**Risk and Reliability of Offshore Structures**
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**Module - 02**
**Reliability theory and Structural Reliability**
**Lecture – 07**
**Reliability Methods – III**

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Welcome friends to the seventh lecture on module 2 on the online course on Risk and Reliability of Offshore Structures. So, we are talking about lectures on module two, where we are focusing on reliability theory and structural reliability. We are talking today on lecture 7, which is continuation of the last lecture on reliability methods, so I put this as three.

In the last lecture, we have been discussing about first order second moment method; we also said a case specifically where when the variables R and S are normally distributed. We gave equation in expression of finding out probability of failure, we also continue with the discussion on if R and S are log normally distributed we said that the limit state function Z is now going to be given by l non of R by S let us call this equation number one, where Z is a normal variable. And therefore, the probability of failure can be given by 1 minus phi of which we gave in the last lecture.

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Now, in this case, delta R and delta S are coefficients of variation of R and S respectively. Now, we have been discussing for about two variables R and S, may be one is strengthen one is the load; one can also apply this in a more generic form, one generalize this for n number of random variables which should denoted by x. So, let the random variables instead of two be denoted by a vector x; in that case, performance function is given by Z is equal to G of x.

Now, one can now apply Taylor series expansion of the performance function about the mean values, which is of course known to us Z will now become let say G of x bar where x bar is the mean plus summation of i equals one to N partial derivative with respect to ith variable x i minus x bar i plus half i is equal to 1 to N second derivative dou square G dou x I dou x j 1 more summation, let us change this to j x i minus x bar i x j minus x bar j. There are two summations here; call this equation number 4.

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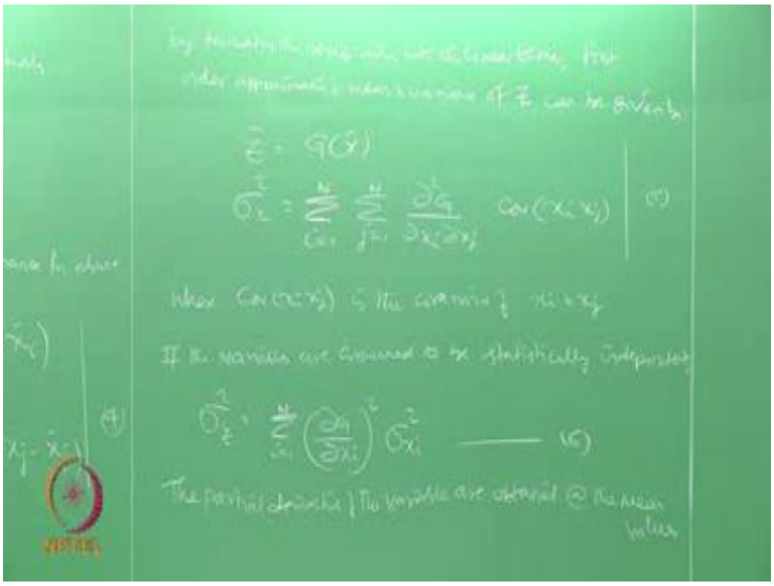

Now, truncating the series only with the linear terms; the first order approximation of mean and variance can be obtained of Z can be given by so Z bar that is the mean which is G of x bar, and variance of the variable equation number 5, where x j is co variance of t x i x j. Now, if the variance is assumed to be statistically independent, in that case, the variance will be given by which we call as equation number.

Now, in this case, the partial derivatives of the variable are obtained at the mean values of the variables. This is one of the comfortable method one of the reliability methods which can used either for a two variable R and S, usually they are found to be normally distributed or log normally distributed for the data or n symbol cases in offshore structures is also generally most of the cases. So, we are given is equation for both cases to find the probability of failure. Suppose if we really wanted to have a set of variables as vector x then one can follow this procedure to find out the expansion or the limit state function and so on.

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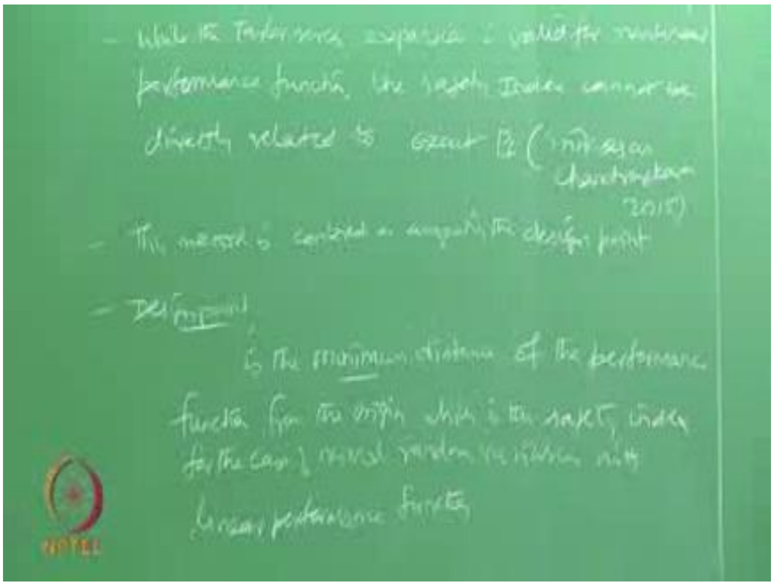

Now, let us look into the next method, which is also very useful in terms of finding the probability of failure. This method is called Hasofer-Lind method. Interestingly, while the Taylor series expansion is valid for non-linear performance function; the safety index cannot be directly related to the exact probability of failure, there is problem here. The estimated safety index now shall be more accurate when all random variables remain statistically independent. Now, given the fact that the normal variables and the performance function are actually a linear combination of random variables Hasofer-Lind has provided an improved method to find the safety index. This is found to be better than that of the safety index obtain from the first order second moment method.

So, now there are some salient points about the Hasofer-Lind method. This is method is centered on computing what is called as the design point. Now the question comes what is the design point mathematically; design point is the minimum distance of the performance function from the origin, which is actually safety index for the case of random variables with linear performance function. Now, let us try to explain the physical difficulty in this whole technique.

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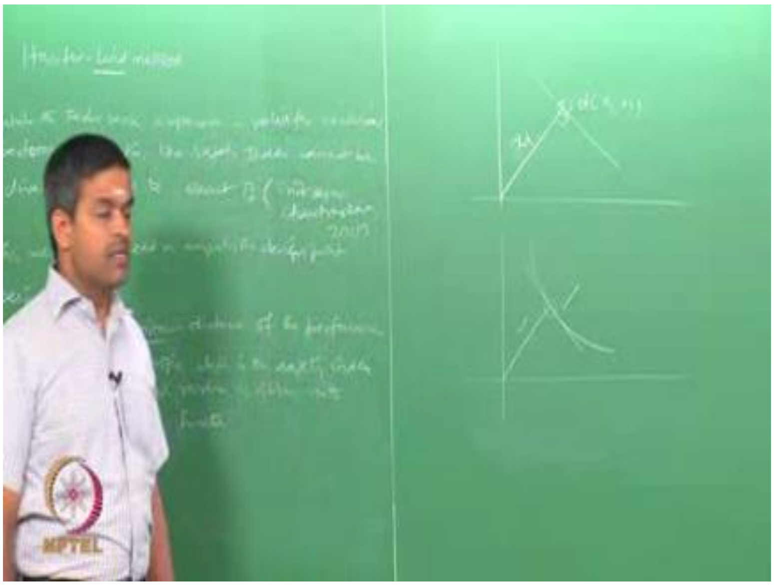

Now, let us take for two variables a performance function, which is linear. For a performance function to remain linear, when the random variables are normalized; it is very easy to locate the design point because design point is the point on the performance function which will give you the minimum distance from the origin to the performance function because that is how design is defined. Design point mathematically or graphically is the minimum distance of the performance function from the origin. Obviously, a line which is normal to the function will be giving you the minimum let say I call this as x d where x stands for the distance and d stands for the origin point, and I call this point as d with some coordinate let say x 1 and x 2 are two variables.

If have a linear function one can easily find out the point at which the distance will be minimum using a simple mathematical relationship using an equation of line passing through the point d, one can always find x d and that will directly give me the safety index. So, there is no confusion if the performance function is linear. Now let us take graphically a function which is non-linear, now I want to find the minimum distance of this function from the origin, one may say let us draw a tangent, and draw a normal to the tangent and this will be the minimum point.

Now, the whole difficulty here is which is this point, through which a tangent will be drawn which will otherwise give me the minimum distance from the origin. So, locating or identifying or determining the design point on the performance function itself is actually a task. So, Hasofer-Lind method is actually centered on computing actually this point if you are able to compute this point locate this point graphically finding the safety index or the reliability index is very easy because that is going to be the minimum distance from the origin. So, identifying or locating or estimating the design point on the performance function is actually a given task.

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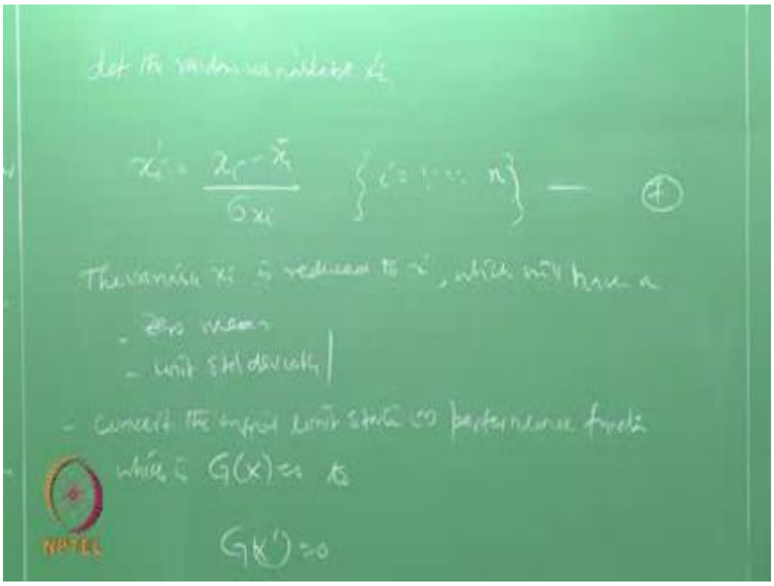

Now the method actually uses a random variable let say relate to the random variable d x i where x i is actually will be x prime i, where x prime i is actually x i minus x bar i by sigma x i, where i is varying from 1 to N let us call this as equation number 7. Now what actually we are done is we have reduced the variable to have a zero mean. So, the variable x i is reduced, so x prime which will have a zero mean and unit standard deviation. Now, with help of reduced variable original limit state function or limit state performance function will be also converted. So, now, I have to also convert the original limit state or performance function which is G X is 0 to G X slash 0 is not it that is what I have to do.

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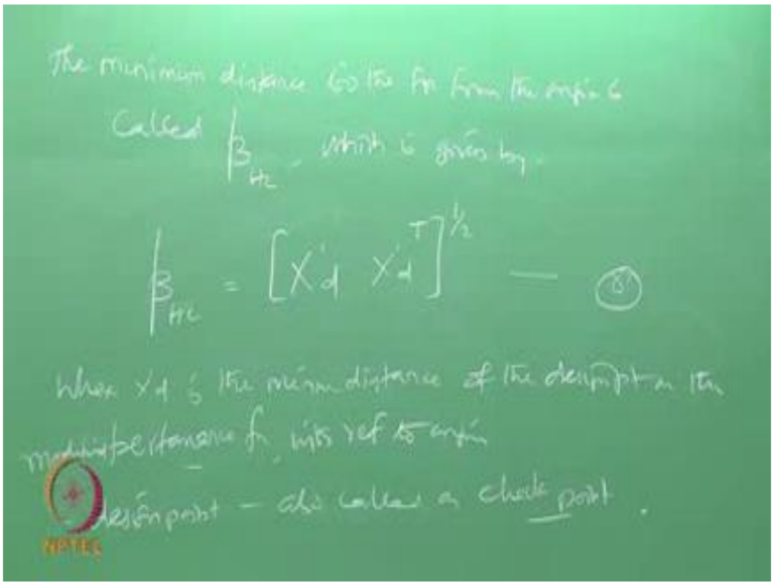


Now the minimum distance to this function from the origin will be called as to the function from the origin is called or is named as beta H L, beta stands for the safety reliability index and H L stands for Hasofer-Lind, which is given by equation number 8. Where X slash d is a minimum distance on the limit state function of the design point on the performance function with reference to the origin.

Of course, whereas the performance function is a modified one on the modified performance function because the original performance function is G of X where is modified one is G X prime X, so I am using a X prime d and X prime d here, so it is a design point. Design point is also called as check point sometimes is also called as check point in some literature so they one and the same. Now, the whole exercise is circumscribed to find out this b H L or beta H L. So, the importance of finding beta H L can be explained with the help of linear limit state function of two variables for better understanding.

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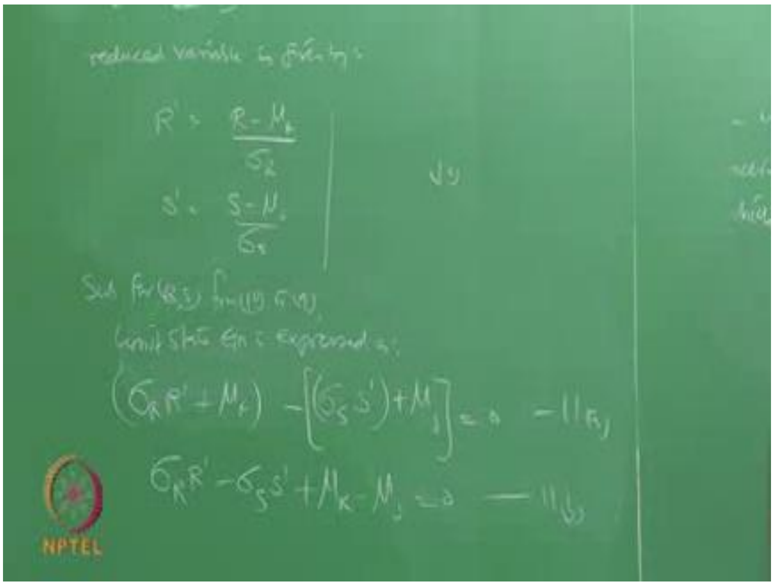

So, the objective is to estimate beta H L. We will take a simple case. The limit state function is linear or performance function is linear, and numbers of variables are two. So, then in that case we know Z is R minus S which is 0, the reduced variable is given by let say R prime is R minus mu R by sigma R which are known to us. Similarly, S prime is S minus mu by sigma S, I call this equation number 10.

Now substituting for R and S from this equation in the original one that is substituting for R and S from 10 in 9, so limit state equation can be now expressed as sigma R R dash plus mu R minus sigma S S dash plus mu S 0 - 11 or 11(a). We can rearrange this equation slightly, so sigma R R dash minus sigma S S dash plus mu R minus mu S is 0, I call this 11 (b). Now, in the space of reduced variables reduced variables are R prime and S prime or on the other hand Z prime; in the space of reduced variable, limit state function then now give plotted.

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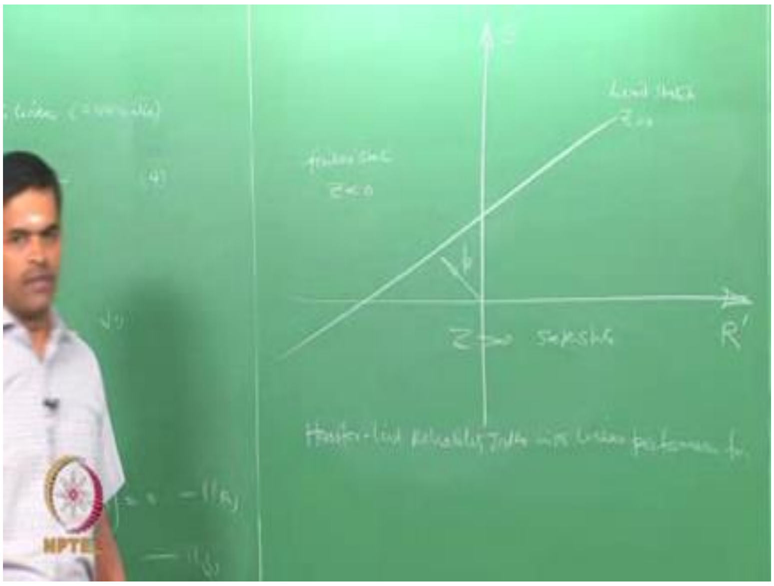

So, I can see the plot here let say I am now plotting it on the reduced variable let say this is going to be my R prime which is going to be my S prime. Let say this is my linear limit state function because it is R minus as linear equation where Z is 0 or Z prime is 0 that is a limit state function which is taken for the problem. Now, we know this is going to the failure state where Z is less than 0.

Let us see assuming a negative value because Z is actually given by R minus S and of course, this going to be the zone where Z is greater than zero so I can say it is a safe state. Now, what we are worried out is the distance beta from the origin to limit state function that is what the worries. So, it is actually they has Hasofer-lind reliability index with linear performance function that is one difficulty here. Hasofer-lind reliability index is useful to calculate the first sort of approximation of failure probability.

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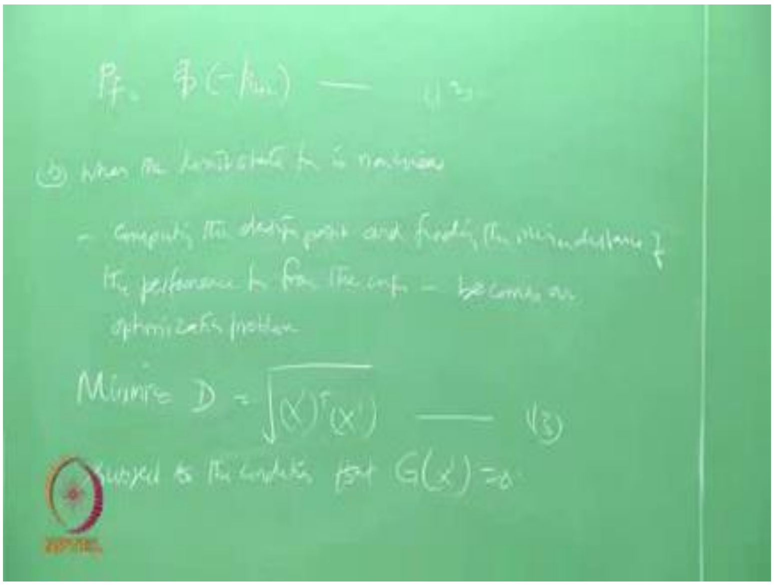

So, beta H L actually is useful the first order approximation of the probability fail so probability of failure nothing but phi of minus beta H L equation number 12. Now, interestingly this is nothing but integration of the standard normal density along the ray joining the line or joining the origin and X dash of D.

Now, when the limit state function is non-linear let say the performance function is non-linear then locating or computing a design point, computing design point and finding the minimum distance of the performance function or the limit state function from the origin actually becomes an optimization problem. So, one has got actually find D minimize it. So, the equation or condition is minimizing D which nothing but the root of X prime transpose X prime. So, we have to minimize this is what is the condition subject to the condition that because minimization requires condition that G of X dash should be set to 0, Shinozuka has given an appropriate solution for this.

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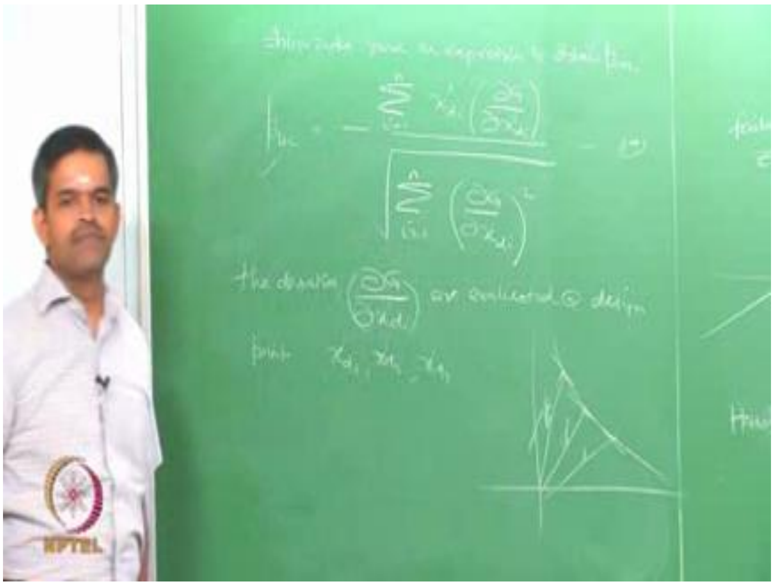

Shinozuka obtained an expression gave an expression to obtain Hasofer-Lind index which will tell me that it is a minimum distance, so that is given by negative i is equal to 1 to N di derivative of this with respect to that design point by root of I call equation number 14. So, Hasofer-Lind reliability index can be obtained with equation given by Shinozuka, there is a negative sign here, given by Shinozuka.

If the performance function or the limit state function is non-linear. Now, in this case, the derivatives that is derivative of G function spect to x dash d i are evaluate at design point which are x dash d i x dash d 2 x dash d 3 so we keep on finding various design points keep on finding various design points and various betas and minimize them. Now the question comes here is (Refer Time: 29:52) at the design point.

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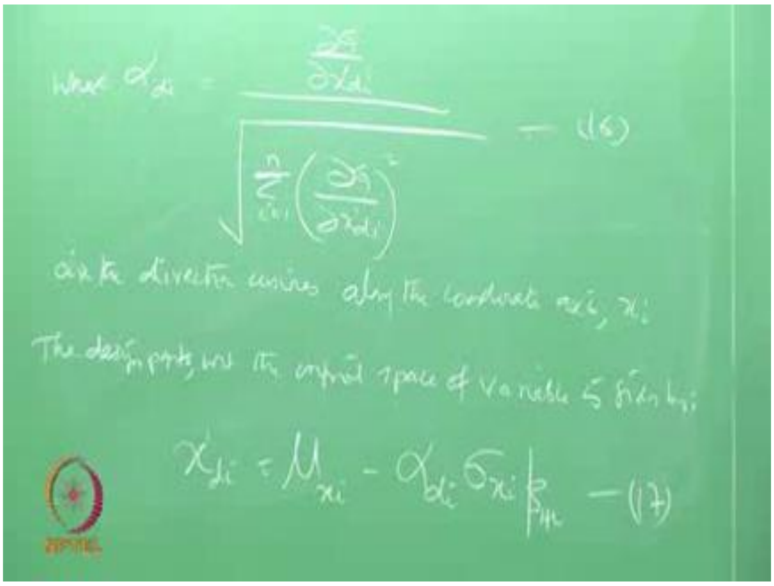

Now, the design points is given by equation number 15, where alpha di is actually is let say because there are meaning are direction cosines along the coordinate access let us say. Now, in the space original variables we have now modified this to the different set of variables is x dash or R dash and S dash but correspondingly in the space of original variable the decide point should be also given. Now, the design point with respect to the original variable or original space of variable is given by the following equation which connects x dash di which is mu x i minus alpha di sigma x i beta H L, this method actually linear approximation limit state function.

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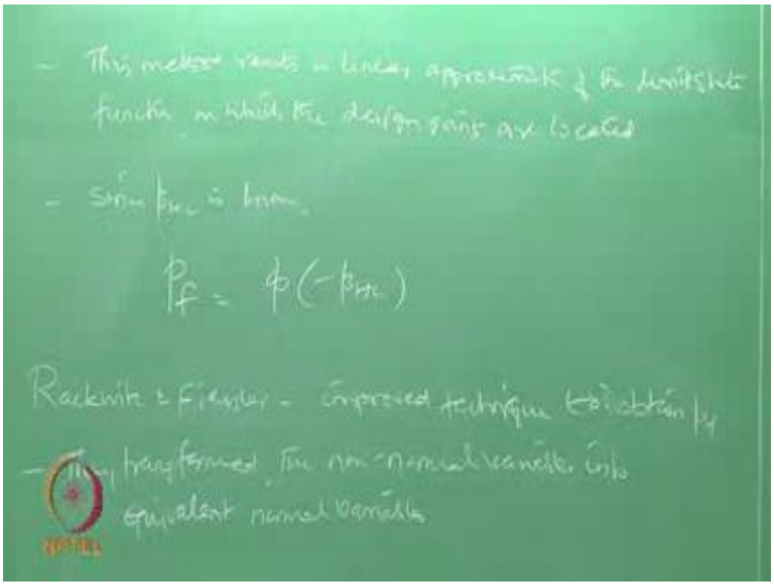


This method what we just now saw results in linear approximation of the limit state function; even though the function remains non-linear, so it is an approximation, which the design points are located. Now, with the help beta H L as obtained from the equation given earlier 14 let us say, probability failure can be estimated. Since, beta H L is now probability of failure can be estimated as the phi function of minus beta.

Now, in case if all the variables are not normally distributed then rather it is more difficult to relate the exact probability of failure this is true only when the variables are normally distributed. If the variables are not normally distributed, then it because more problematic to estimate actually the probability of failure even though you know beta H L Racknite and Ficisler an improved technique for this. Suggested an improve method to obtain the probability of failure; they actually transform the non normal variables into equivalent normal variables. They actually transform the non-normal variables into equivalent normal wedge that is what they have done.

We will see this in detail in the next lecture. So, in this lecture, we have understood details about first order second moment method if the performance function is linear if the performance function is non-linear. We have also seen one more method suggested by Hasofer-Lind it is very easy if the performance function is linear one can find the design point; design point is the point on the performance function whose distance is minimum from the origin.

So, it is very easy for (Refer Time: 35:40) a point whose distance is minimum from the origin if the point is lying on straight line or linear function. If the performance function is non-linear then we do some approximation to convert the space of variables to some other form and approximately convert the non-linear performance function to equivalent N number of linear functions and find design points of each one of them and then find out distance each one of them and minimize this that is what given by Shinozuka which we already seen in the lecture today.

However, if the variables are normally distributed then one can easily find the performance of I mean probability failure if you know the reliability index given by Hasofer-Lind. (Refer Time:36:34) not normally distributed then it becomes more complicated because probability of failure will not then given by a simple equation like this; in that case Racknite and Ficisler, suggested a method by which we can convert the non-normal variables to equivalent normal variables which will see in the next lecture.

Thank you very much.