Risk and Reliability of Offshore Structures
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Module – 02
Reliability theory and Structural Reliability
Lecture – 10
System Reliability-II

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Welcome friends to the 10th lecture on System Reliability. This is an online course on Risk and Reliability of Offshore Structures. We are talking about lectures on module 2; we are focusing on structural reliability and reliability theory in general. This is lecture 10, where we will continue with the discussions on system reliability the discontinued from the last lecture. So far we have being discussing about different methods of reliability; different levels of reliability analysis, which can be applied to component level reliability analysis.

So, all methods like for example, first order second moment method, advanced first order second moment method, Hasofer-lind method, response method using linear functions for performance theorems etcetera, all these are applicable to component level reliability. Let us extend the discussions on structural level; performance of a structural component or an element is quantified as single, continuous or differential limit state function. So, when you talk about the performance function of a structural component what we can say as an element level can be quantified in three ways may be as a single limit state function, may be your continuous limit state function or limit state function which is differential.

So, the validity of the limit state function g of x less than 0 will show me that it is a failure domain through which or based on which we can extend the discussions to find out the probability of failure as said by Elishak off in 1999. Now, when we have more than one function which may be needed to define the performance of a structure then that function is also lot continuous and differentiable then the structure must be analysed as a system. If we have a performance function which is more than one and it is not continuous or differentiable then the whole discussion applied to component level will get back to a system, because then the analysis should be done at the system level as the system level it should be carried out as a system. So, system reliability does not essentially mean that you are analysing a system which is having combination of elements, it also depends upon the qualifying performance function as you see here.

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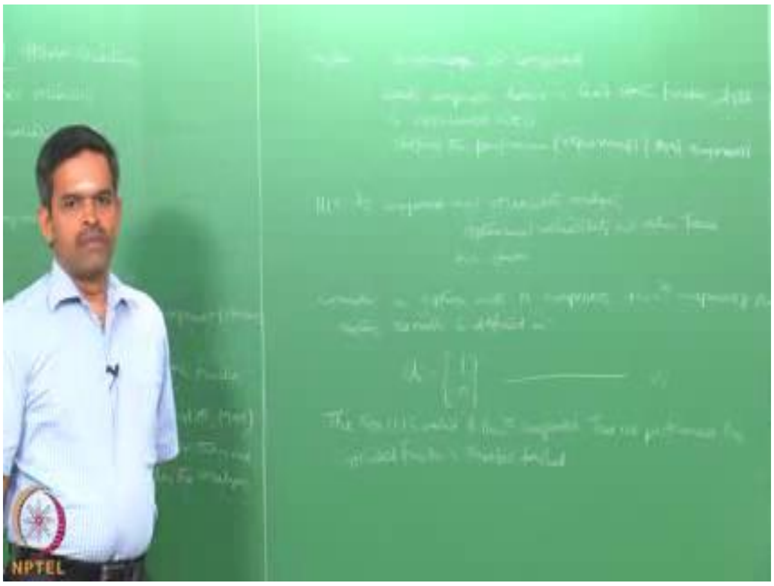

Hence, system can be now defined. System can be defined as we all know an assemblage of components each component having element state function, which is associated with that; and this limit state function defines the performance requirement of that component. Similar to the component level reliability problems, which considered only let us say two variables. What are the two variables the safe and the failure that is a binary variable 0 or

1 system reliability will also have two states similar to component level reliability system level reliability will also have two states. So, let us say for example, consider a system consisting of n components. Now, for an ith component, let us define a variable. The variable is defined as let say we can have only two states, either fail or safe; let us call this equation number 1. This equation is valid if the ith component does not perform the intended function and as fail the equation 1 is valid if the ith component as not performed the intended function and therefore, fails.

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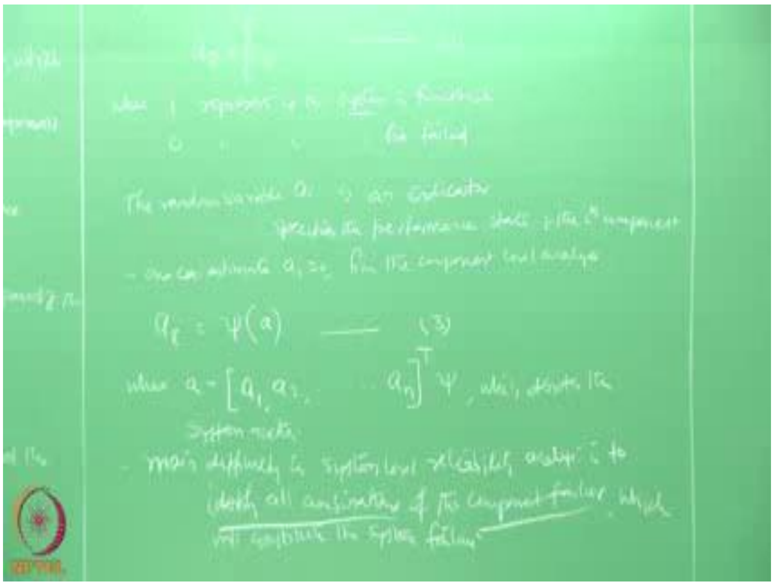

Similarly, for the entire system, we can say set is 1, 0; where 1 represents if the system is functional and 0 represents if the system is as failed in the whole discussion. The random variable a i is actually indicated this indicates a specifies the performance state of the ith component. Therefore, it should be apparent for us to know that the probability a 1 equal to 0 can be estimated by a component reliability analysis. So, I can write here 1 can estimate let say a 1 equal to 0 from the component level analysis and to do this you do not require a system level analysis.

Now, the indicator function a i which specifies the performance state of the system is actually the function of performance of it is components is it not. Therefore, a z can be given as function of a where a is nothing but the set of a 1, a 2, a n where n is the total number of component present in the system. Let us say transpose the function, which denotes the system auction while the performance criteria for an individual element are usually easy to determine and fix.

You can easily quantify this through a clearly defined simple limit state function performance criteria to be fixed for entire system is generally complex therefore, an important step in system reliability analysis is identifying all combination of the component failure, which constitutes the system failure. So, the main problem or the main difficulty in system level reliability analysis is to identify the combinations all combinations of the component failure which will constitute the system where as reinsist that determining the system failure is far simple in terms of assuming a limit state function appropriately and finding out the component level failure. Whereas a system level failure is difficult because you need to identify all possible combinations of the component failure to really declare the system as failed. Therefore, system needs to be idealized; system can be idealized in two ways series system and parallel system.

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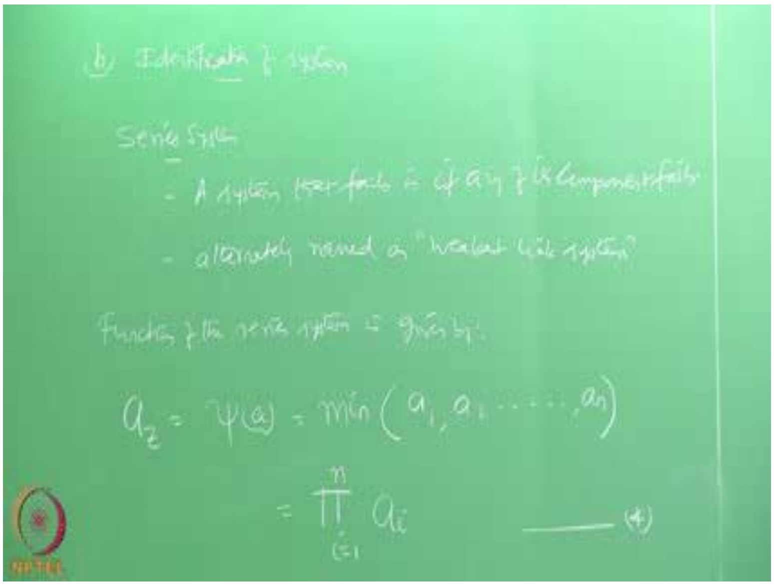

A system that fails need to be identified let say series system series system is that system that fails if any of it is components fails such systems are often called as the weakest link system alternatively name as weakest link system that is the system with weak links. We can give an example let us say chain is considered to be strong as strong as the weakest link. Now the function of the series system, now can be given by let see a z psi of a which is minimum of a 1, a 2, a n which can be expressed as i equals 1 to n a i call equation number 4. Clearly, if any one component in this system fails which is the series system then the product terms in this equation will become 0 which amounts that if any 1 component fails in the system; obviously, that is amounting to the system fail first systems are called series system which otherwise called as weakest link systems.

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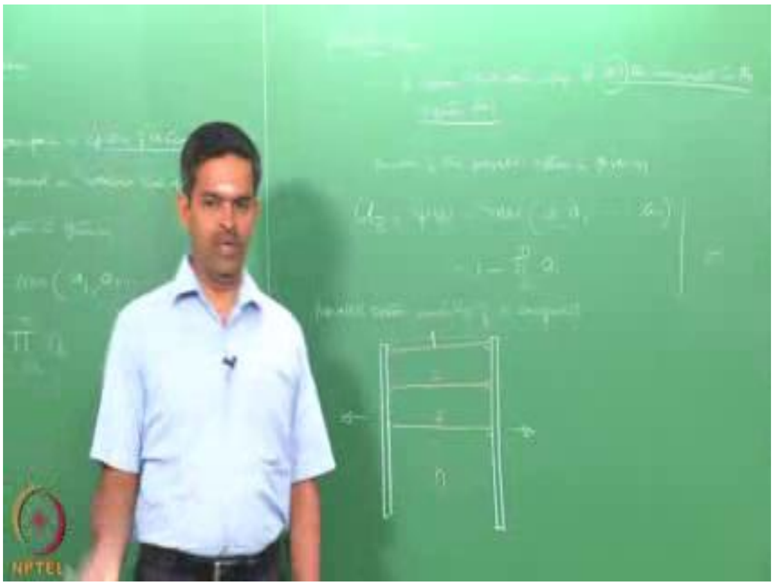

Alternatively, one can go for a parallel system. Parallel system is that system that fails only if all the components in the system fail. It is easy to now understand the difference between the parallel and series system. In series system, if any one of it is component fail system fails. In parallel system, if all the components fail system fails. Function of the parallel system is given by z function of a which is maximum a 1 a 2 a n which is equal to 1 minus i call this as equation number 5.

Let us schematically see how a parallel system looks like we will draw a parallel system consisting of n components let say I call this as link 1. Similarly, I can have link 2. Similarly, I can have 3, and it can continue till n. It continues further also is a typical graphical representation of a parallel system with n components. One can see the equation 5, which is expressing the performance function for the parallel system of n components, it is seen that equation 5 will be set to 0 will become 0 only when a i is set to 0; for i is equal to 1 to 2, 3, 4, n that is when all the components failed then only the system failure will be shown. If even one component functions then the product in the term of equation 5 will be equal to 0, and the system function will becomes unity it is not fail. So, is a binary value 0 or 1; 0 means fails, 1 is functional.

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Now, one can also look at alternatively k out of n systems is another way of looking at what is it mean. When k of the n components given in a system or of a system functions. If k of the n component of the system functions then it is called as k out of n system. Function for the system or the expression for the performance function of the system is given by a equals 1, if k, otherwise zero.

So, let us talk about generic form of the systems general systems are essentially combination of the above. Therefore, a general system is a combination of the above a general system is set to a fail if a force or a deformation can be transmitted from one end of the system to the other. Now, to analyse the general system, it is useful to introduce a concepts of cut sets and path sets.

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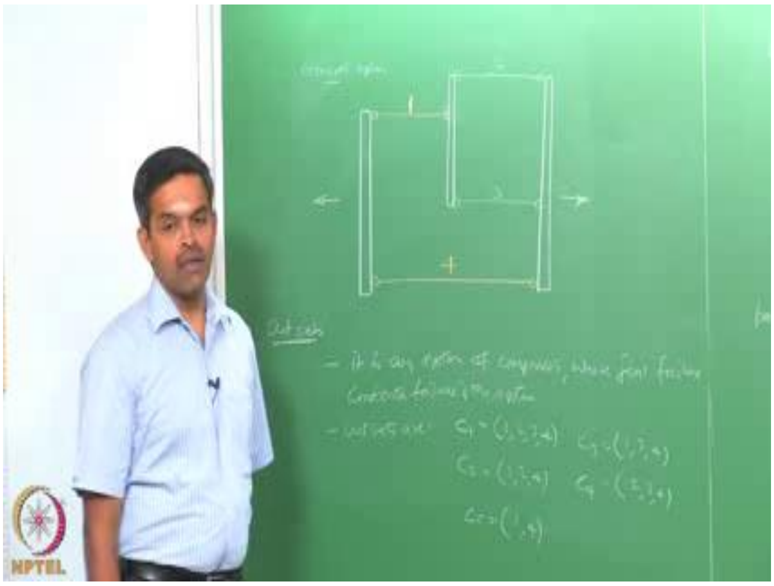

Let us typically draw a general system, which is a combination of series and parallel system. So, I have links here and call this as 1, I have links here I call this as 4. I have links here I call this as 2, 3, 4. So, the system can expand either ways. So, it is a combination of series and parallel systems. With reference to this, let us try to understand what is a cut set, because to understand the failure of a general system. we need to analyse a system in such a manner that if the force or the deformation is able to pass through the system from one end to another end, the general system is appear to be or set to be failed. To understand this analysis in detail we have got subdivide the general system for our understand say into cut sets and path sets.

So, what is a cut set. A cut set is a system of components is any system of components, whose joint failure constitutes a failure of the system. Let us take this example and try to mark out the cut sets for the system. The cuts sets therefore are let say C 1 - cut set 1 can be 1, 2, 3 and 4. Cut set C 2 is let say 1, 2 and 4. Cut set C 3 can be 1, 3 and 4. Cut set C 4 can be 2, 3 and 4. Cut set C 5 can be simply 1 and 4. These are nothing but the paths which enables the transformation of the force or the deformation from one end to another end of the system. Now, we will talk about the failure we already know for a component failure for a system failure, which is series if any one component fails system is fail. For a system in parallel when all components fail then the system fails one is a minimising function other is a maximising function. So, I should be able to identify now a minimum cut set from this set of C 1 to C 5.

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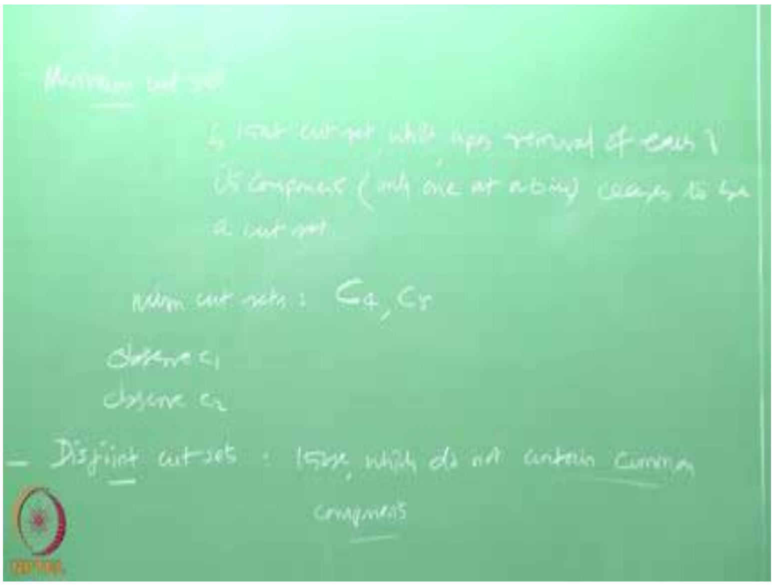

So, now, we are talking about a minimum cut set is that cut set which is upon removal upon removal of each of its components. Please understand only one at time can be removed ceases to be a cut set, therefore referring this to the figure on the board here the minimum cut sets could be C 4 and C. These are the minimum cut sets. So, cut sets is their one minimum cut sets is that which upon removal of any 1 it is components of each 1 of the component 1 at a time we will C to become a cut set, so C 4 and C 5. For example, let us look at C 4, if you remove C 4, it will not be able to connect the members is it not. You can see here C 1 as got 1, 4 connecting these two ends C 2 as got 1 4 connecting these two ends in addition to two that is different. C 3 has got 1, 3 and 4 connecting these two ends. C 4 is got 2, 3 and 4 at least one member connecting two ends. C 5 of course, as got 1 and 4 connecting two ends. So, of any one components is removed it will not be a cut set for example, if I remove 4 it will not be a cut set, if I remove 4 it not be a cut set C's are to be a cut set that is what we call as minimum cut set. So, C 4 and C 5 from this list are the minimum cuts sets.

Let us look at C 1 for argument. So, let us say observe C 1 C 1 is not a minimum cut set because if you remove the component 1 still we can see here 2, 3 and 4 will connect this ends. So, 4 is present of course, if you remove 4 then it is a problem because of course, it can be still because 1 will connect these 2 and 2 and 2 further we are connect these two. So, there is connectivity between this end and this end. So, C 1 cannot be your minimum cuts set similarly let us observe C 2. So, observe C 2 on removal of component 2 from C

2 or 2 from C 2 do not cause either of the sets to actually Cs to be a cut set is it not. So, let us then define what is disjoint cut sets disjoint cut sets are those cut sets which do not contain common components let us refer these two this example again these are the cut sets we have C 1 to C 5.

For this example there are no disjoint cut sets because each cut set contains component 4 is it not all of them contain component 4 disjoint sets are those which do not contain a common component where as C 1, C 2, C 3, C 4, C 5 all of them have 4 is it not. So, I do not have a disjoint set in this example that is about the cut sets which is require to analyse or understand the analysis of system reliability for a general system.

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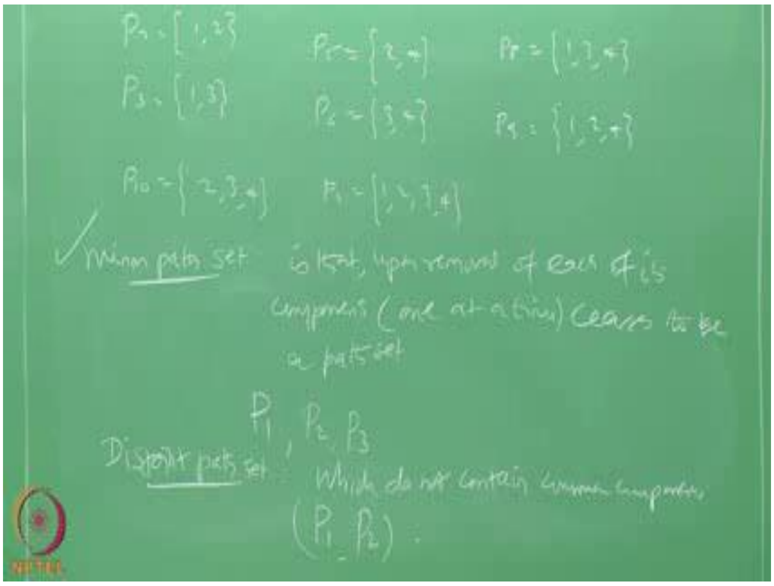

The next is going to be the path set path set is an alternative to the cut set path set is one which is any set of components whose joint survival constitutes survival of the system. Let us take the same example again here let us write down the path sets P 1 let say is going to be 4 that is this connection there is 1 path. P 2 can be the other path which is 1 comma 2 which going to connect 1, 2. P 3 can be another path which connects 1 and 2. P 4 can be another path which connects 1 and 4 which constitutes 1 and 4. P 5 can be another path, which constitutes 2 and 4. P 6 can constitute 2 and 4. P 7 can constitute 1 2 and 2. P 8 can constitute 1 2 and 4. P 9 can constitute 1 2 and 4. P 10 can constitute 2, 3 and 4. P 11 can constitute 1 2, 3, 4.

So, what is a path set path set is any set of components whose joint constitutes survival of the system. Let us take for example, 4 P 1; P 1 is having link 4 which constitute survival of the whole system. Look at P 2, 1 and 2; 1 and 2 connected. Look at P 3, 1 2 and 3 connected may be redundancy, but disconnected. P 4 1 and 2, 1 and 4 let say P 4 is 1 and 4. P 5 is 2 and 4. So, you can easily see this. Now again let us talk about a minimum path set why we are bother about minimum path set because you know the system comprises both series and parallel series one which any one component fails system fails, parallel is one between all component fails system fails. So, one should identify the minimum series path minimum path set for cuts sets and minimum path set for general system.

So, let us say a minimum path set minimum path set is the path set that upon removal is that upon removal of each of it is components of course, one at a time ceases to be a path set. Let us look at this list of path sets available in the screen. Now or in the black board the minimum path set in this case are going to be if you remove 4 system cannot exit. So, P 1 if you remove either 1 of them in P 2 let say I remove 1 or 2 it cannot connect. So, P 2 if we look at P 2 if you remove 1 it cannot connect. So, P 3. So, minimum path sets are going to be P 1, P 2 and P 3. Let us look at P 4 if you remove 1 it does not matter because 4 is connecting look at P 5 if you remove 2 from P 5 still 4 is exiting so, connected.

If you look at P 6 3 and 4 if you remove 2 4 is still connected. If you look at P 7 1 2 and 3, so if I remove 3, 1 and 2 are connected. If I remove 2, 1 and 3 are connected, so no problem. Look at P 8 - 1, 3 and 4 if I remove 3 1 and 4 are there connected. P 9 1, 2 and 4. So, remove 1 still 4 is can connected; remove 4 still 1 and 2 are connected and so on. P 10 2, 3 and 4, so if you only if you remove either 2 or 3 4 are connected. P 11 of course, 1, 2, 3, 4 is a maximum number of component, so no issue on.

So, one can easily diagnose the set of path sets what you have prepared for a given system and try to find out what would be the minimum path sets and what would be the minimum cut sets let us looked at the disjoint path disjoint path sets are those which do not contain common components. So, for this set one can write out are P 1 and P 2. So, friends, we are continuing the discussion on system reliability, we are identifying the series and parallel system; we are also identifying the combination of series and parallel system. To understand the analysis of series and parallel system in particular or general system to be very specific, one should be able to understand the analysis through some definitions like cut sets and path sets for a given general system which is a combination of series and parallel system as you see in the blackboard here.

We are extend a discussion to really understand or list out the cut sets and path sets from a general system from the identified cut sets and path sets we identified also the disjoint sets and the minimum sets for both cuts sets and path sets as we now discussed. We will continue the discussing in the next lecture to really understand how this system functions for general system can be extended more in detail to really find out the failure or probability failure of the general system based on the discussions what we have so far.

Thank you very much.