Risk and Reliability of Offshore Structures

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Module – 02
Reliability theory and Structural Reliability
Lecture – 08
Reliability methods IV

Welcome friends to the 8 lecture on Risk and Reliability of offshore structures. This is the eighth lecture in module 2.

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Here we are focusing on structural reliability. This is lecture 8, where we will continue with reliability methods. So, this is the forth lecture in reliability methods. We already said that in first order second moment method, one can easily compute the reliability index, provided the variables are normally distributed. We illustrated the equations and derivations for 2 variables r and s. If the equations are log normally distributed, we again gave the expression of finding what the probability of failure in reliability index.

Again when there are two variables which are log normally distributed, if they are general system of variables which is of vector x, then also we can find out the reliability index using the first order second moment method and improvement is done using Hasofer Lind Reliability method of finding out reliability index again. In this case, we assume first derivation saying that the variables are linear or let us say normally distributed. Now, if the variables are not normally distributed then Rocknites and Ficseler suggested an improved method, what exactly they did is they transformed the non-normally variables into equivalent normal variables. So, the non-normally variables are transformed into equivalent normal variables. They estimated the parameters of equivalent normal distribution which we call as.

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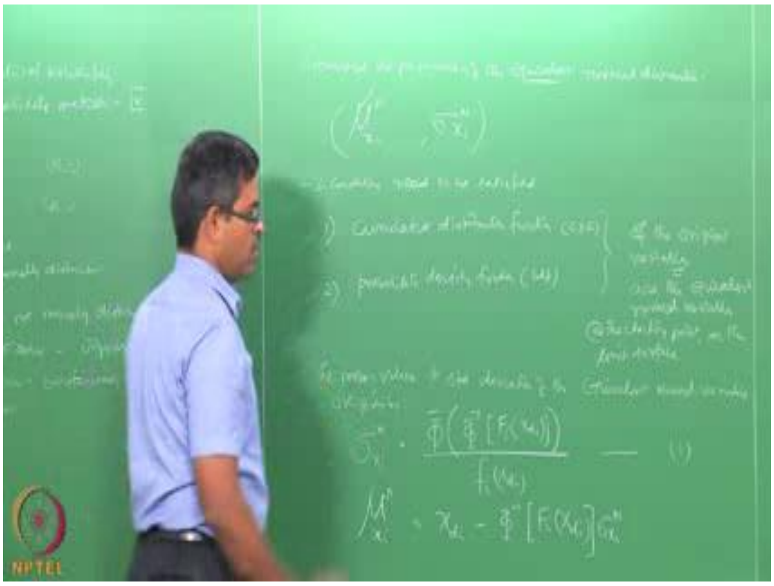

So, then they estimated the parameters of the so called equivalent normal distribution. What are those parameters required mu x i I put normal and of course, sigma x i again I put normal because they are equally, I mean equivalent normal distribution to do this. They imposed two conditions, what are these two conditions? One; the cumulative distribution function and two; the probability density function that is c f d and p d f of the original variable which are known to us and the equivalent normal variables should be equal at the checking point of course, on the limit surface. So, this is the condition what they have imposed.

With the help of these two conditions, feeling satisfied between the original variables and the equivalent normal variables then the original state of variables will be transformed into equivalent normal variables, of course, the original variables are non-normal in that case the mean value at standard distribution which we wanted here and standard deviation of the equivalent normal variables will be given by let us say, sigma x i normal yes phi bar phi inverse of f i of x d i, where x d i at the design points or the check points divided by f of i x d equation number 1.

And mu x i normal which is the mean value which is nothing, but x d i minus phi inverse of f of i at x d i of the top normal, which you already know from this equation 2, where f i is the non-normal cumulative distribution function.

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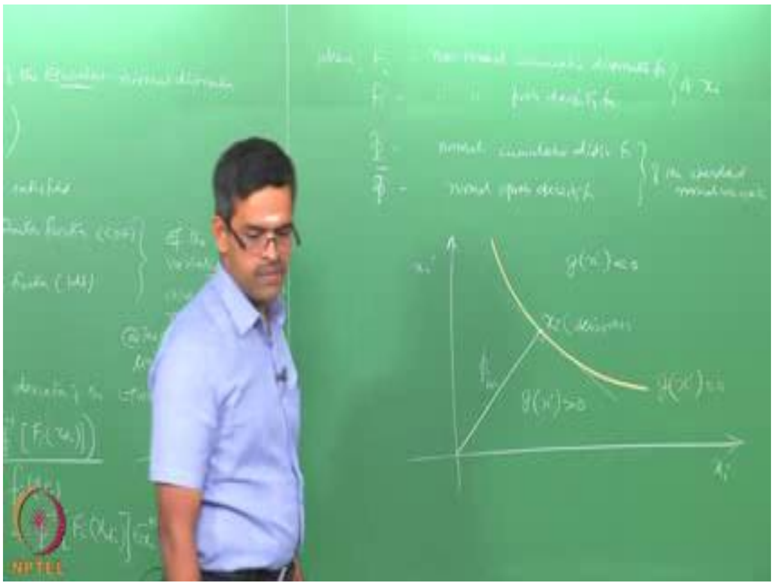

And f i is the non-normal probability density function of x i of course, phi is the normal cumulative distribution function and phi bar is the normal probability density function of the standard normal variate. Let us try to graphically look at this plot and see, how does it vary? Kindly pay attention to the figure shown on the screen now.

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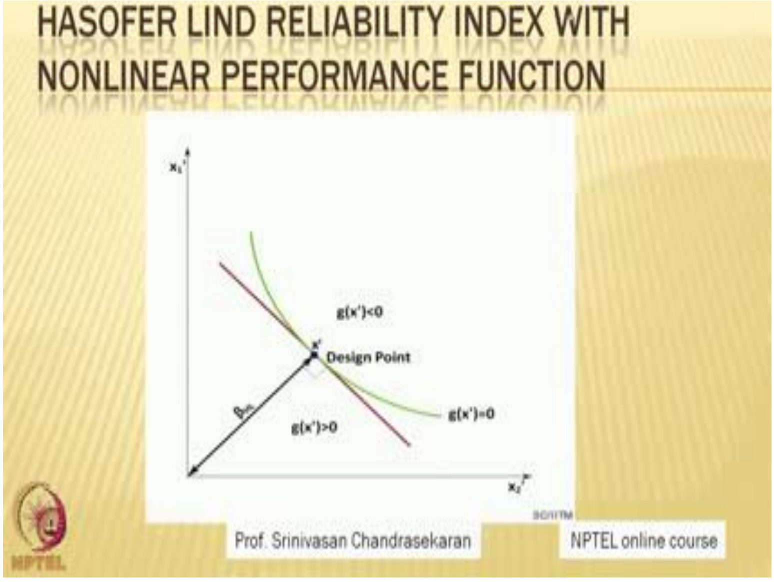

The screen shows the graphical representation of 2 variables; x 1 bar and x 2 bar, which is going to govern the limit state function and the performance function as suggested by Hasofer Lind reliability index of course, you can see here the performance function is non-linear which is shown in the green colour. So, g x prime is equal to 0 that is my limit state function or performance function.

Actually, I am interested in locating the design point that is called the checking point, may be the design point is drawn at this level as a tangent and this is my x i, which I call as a design point or the checking point and we all know the reliability index will be the shortest distance from the design point between the origin and the design point. Therefore, this is going to be my reliability index which is given by Hasofer Lind. Therefore, beta h l and; obviously, can see here this area will indicate the performance function greater than 0, which is going to the safe domain and this area will indicate the performance function is less than 0, which is failure domain where as the variables are x 1 dash and x 2 dash in this case.

Now, with this modification of the random variable which is shifted from non normal to equivalent normal variables one can use the following algorithm to compute a reliability index beta l.

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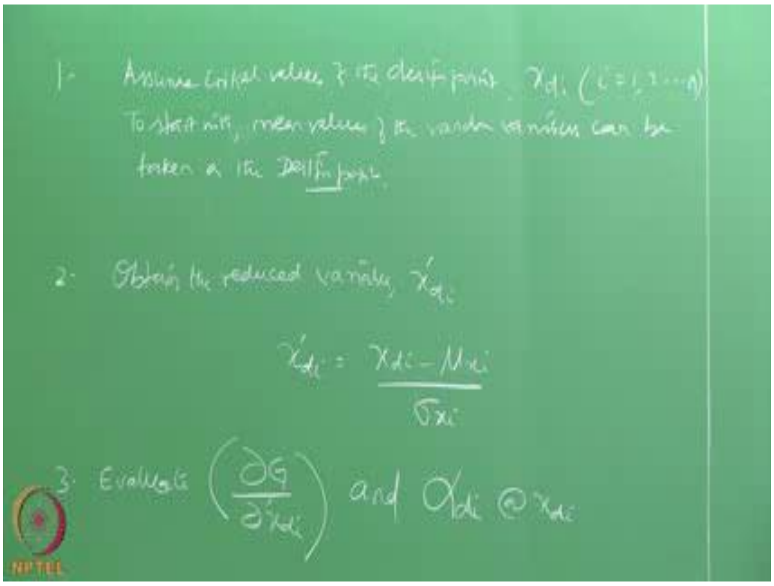

So, now algorithm to compute beta h l, let us first assume the initial values of design points because design points are not here. So, assume initial values of the design point which will be x d i, where i is 1 comma 2, etcetera till n to start with the mean values of the random variables may be taken at the design point, step number 2; obtain the reduced variable x d i dash, we are now transforming the variable which will be given by x d i by sigma x. Let us say equation, we do not give the number here, the third step evaluate the derivative and the weighted functions at x d i. Therefore, now you will get a new design point x d i in terms of beta h l.

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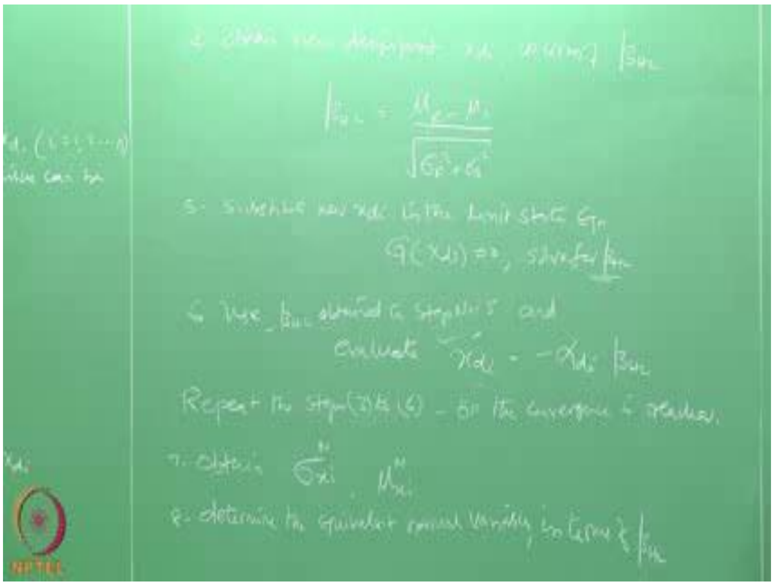

So, we all know in general beta h l is simply given by mu of one variable minus mu of another variable divided by sum of square of this square root of sum of squares of this one, can find out beta h l. Therefore, you get a new design point x d i. Now, after getting this substitute mu x d i, substitute the new point in the limit state equation g x d i is 0 and solve for beta h l again.

Now, use beta h l obtained in step number 5 that is this beta h l use beta h l obtained step number 5 and evaluate x d i again in terms of x d i dash, which can be given by minus alpha d i beta h l, this is done you are already got 1 x d i dash. Now, you refine x d i dash again. So, keep on iterating this, repeat the steps 3 to 6, till the convergence reached. Once you do this then obtain sigma x i for the normal variable and mu x i for the normal variable, where we have already given you the equations for computing this in the beginning of this lecture.

After doing this, find the equivalent normal variable in terms of beta h l. While doing so, you should replace in doing this replace the mean and sigma with what you have computed here which will be mean and sigma.

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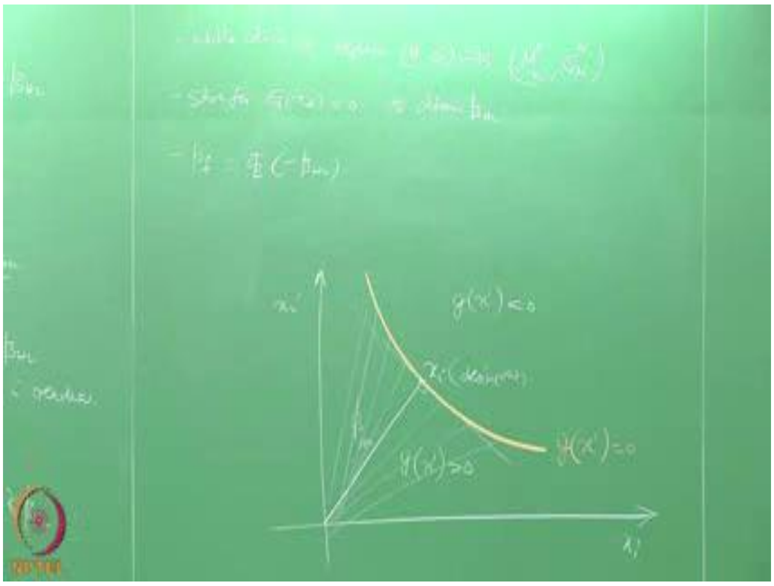

What you have computed from this step now, again solve for g x d set to 0 to obtain beta. Then one can find the probability of failure because now, this beta h l will be in terms of normal variables. So, you converted them into equivalent normal variables. So, the beta actually what you got here before convergence is all meant for non-normal variables. We are converting them equivalent normal variable therefore, probability of failure simply is nothing, but the phi function of minus beta h l.

Now, can be computed this equation is valid only when it is normal variable variate, if it is non-normal variate you cannot use this. So, we are converted this into equivalent normal variates therefore, I am using this graphical. If you look at this figure again it is nothing, but finding out various x is or along the performance function finding out various beta h l along the performance function with respect to that of the origin and trying to minimise or find out, what is the shortest possible distance that is, what it is an optimisation problem or minimising this distance to the maximum possible solution. So, that is what you are doing it in this case. So, that is how you get the reliability index for non-normal variate functions by converting them into equivalent normal variables.

Let us move further to do second order reliability methods. So, far we have been looking at the first order, why first order? We are looking only at the mean and standard deviations and variance. So, let us move ahead and do so.

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Now, in this case we all know that the limit state function could be non-linear because of many reasons, what are these relationships? It may be due to non-linear relationship between the random variables and limit state function. Secondly, you are transforming a non-normal variable to standard normal variable that can be a reason transformation of non-normal variables into equivalent normal variables this can also add non-linearity to the limit state function.

Thirdly, the transformation from correlated uncorrelated variables, transformation from correlated to uncorrelated variables. So, these are the reasons which can lead to non-linear limit state function. In such cases, what we actually do the joint probably density function does not decay rapidly that is very important the joint probability density function does not decay as it moves, I mean does not decay rapidly as it moves follow or as it moves away from the so called design point. So, that is the problem what we have when the limit state becomes non-linear as we can see from this figure when the limit state function becomes non-linear as a design point as the limit state function moves to away from the design point, for example, an ideal design point. Let us say, but your design point is somewhere here which is moving away you will see that the joint pdf will not decay faster.

Therefore, one has to do higher order approximations to compute failure probabilities. So, one requires to use higher order approximation for failure probability. So, in this method the curvature of the limit state function around the minimum distance point is approximated.

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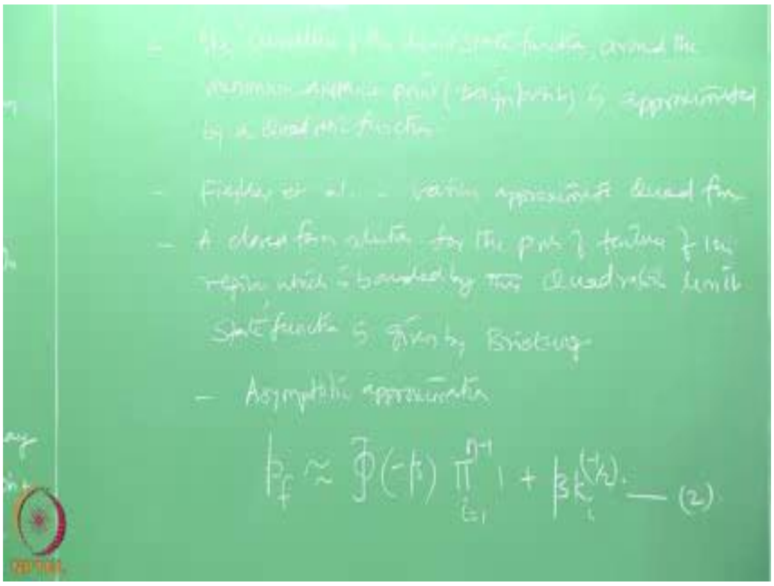

So, what is the procedure followed in such methods. The curvature of the limit state function around the minimum distance point, what is the minimum distance point nothing, but the design point is approximated by a quadratic function Fiesler et al has given some solution for this has given various approximate quadratic functions. Therefore, with the help of these approximate quadratic functions your close form solution for the probability of the failure of the region which is bounded by this quadratic function, I should say quadratic limit state function because talking about failure limit state function is given by brietning he as used an asymptotic approximation he says that probability of failure is approximately given by minus beta minus half equation number. Let us say, 2 where k i denotes the ith mean curvature of the limit state function at the design point.

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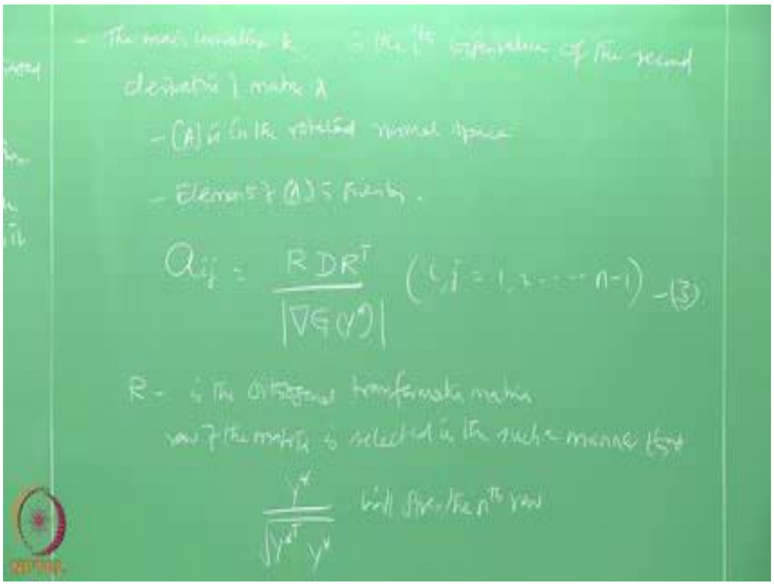

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Then the main curvature k i is the ith gain value of the second derivative of matrix a of the limit state surface of the design point matrix a is in the rotated normal space elements of this matrix is given by, let us say i j any elements is given by r d r transpose by where i comma j is 1 comma 2, till n minus 1. So, that is the order what behave here that is the order, what we have here equation, number 3. In this case, r is actually the orthogonal transformation matrix. The row of this matrix is actually selected in such a manner that y star by y star transpose y star to the power half will give the nth row. Now, one can use Gram Schmidt algorithm used to determine order.

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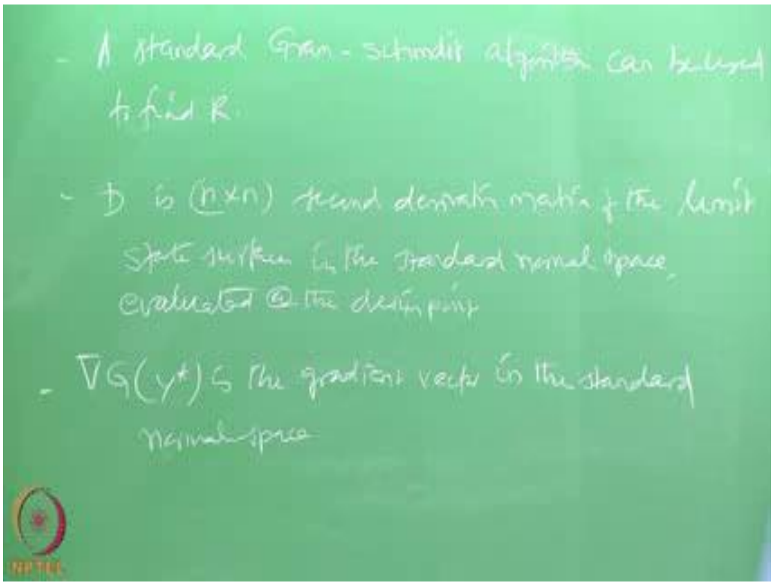

D is actually the n cross n second derivative matrix of the limit state surface. In the standard normal space evaluated at the design point del g y bar or y star is a gradient vector in the standard normal space. Having said this, one can also use simulation based reliability methods.

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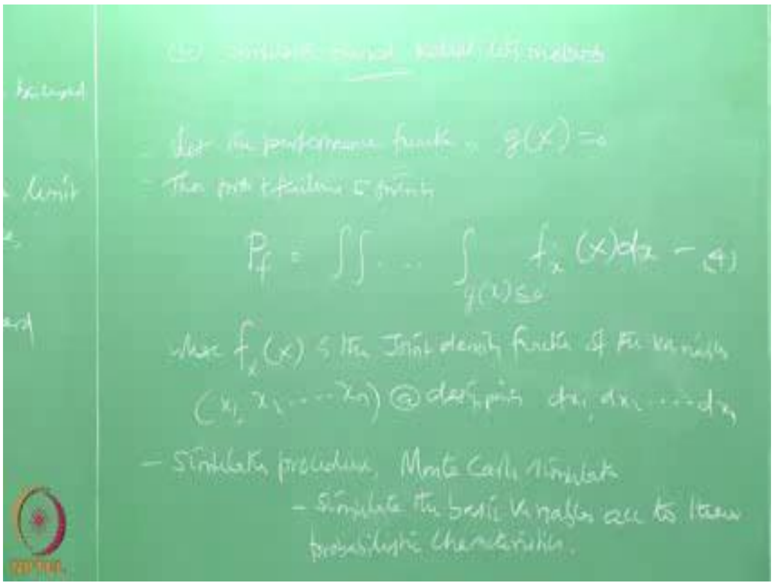

Now, let us say the performance function is given by g of x set to 0, then probability of failure is given by in general integral of function which going to be g of x less than 0 that is a failure function f of x, f of x d x called equation number 4, where f of x, x is the joint density function of the variables x 1, x 2 till x n and evaluated at design points d x 1, d x 2, d x 3 one can use a simulation procedure as suggested by Monte Carlo. For the basic variables, one can simulate the basic variables according to their probabilistic characters. Now, the number of failures will be given by which is N f.

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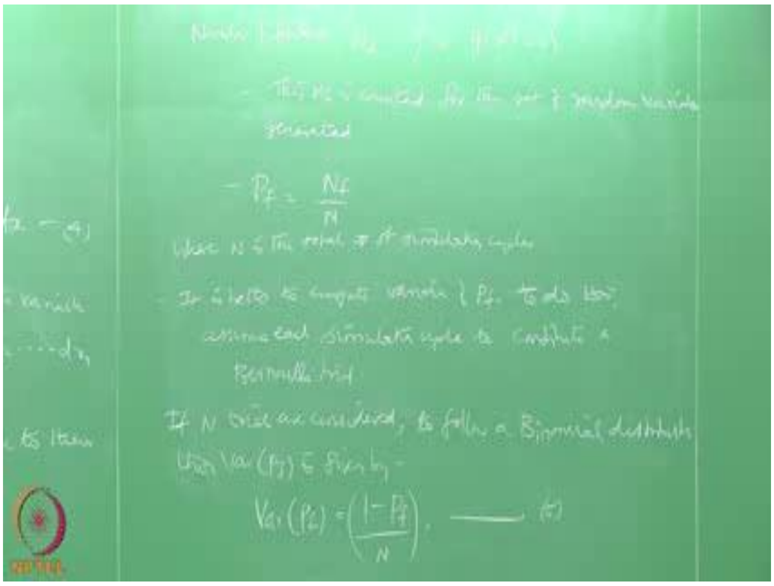

What do you mean by N f, N f means g of x is less than or equal to 0, not equal to less than 0 is then counted. Actually, this N of f is counted for the set of random variable generated then the probability of failure can be simple given by N f by your data n, where n is the total number of simulations cycles. Now, the estimated probability of failure in this case depends of course, on the number of cycles simulated.

So, for sufficiently accurate result, large number of simulations may be required because the denominator should be very high to have more accuracy. Therefore, it is always better to approximately compute the variance of the estimated probability of failure instead of going for directly probability of failure. Now, to compute the variance of probability of failure each simulation cycle is to be constituted way of Bernoulli trail. So, it is better to compute the variance of probability of failure to do. This one should assume each simulation cycle to constitute a Bernoulli trail.

Therefore, if n trails are considered to follow, n in a binomial distribution then the variance of probability of failure is given 1 minus probability of failure by n. Equation number 5, the statistical accuracy of the estimated measure is checked using the coefficient of variation which is given by equation number six.

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Friends' it is important to note that smaller the coefficient of variation better the accuracy. So, for smaller the coefficient of variation, accuracy is better. Accordingly, one can decide the trails for the simulation. So, friends in this class we learnt about extension of Hasofer Lind method for non-normal variables, how they are converted to equivalent normal variables. How Hasofer Lind method can be obtained, index can be obtained by iterative scheme. We have given an algorithm; we have also extended the study for second order reliability methods. We also discussed something on simulation based reliability methods. In the next lecture, we should continue to discuss on reliability estimates using higher order response surface methods, which we further continue in the next lecture.

Thank you.