Risk and Reliability of Offshore Structures
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Module – 02
Reliability theory and Structural Reliability
Lecture – 09
System Reliability – I

Welcome friends to the 9th Lecture on the online course on Risk and Reliability of Offshore Structures.

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We are continuing with the lectures on Module 2, where we are focusing on Structural Reliability. In this lecture which is the 9th Lecture we will talk about System Reliability. Now there are two issues here; one can do a component level reliability analysis, one can do system level reliability analysis let us look into detail about this more in this lecture.

Now, reliability estimates can also be done using higher order response surface methods. So first we will start with reliability estimates using higher order response surface methods. Now structural reliability analysis is perform actually to obtain the safety of the system, in terms of its design under various load combinations material characteristics and geometric form of the given structural system for a large and complex systems like, offshore structures approximate techniques are also available to reduce the computational efforts to an acceptable level of degree.

Response surface methods are one amongst them which is very recent development. So, response surface method is an approximate method which should include various complexities arising due to material characteristics due to various load combinations and complications arising from geometric form of the structure which are all present very much in offshore structures as said by Tred 1990.

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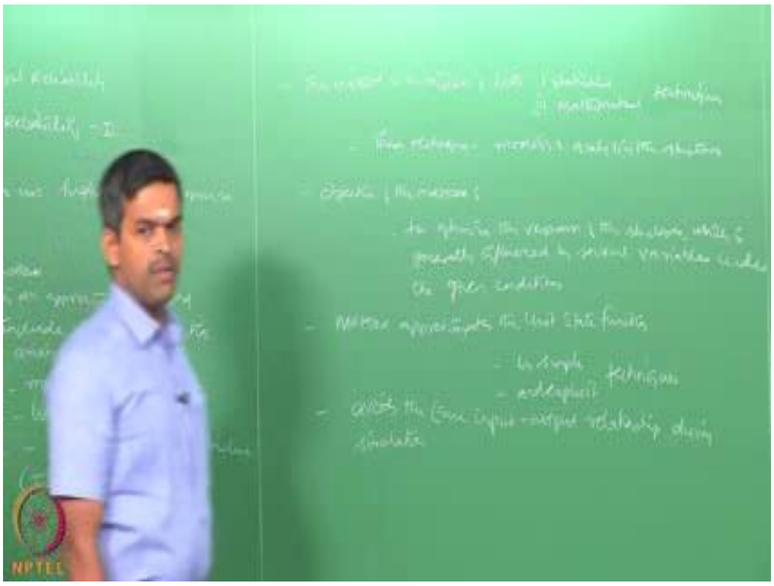

Of course, this method is a mixture of both statistical and mathematical technique. It is useful in modelling and analysing the structure, so these techniques are useful in modelling and analysing the structure. And we all agree and know that the response of the structure is influence by several variables. Now what should be actually the objective of this method? The objective is to optimise these response under given conditions, the response of the structure which is generally influence by several variables under the given conditions.

Actually, the method approximates the limit state function by simple and explicit techniques. Therefore, it actually reduces or avoids the true input output relationship during simulation. So one is advantages, this method avoids the true input output relations during simulation.

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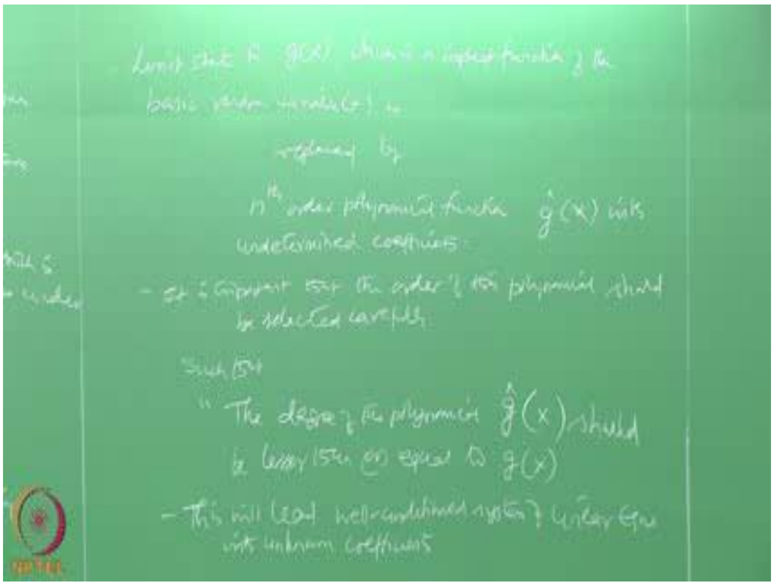

Now the limit state function g of x which is an implicit function of the basic random variable x is replaced by nth order polynomial function, which is g hat of x with undetermined coefficients. Now interestingly selection of this order of the polynomial should be done carefully. So, it is important that the order of this polynomial should be selected carefully, such that what is the condition. The degree of the polynomial or of the polynomial function g hat of x should be lesser than or equal to g of x, because if you do this the advantages will lead to well condition system of linear equations with of course unknown coefficients.

Friends in reliability analysis it is quite common that neither the limit state function nor the design points are known, therefore this will general lead to confusing state of choosing the degree of polynomial because we neither know the limit state function nor the design point in general in reliability analysis. When these both things are not known rather you cannot select or you cannot fix the degree of polynomial, typically a quadratic with or without cross terms is generally used for approximation.

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Generally a quadratic with or without cross terms is used for approximation, so let us write a quadratic like this for example, g hat of x is a 0 plus sum of i equals 1 to m a i x i plus sum of i equals 1 to m double i x i square plus sum of i equals 1 to m sum of j equals 1 to m the cross terms a i j x i x. Of course, you can have a polynomial without the cross terms also or with the cross terms also for i less than j, equation number 1.

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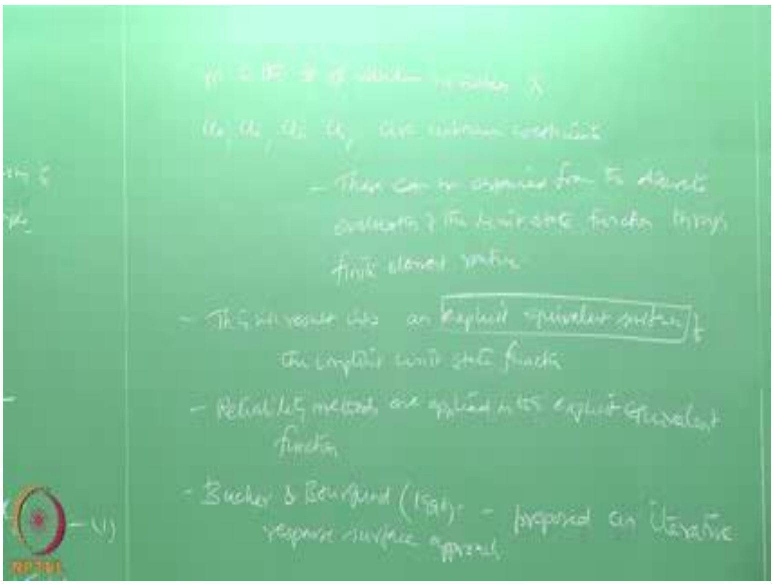

Now in this case m is a random variable of random variable x let us say a 0, a 1, a 1 1, a i j, a 0, a i, a i i, a i j are unknown coefficients which are obtained from the discrete evaluation of the limit state function. So, these can be obtained from the discrete evaluation of the limit state function though finite element routine.

Now, the resulting fitted surface will be an explicit equivalence of implicit limit state function, so this will result into an explicit equivalent surface of the implicit limit state function, because the original function is an implicit function. Now I will apply the reliability methods on the explicit function. So, reliability methods are applied on this explicit I should say equivalent function, one can also say limit state surface itself. Now more interestingly the importantly selection of design points where this function we evaluated is very crucial. This is advised by Busher and Bourgund in the year 1990, proposed an iterative response surface approach. What this actually do is, the successive iteration shifts the design point towards the proposed limit state function.

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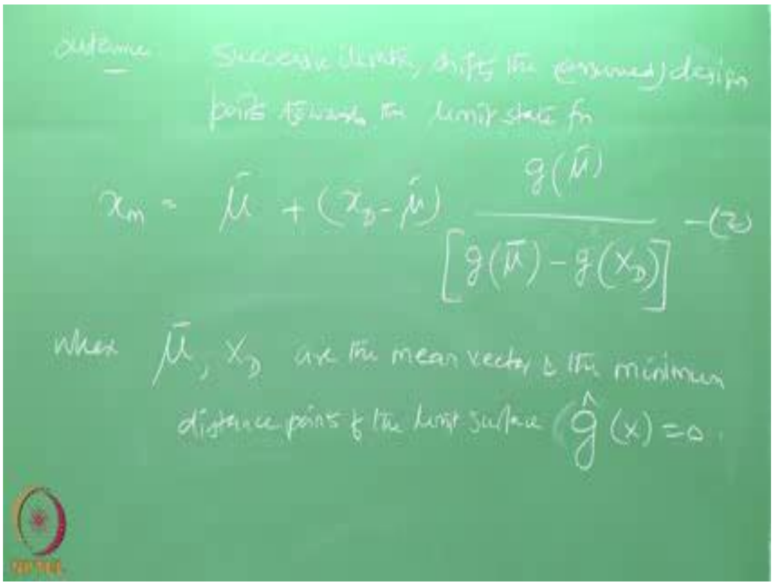

Outcome of the scheme is successive iteration shifts I should say assume design points towards the limit state function, because we all know and agree that the design point should lie on the limit state function and reliability index is nothing but the shortest distance of the limit state function or the surface performance failure from the origin. So, let us see how we can find out this.

That is important let say, the mth iteration is given by mu plus x d minus mu bar of g evaluated at mu bar by g mu bar minus g evaluated x d. Call equation number 2, where mu bar x d are the mean vector and minimum normal points for the limit surface of the limit surface which is explicit g hat x 0. It is not the original function; we have transpose the function to explicit function as you can see from this equation 1 using a polynomial or using a quadratic form where the cross terms may or may not be present as proposed by the researchers.

There is one more method by which we can still do this that is called higher order stochastic response surface method.

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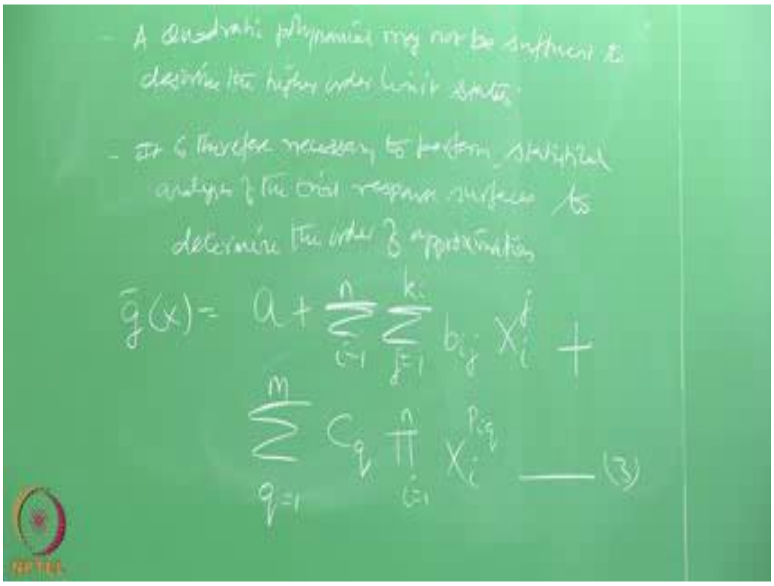

In this case your quadratic polynomial may not be enough to describe the higher order mistakes. All the time you know your quadratic polynomial may not be sufficient to describe the higher order mistakes, because the limit state function becomes now linear due to many reasons as we saw very recently. In such cases it is necessary to perform statistical analysis of the trial response surfaces.

Therefore, necessary to perform what we call statistical responses analysis of the so called trail response surfaces to determine the approximate order of approximation. We call the second order which is a quadratic may not be sufficient. Now in that case g bar of x will be given by a plus i equals 1 to n j equals 1 to j i b i j x i j plus summation of q equals 1 to m c q the pi function i equals 1 to n x i to the power p i q, call as equation number 3.

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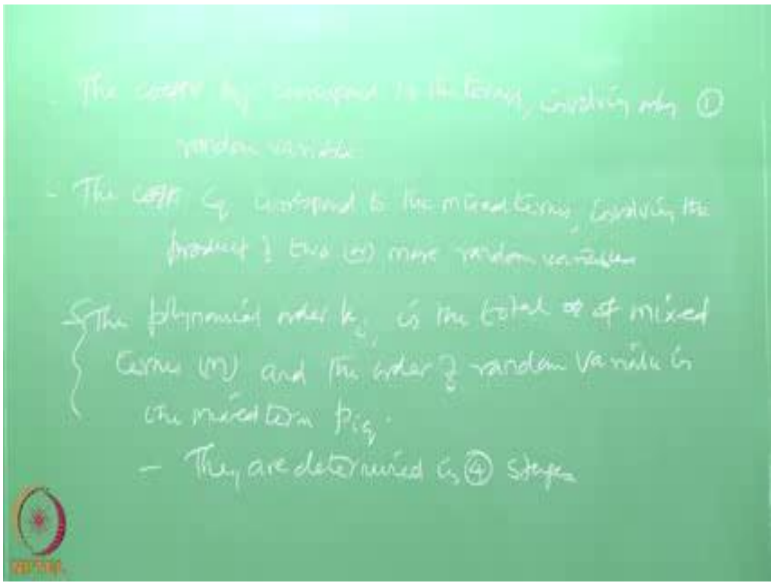

Now, in this case the coefficients b i j actually correspond to the terms involving only one random variable. The coefficients c q that is here corresponds to the mixed terms involving the product of two or more random variables therefore higher order comes into play here. The polynomial order k i here is the total number of mixed terms m and the order of random variable in the mixed term p i q. Now they are determined in 4 stages, let us see.

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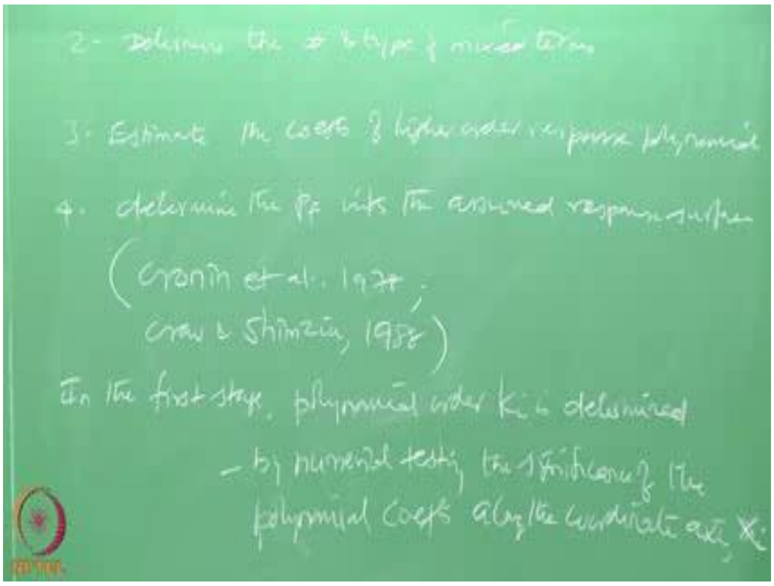

Step 1; identify the order of response surface. Step number 2, determine the number and type of mixed terms. Step number 3; estimate the coefficients of higher order of response surface polynomial. And forth we determine the probability of failure with the assumed response surface. The algorithm is supported by Cronin et-al 1978, Craw and Shimizu 1988. Obviously, Mote Carlo simulation is carried out on the response surfaces to determine the probability of failure.

So in the first stage, polynomial order k i is determined. How? By numerical testing the significance of polynomial coefficients along the coordinate axis; by numerically testing the significance of the polynomial coefficients, that is whether the b i j values are getting decreased or increase or becoming 0 10 into become 0 etcetera, so numerically testing the significance of polynomial coefficients along the coordinate axis. So, one can interestingly also use Chebyshev polynomial.

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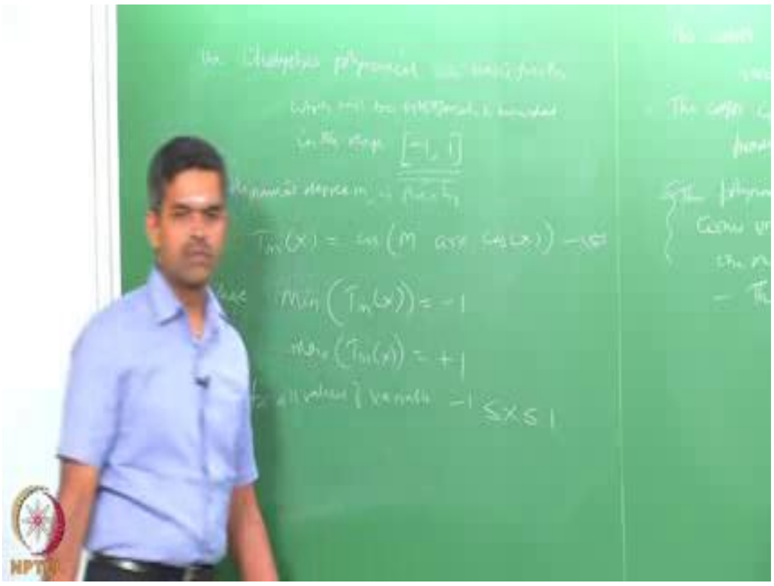

One can use Chebyshev polynomial as a basis function, which will be orthogonally and bounded in the range minus 1 figure. Now, a polynomial degree is given by a simple expression T m of x is cos m are cos x, where equation number 5. Where, minimum of T m of x is minus 1 that is the range we have and maximum of T m of x is plus 1 that is the range we have for all values of variables in the range minus 1 less than x less than 1.

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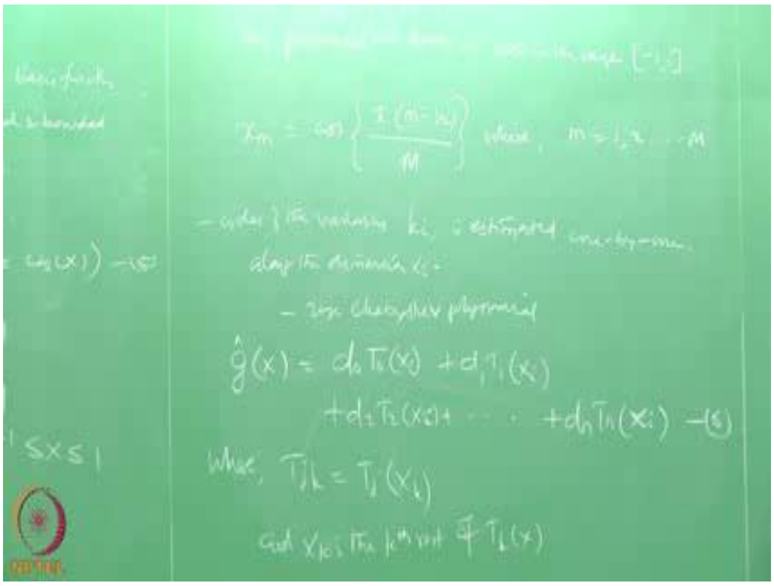

This polynomial will have m roots in the range minus 1 to 1, and that is given by the following equation, let say we will use capital M here where small m varies from 1, 2, 3. Order of the variables k i is estimated one by one along the dimension x i to do this one can use the Chebyshev polynomial which can be given by g hat of x is d 0 T 0 x or let say x i d 1 T 1 of x 1 plus d 2 T 2 of x i going till d n T n x i; equation number 6. Where the value T j k is T j of x k and x k is the kth root of T k of x.

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X i is the interpolated values of x i from the interval mu minus h r sigma i to mu plus h r sigma i that is the interval. Where, h r is a domain of sampling points used to determine the polynomial degree of approximation. Now the Chebyshev coefficients which are d j are determined by least square approximation which can be given by D is equal to T transpose T inverse T transpose of g i of x i; equation 7.

Since, all Chebyshev polynomial coefficients are bounded within the limits minus 1 to plus 1, contribution of t j to the limit state function will be governed to the order of d j only. Having understood different methods of linear first order, second order, for normal, non-normal variate, using simulation methods, using higher order response methods, using Chebyshev polynomial, where we are assuming or predicting the order of the polynomial which is going to represent the limit state function or the performance function based on which failure probability can be obtained. One can now apply these algorithms very clearly to the system reliability.

So far we have been talking about the component reliability where each component or a member of a given system is analysed for a safety of failure in a given domain. So, when this components gets assemble to make a system then one is not interested in component reliability alone, but also in a total interested in system reliability.

So, one should talk about how the component reliability will influence the reliability of the whole system, what are the different methods available, parallel system, series system, etcetera. How we can estimate the probability of a failure of a given system knowing the significance of the probability of failure the components present in the system, which we will discuss in the next lecture.

Thank you very much.