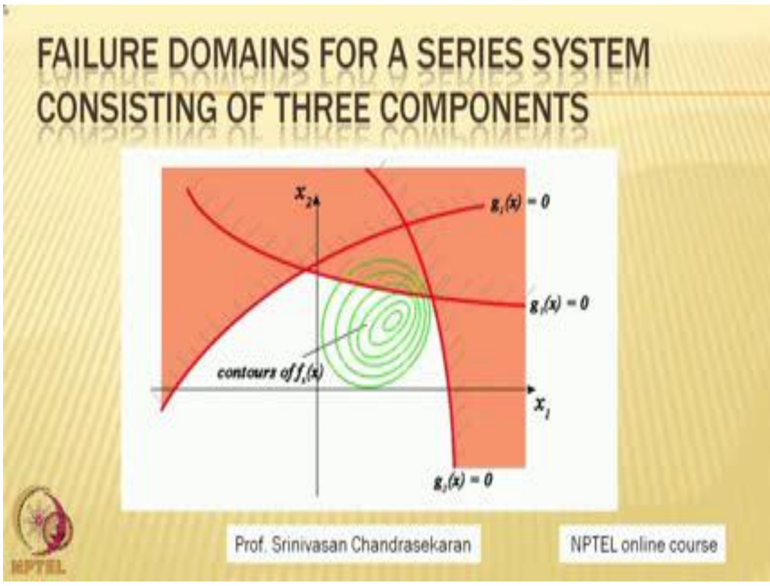

The figure shows the failure domains for a series system consisting of 3 components 1 can see here g 1 of x is equal to 0. Which indicated by this line the failure domain is this, this area which is the failure domain.

Similarly, g 3 of x is this line the failure domain is indicated here, and g 2 of x is this red line the failure domain is indicated here we know for a series system consists of 3 components it is union of all of them. So, the hatched portion actually shows the failure domain of a series system with 3 components; x 1, x 2 and x 3 or g 1 of x, g 2 of x, g 3 of x being the limit state functions of 3 independent components. Similarly pay attention to the figure shown in the screen now.

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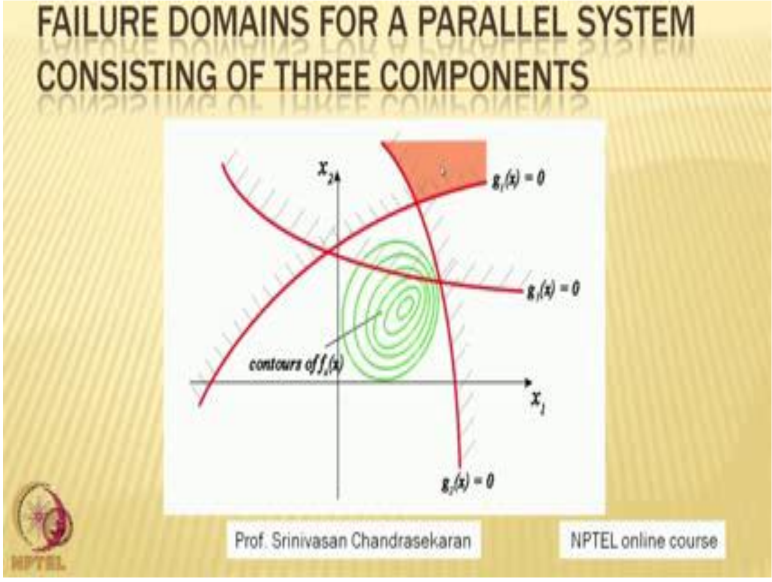

This figure shows the failure domain for parallel system consisting of 3 components and see here g 1 of x is the limit state function for the first component. g 2 of x is the limit state function for the second component. The hatched portion shows the failure domain. g 3 of x is the failure domain for the third component the hatched portion shows the failure domain. Just now as we saw from the equation 1 in the blackboard here the failure domain is an intersection of these failure components. So, the hatched portion becomes the intersection of this failure domain, this failure domain and this failure domain. So, this becomes an intersection of all the 3.

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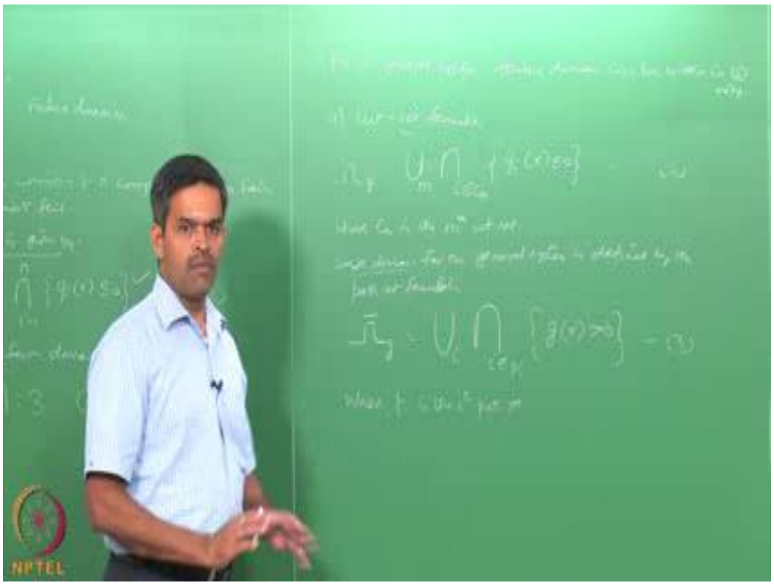

Therefore for a general system failure domain can be given by cut set formulation that is a failure domain can be written in 2 ways, can be written in 2 ways. Let us use the cut set formulation. For the cut set formulation the failure domain is given by u of m, intersection of i element of c m and failure function is a g i of x less than or equal to 0. Where c m is mth cut set. Now the safe domain for the general system with cut set of representation is define by the path set formulation. So, that is going to be the safe domain which is going to be over i intersection of i element of p i with g of x greater than 0, because I am talking about the safe domain equation number.

Where p i is the ith path set. Now let us try to understand this dialogue or argument with minimum cut sets. So, for a general system failure domain is given by the union of minimum cut sets.

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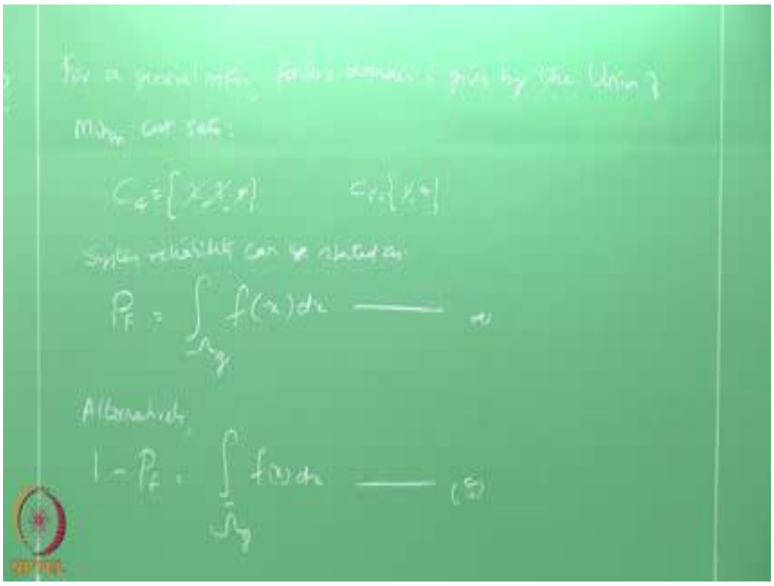

What are the cut sets in this case for our example 1 is, c 4 which is 2, 3, 4 other is c 5, Which is 1 and 4, these are the minimum cut sets. We have the failure domain was shaded in the figure as we you saw in this screen for some assume component, limit state surfaces.

Therefore, the problem of system reliability now can be stated. Now your failure can be given by g f of x, which you call as equation number 4. One can always say alternatively probability of failure you can also be given as r 1 minus probability of failure can be given as integration of bar g f of x. So, pay attention to the figure shown in the screen now.

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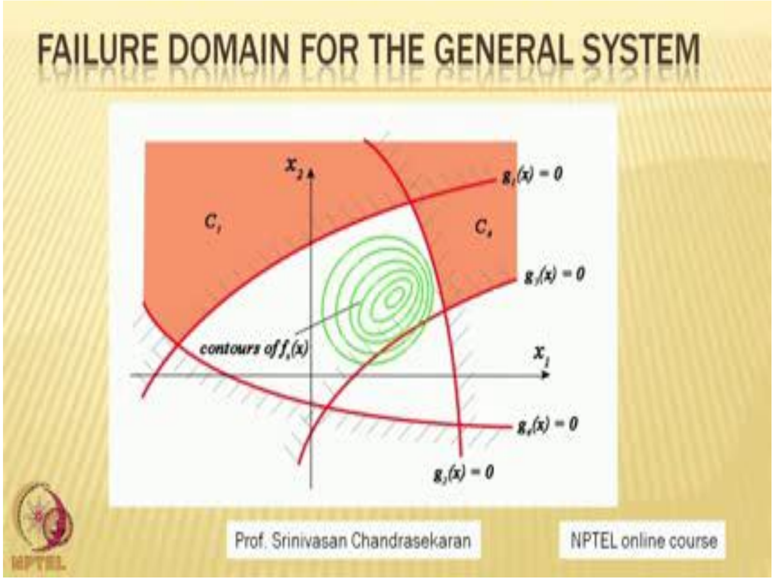

We are now marking the failure domain for the general system, which includes the minimum cut sets c 4 and c 5. So, one can see here the shaded region, which includes c 4 the failure domains and g 1 of x, g 2 of x or g 3 of x and g 4 of x and g 2 of x or the limit state functions assume for events 1, 2, 3 and 4. So, the shaded region shows the failure domain we can see here; g 1 of x is got failure domain the surface, g 2 of x has got a failure domain in this way; g 3 of x has got a failure domain this way and g 4 of x has got a failure domain this way. So, the intersection i mean the union of this is going to give me the failure domain of the general system.

Having said this lets us explain the discussion for first order estimates of this way.

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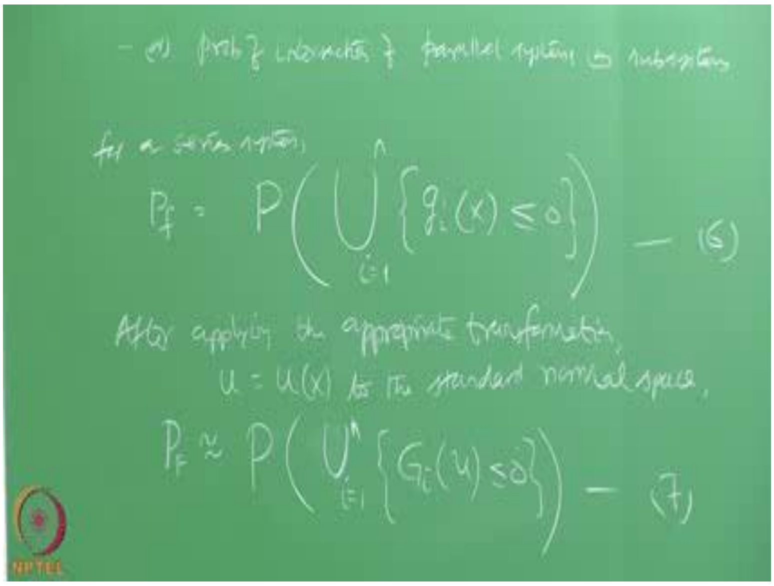

Now, in order to compute the system reliability, one must be able to compute the probability of the union of events beat system series is a series systems of sub systems. So, if you really want to find the probability of failure, or to compute the system reliability. One needs to estimate the probability of union of series systems or sub systems or probability of intersection of parallel systems or sub systems. Now for a series system, probability of failure is given by probability of union i equals 1 to n g i of x less than or equal to 0, call equation number 6, after applying to the approximate transformation u t of x, after applying the approximate, let us say appropriate and after applying the appropriate transformation, u as u of x to the standard normal space.

One can find the probability of failure approximately as probability of union of i equals n g of i of u less than or equal to 0. When there is an approximation due to mapping of non normal variable occurs. When there is an approximation of mapping of non normal variables to the standard normal variable space occurs then, one has to linearlize g of u equals 0, at the design point for the ith limit state function.

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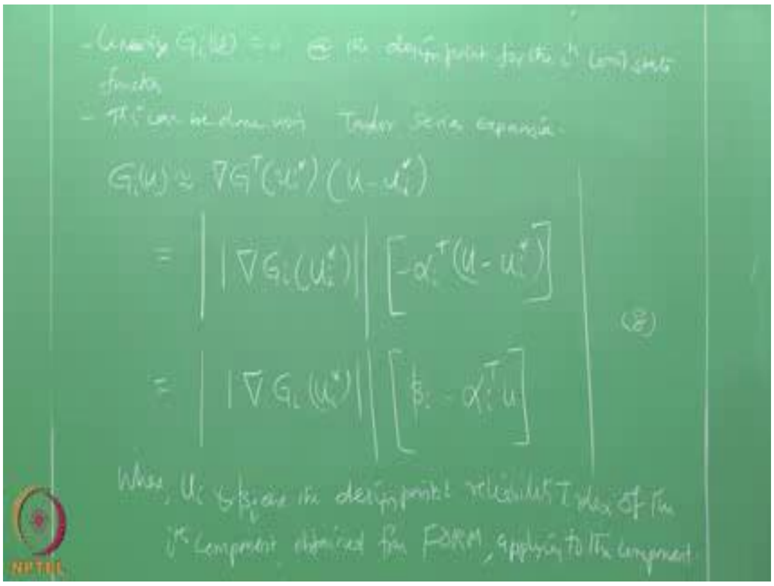

Now, this can be done using Taylor Series Expansion. So, in that case g i of u will be approximately equal to g transpose u i star u i u minus u i star, which is further equal to g i of u i star minus alpha i transpose u i or u minus u i star, which can be further written as; g i u i star beta i minus alpha i transpose u. It say set of equation as where u i and beta i are the design points and reliability index of the ith component of time from first order reliability methods applying to the component.

The corresponding unit travel vector to limit surfaces.

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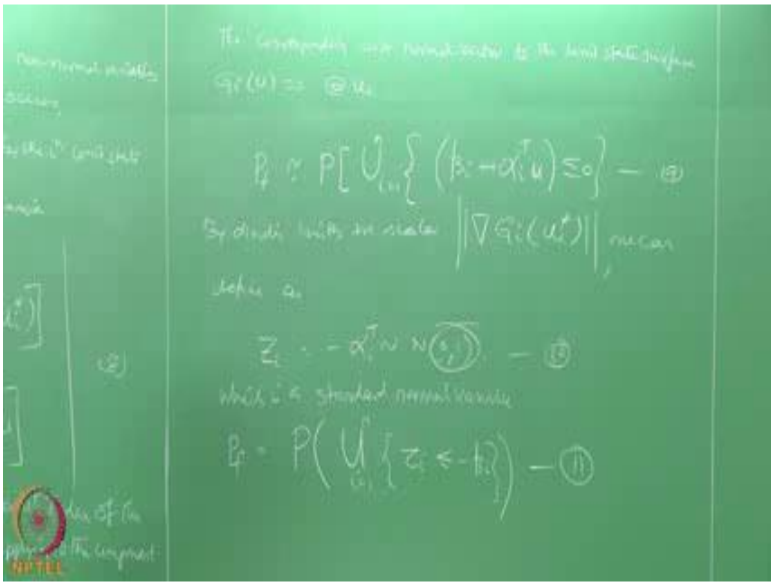

Your surfaces is actually g i of u set to 0 at u i. Now, in that case the probability of failure will be approximately equal to probability of intersection, union i equal 1 to n beta i or beta i minus alpha transpose u. Which is less than or equal to 0 equation number 9. Now dividing both sides of the inequality whether positive scalar, let say my dividing with positive scalar which is g i u i star. One can define as z i minus alpha i transpose which is approximately n of 0, one which is a standard normal variable which 0 mean process in that case probability of failure is given by probability of u i equals 1 to n z i less than equal to beta i.

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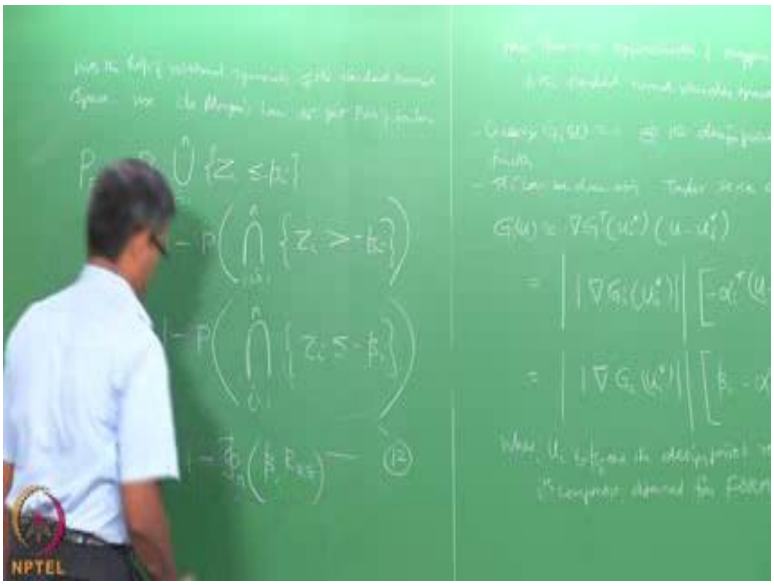

So, with the help of rotational symmetry of the standard normal space which is actually used, when you are transforming the non normal variable to the normal variable, one can use the de Morgan's law. So, in that case to get probability of fail in that case probability of failure is given by probability of z i less than, beta i which is equal to 1 minus p of section of i equals 1 to n z i greater than minus beta. Which is further equal to 1 minus p of intersection of i equals 1 to n z i less than or equal to minus beta, which I call as equation number 12, which can now result to easily because i have a very interesting argument of normal variability here which can be simply 1 minus beta and r z.

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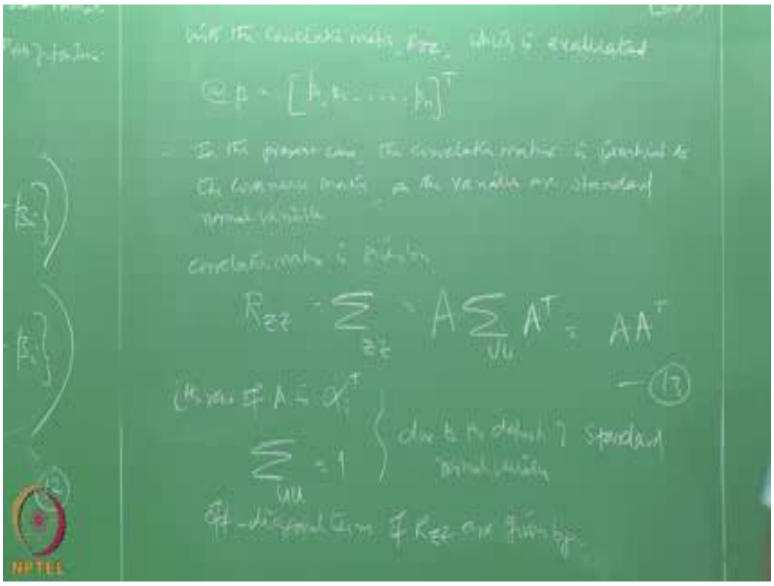

Where phi n of beta r z is actually the joint normal cumulative distribution function joint normal cumulative density function, CDR with the correlation matrix r z which is evaluated at beta which is given by different indices beta 1, beta 2, beta n transpose in the present case the correlation matrix is identical to co variance matrix. Because of the normal variables in the present case the correlation matrix is identical to the co variance matrix because variables are standard normal. Therefore, the correlation matrix is given by r z z is nothing, but which is a u, u a transpose nothing, but a a transpose equation 13 the off diagonal terms in this case the ith row of a is actually alpha i transpose and actually unity this is due to the definition of the variables.

The off diagonal terms of r z z are given by z I, z j is alpha i transpose alpha j equation number 14 which quantifies the correlation between the failure modes i n j.

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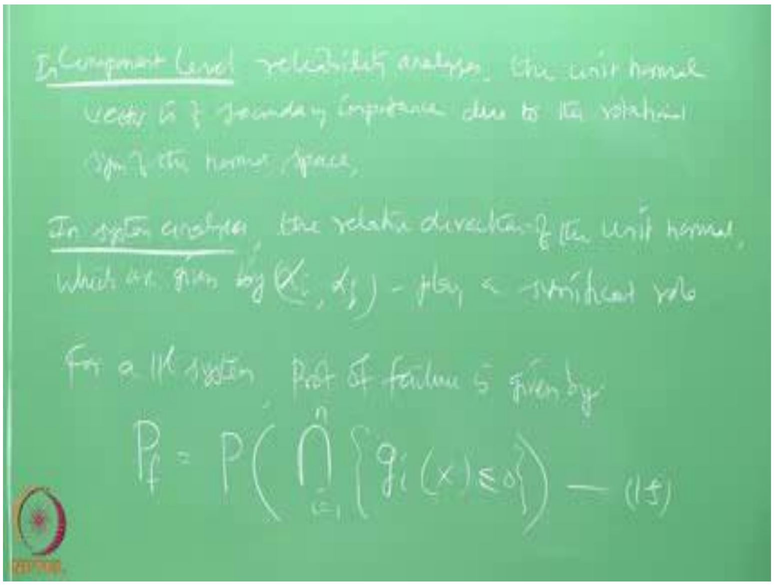

So, equation 14 quantifies the correlation between the failure modes i n j, now also friends please understand the unlike the reliability analysis component level. So, in component level reliability analysis the unit normal vector is of secondary importance due to the rotation symmetry of the normal space in system analysis you will see there the relative directions of the unit normal places rule which are given by alpha i and alpha j, play a significant role that is a difference actually between the component level and reliability level in analysis.

So, for a parallel system the probability of failure is given by probability of section of i p equals 1 to n g i x less than equal to 0 - equation number 15.

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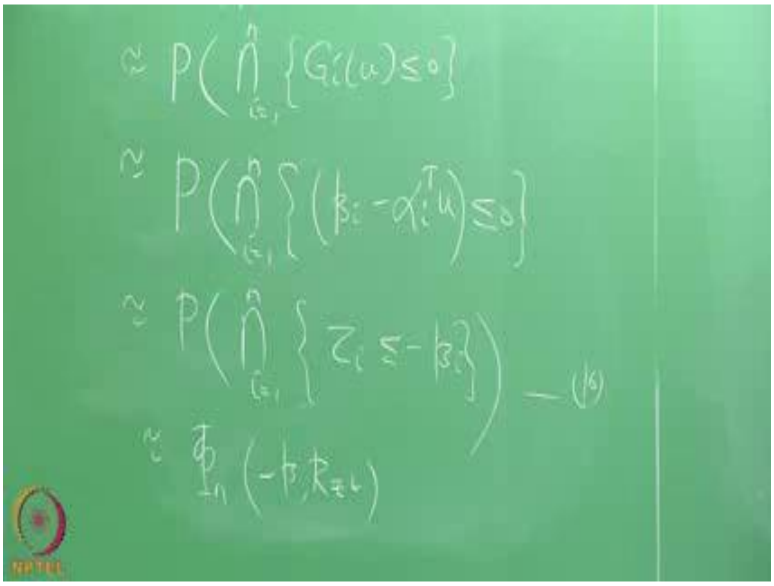

Now, this can be approximated as follows. So, I say probability of failure is probability of intersection of i equals 1 to n, which is g i of x less than or equal to 0. We just approximately equal to probability of intersection of i equal to 1 to n of another set of variable which is u less than 0. Which then can be approximated as probability of intersection of i equals 1 to n, which tells me the reliability index beta i minus alpha i t into u variable less than or equal to 0. Which can then approximated as probability of intersection of i equals 1 to n, which is z i less than or equal to minus beta i because i am transforming this into an equivalent normal variable space equation number 16, which will amount to phi of n of minus beta r z.

So, we already defined beta and r z z in the previous explanation already said beta and r z z z correlation matrix. So, the same definition applies here so, since we are transform the variables from x to a normal space u i can apply this algorithm and get my probability of failure is nothing, but the phi function of the 2 variables beta and r z z.

Therefore friends from these 2 lectures, one can easily understand behaviour of a general system can be model either as a parallel system composed of path sets with each path set acting like a sub system of components in series or vice-versa first order approximation for general systems reliability is based on the cut set formulation, which is now discussed a similar approach can also be developed using a path set formulation, which i live it to you for the self study in order to estimate the probability of failure system using the cut set formulation 1 need to evaluate the probability of failure of each system. So, let us talk about that.

So, we are trying to estimate the probability of failure general system using cut set formulation. So, 1 need to know the probability of failure of the system which is given by probability of u m section i the element of c m of g i x less than 0 equation 17.

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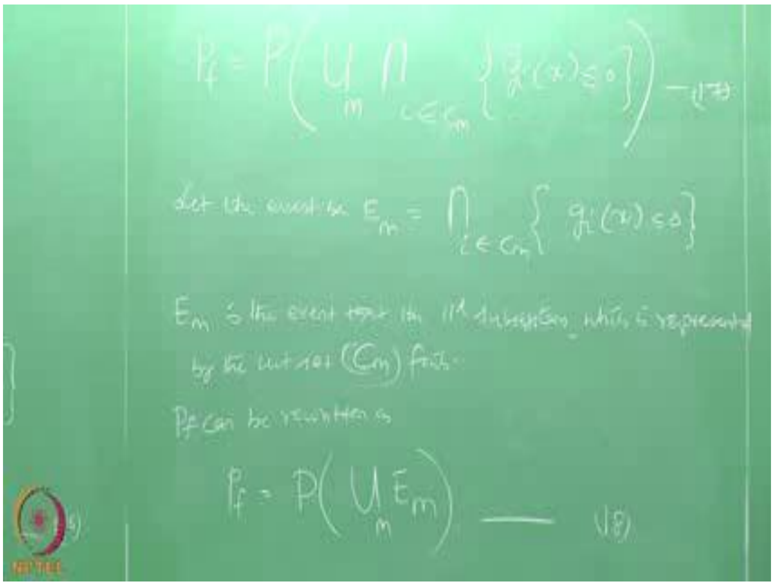

Now, let the event be e m which is intersection of i element of c m which is then extended on the failure domain g of x less than 0. Now e m is actually the event that the parallel sub system which is represented by the cut set when c m fails.

Therefore probability of failure can be now rewritten as probability of u m e equation number 18. So, as per the inclusion exclusion of set theory 1 can derive the following statements.

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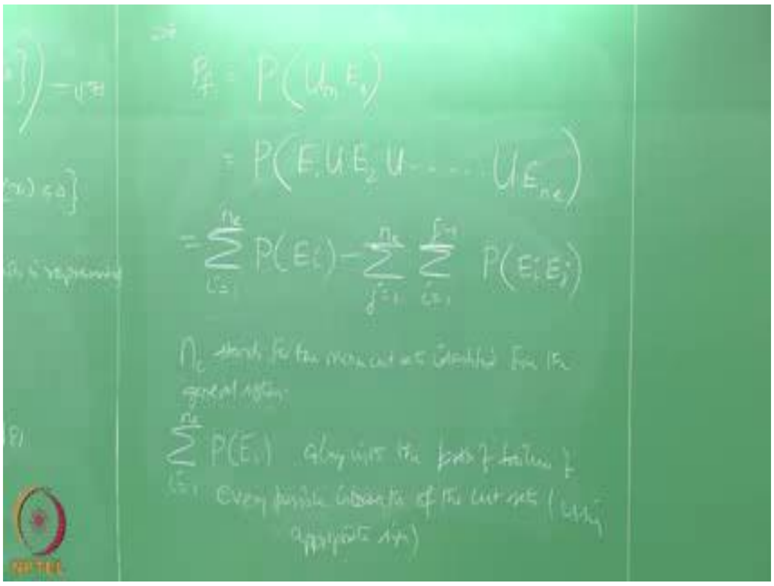

Further probability of failure can be then written as probability of u m e. One which can be further read as probability of e 1 union, e 2 union keep on going as e n e which then can written as i equals 1 to n e of probability of e i minus summation of j equals 1 to n e summation of i equals 1 to j minus 1 of probability of e i e j.

In this argument n c stands for the minimum cut sets identified from the general system. Now the above equation can be solved with summing the probability of failure of each cut set. Let say summation of i equals 1 to n e probability of e i along with the probability of failure of every possible intersection of cut sets. So, the above equation is now solved by summing the probability of failure of each subset each. Sorry each cut set given by this expression along with the probabilities of failure of every possible along with the probability of failure of every possible intersection of the cut sets identified of course, using appropriate sign that is very important it is an algebraic summation.

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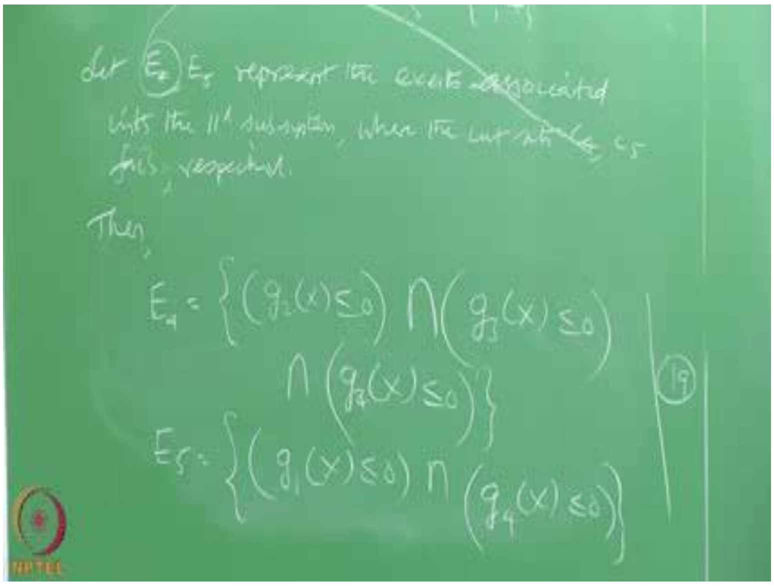

For the example we are discussing we are discussing 2 cut sets where identified. What are they? They are c 4 which is 2, 3 and 4 and c 5; which is 1 and 4, therefore let e 4 and e 5 represent the events associated with the parallel sub system where the cut sets c 4, c 5 fail, now I should say representatively because e 4 is also with c 4 and e 5 with c 5 respectively.

Then one can say e 4 is nothing, but there is nothing, but related to c 4 therefore, 2, 3, 4 are there. Therefore, i can say g 2 of x less than equal to 0 intersection g 3 of x less than equal to 0 intersecting g 4 of x less than equal to 0 that is going to be e 4 and e 5; obviously, in the similar pattern is e 5 compress of c 5 is a 1 and 4. So, 1 can say g 1 less than 0 intersecting g 4 less than 0 calls as equation number 19.

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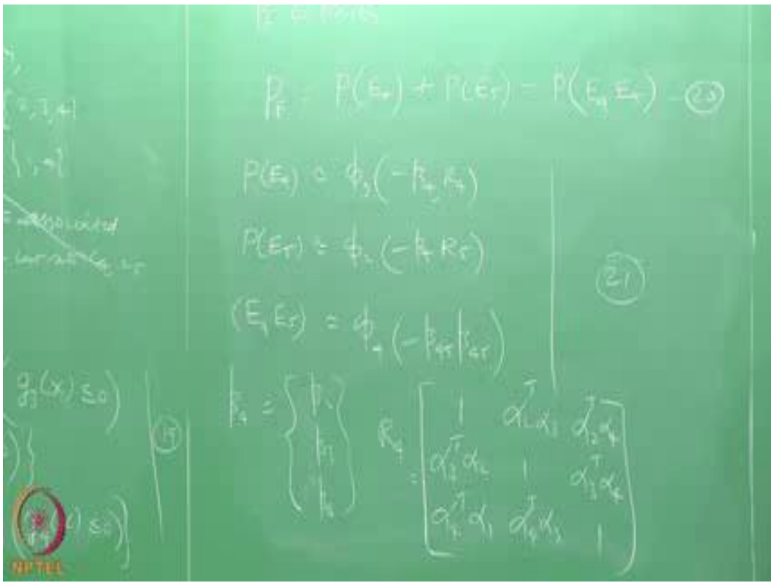

Therefore, probability of failure is given by probability of failure is p of e 4 plus p of e 5 because, these are the 2 events from the cut sets minus p of e 4, e 5 which is essentially derive based on the rules of probability, what we studied in the first model call equation number 20. So, for explanation p e 4 is nothing, but phi of 3 which has arguments of reliability index b 4 and r 4 p of e 5 is approximately phi of 2 of minus beta 5, r 5 and e 4 e 5 is probability of phi 4 minus beta 4, 5 beta 4, 5 equation number 21.

Where beta 4 is; beta 2, beta 3, beta 4; r 4 is the correlation matrix which is going to be 1, alpha 3, alpha 2, alpha 4, alpha 1, alpha 2, alpha 3, 1; alpha 4, alpha 3, alpha 2, alpha 4, alpha 3, alpha 4, and 1.

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And in this case alpha 2, 3 and alpha 2, 4 again 1 alpha 4, 3 and 1 alpha 3, 4 that is going to make r 4, 5.

Friends, in this lecture we are able to estimate the probability of failure for example, of a minimum cut system taken from the general system, applied the mathematical simplification of converting the non normal variables. So, a normal variate space and variable to estimate the probability of failure using the rules of probability theory what we studied in the first module.

We will extend this discussion further and try to understand how this can be further in detail being done, and then we will take up this application later with a numerical example.

Thank you very much.