Risk and Reliability of Offshore Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Module- 02
Reliability Theory and Structural Reliability
Lecture – 06
Reliability Methods II

We will talk about the 6th lecture on module 2.

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In the online course on risk and reliability of offshore structure, in this lecture we will continue with the reliability methods which we started in the last lecture. This is the lecture on module 2 on the online course on risk and reliability of offshore structures. We are talking about the focus on reliability theory and structure reliability in module 2.

We already said as a part of the design process, we generally introduce safety check measures as a part of the design process which is considered to be one of the levels of reliability level 1. We introduced method of safety checks which is actually level 1 of reliability, but unfortunately you are more interestingly even the method of safety checks also could introduce uncertainties. Let us see that slightly later, but now let us make a statement that human error which is one of the main (Refer Time: 02:00) of most of the accident and mishaps is not included in arriving at these safety checks. So, they still remain as one of the importance source of uncertainty that is very important.

Now, how to avoid this? How to improve on this? One can work on improving inadequacy in knowledge, inadequate training, inadequate experience. So, one has to work on this to really comply the standards of safety practices to all personnel involved in offshore industry. So, this could be one of the improvement techniques to improvise safety or to improve safety which will address partly the errors occurring from or uncertainties occurring from human errors.

Now, let us come back to the first statement we said that method of safety or checking of safety may also introduce uncertainty. We can take a very classical example, let us say we check the joint adequacy what we call joint adequacy checks based on empirical rules of course, one can argue that these empirical relationships are based or derived from model tests, no doubt on that, I will show you later in this module 1 of an applicable study what we did, where the inferences derived from the model tests for stress concentration factor, the load carrying capacity of tubular joints may be KLT, etcetera show a different significant compared to that of which has been derived based on the so called empirical studies, which are referred back in the literature. So, even then this will lead to lot of uncertainties.

So, one method by which this can be improved upon is updated research. So, this is been updated continuously. I can give a classical example on this let us say, for example, about 90 percent of the joints have fatigue life in excess of 1000 years.

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So, once the statement is realized and understood then one can say that the so called SN curve is a very conservative which has a very substantial reserve in estimating the fatigue life.

The other way by which uncertainties arising from the checking process can also be overcome by doing periodic underwater inspections, which can address some issues related to critical joints at least on critical joints because this will be helpful in arresting the crack propagation. So, having said this, let us come back to the formation of reliability problems. Let us recollect, the reliability problem is nothing, but identifying a limit state function then integrating this function in the preferred domain to real find out, what is the value of this function? Whether is positive or negative, to know whether it is going to be safe or unsafe, reliability analysis of offshore structures could be actually formulated in different ways. This formulation probably can be grouped into two; 1 is what is called time invariant problem, the other is time variant problem that is the overall broad grouping of reliability problems as applied to offshore structures.

What is the commonness between these two? We will see them independently, but let us quickly see the commonness between these two. In both, the limit state function should be defined I mean that is common this function of course, should be based on either limit state of serviceability or ultimate stress criteria. So, 1 has got a prefix to this before we do a failure analysis or probability of a failure of any specific function not the structure. So, the limit state function or the probability of the failure of this particular function or probability of this function falling in the failure domain can be assessed, when one of this prerequisite is available and applicable to the problem of our interest. Therefore, reliability problem aims determine the probability of limit state failure. So, the aim of the reliability problem is to determine the probability of limit state failure, which violates the limit state condition, the assumed limit state condition.

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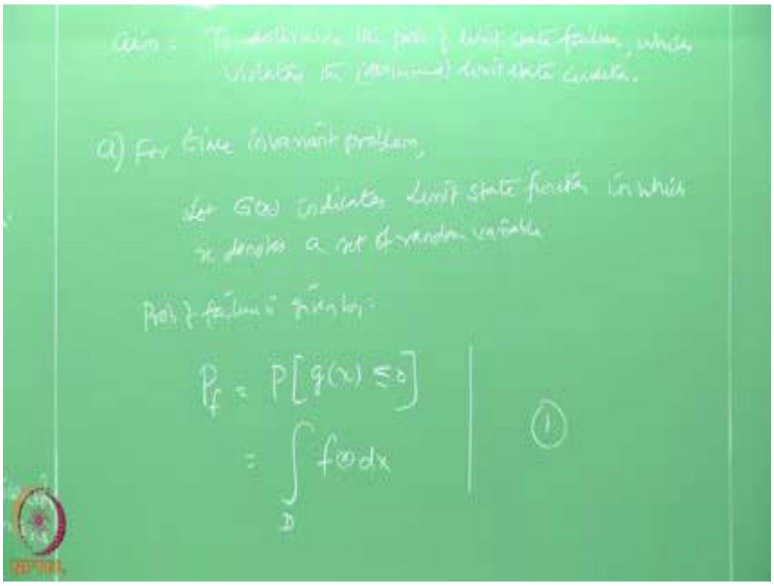

Now, for time variant problem, for time invariant problem, let g of x indicate the limit state function in which, x denotes a set of random variable, probability of failure is given by probability of this limit state function assuming a value less than 0, which is alternatively integration of the probability density function over the domain D. So, let me call this as equation number 1. In equation 1, the joint probability density function and the integration is performed over the domain D, a prefix a joint probability density function which is whose integration is performed over the domain D. Once you know the probability of failure reliability is nothing, but 1 minus probability of failure which is converse of the probability failure. So, reliability can be estimated I request you to please look at the screen.

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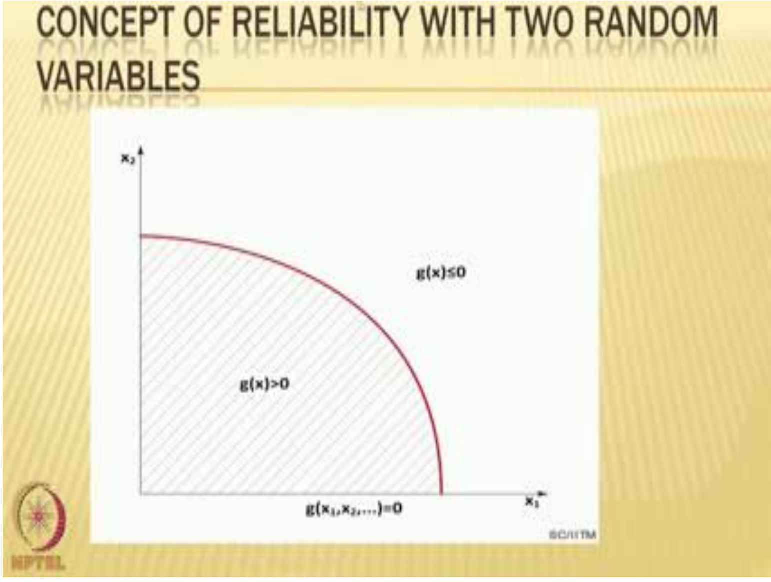

Now, the screen shows a graph between 2 variables, x 1 and x 2 which is indicating conceptually the reliability with 2 random variables x 1 and x 2. Therefore, the plot shows the limit state function indicating or connecting x 1 and x 2, for example, for our understanding x 1 can be the loads and x 2 can be the material strength. So, let us pick up 2 random variables whose connectivity is to be established by a limit state function, let that be g of x as indicated in the black board here. So, g of x 1 x 2 should be equal to 0 is a limit state function. If it is exceeding 0, which is shown in the hatched position is the safe domain. If g of x less than or equal to 0, which is shown here it is a failure domain.

This becomes a simple illustration, graphically this shows me the concept of reliability with 2 random variables x 1 and x 2, having said this integration of a suitable or appropriate probability density function within the shaded area is the reliability against failure, so g of x 1 x 2 and so on x 1 and x 2.

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So, this is my shaded area where g of x is greater than 0, this is the area where g of x is less than or equal to 0 which is the failure domain. So, an integration of suitable probability density function within the shaded area is actually the reliability because that is the safe area. This is the failure area 1 minus failure is going to be the safe area, right. Therefore, reliability analysis amounts to integration of a suitable probability density function within the shaded area a reliability problem is therefore, said to be time variant if the following statement is valid.

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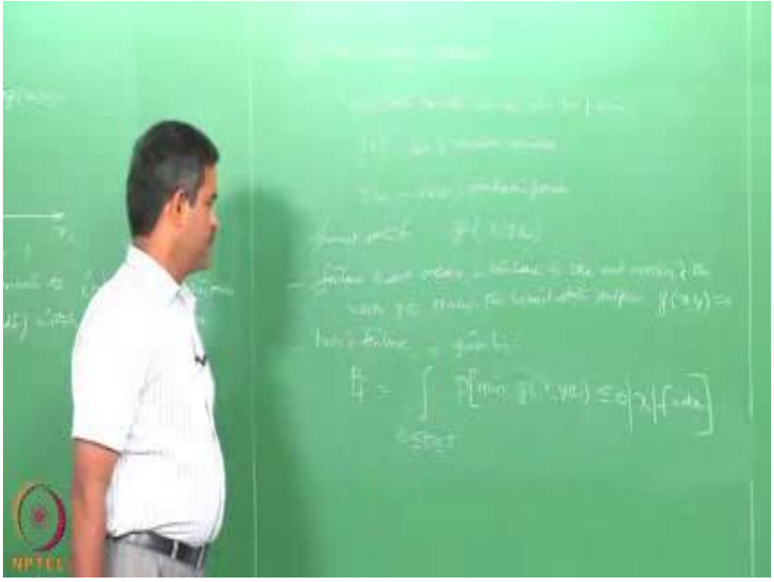

Let us say time variant problem in this case obviously, the limit state function is also going to be a function of time. Function of time in most specific terms limit state function will be defined by a specific set in which the variable x is going to represent set of random variables and y of t at say is going to represent a vector stochastic process.

So, I can say the limit state function for a time variant problem can be expressed as g of x comma y of t. The failure event of such problems may constitute the out crossing of the vector processes y of t through the limit state surface g of xy. So, essentially the failure event means in this case is actually the out crossing of the vector y of t through the limit state surface by just simply g x y 0. So, let us see how this function looks like I request you to please look at the screen which is a graphical illustration of the time variant reliability problem.

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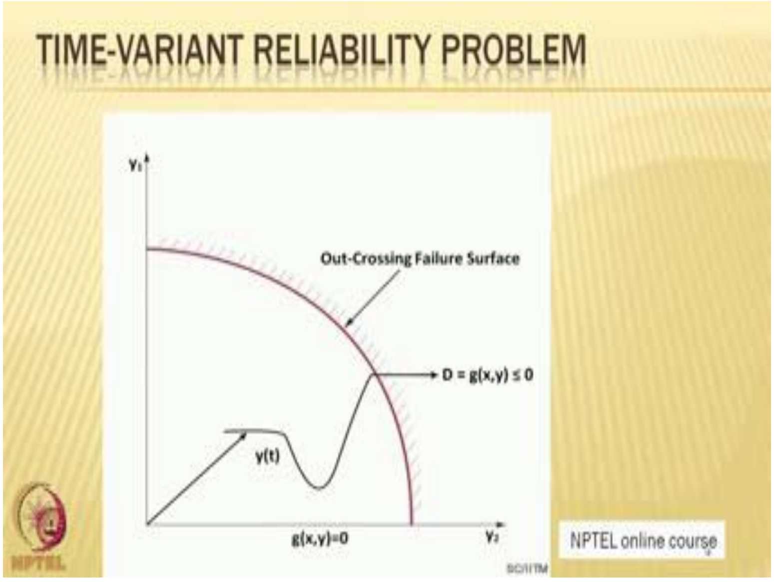

There are 2 variables; let us say y 1 and y 2, g of x y is equal to 0 is what we have the limit state function chosen for this problem. The shaded one indicates the out crossing failure surface, the out crossing failure surface as indicated here and y of t is actually a vector which out crosses this limit state function when it crosses through the value.

So, the probability failure in this case is given by. So, probability failure is going to be limit integral of the value of t between 0 and t you got a set up because I already said failure is defined as non-performance of a function over a specific period of time under specific conditions is it not. So, time has got specific, why time is not important in the earlier case equation, because the limit state function chosen remain to be time invariant whereas in this case, since the limit state function is going to be representing the time variant problem as well therefore, time boundary will also come in to play here. So, probability of minimum g of x y t less than or equal to 0 equal to 0 given x given f of x dx the equation number 2 in equation 2 t actually denotes the capital t denotes the life time of the structure and small t denotes the time.

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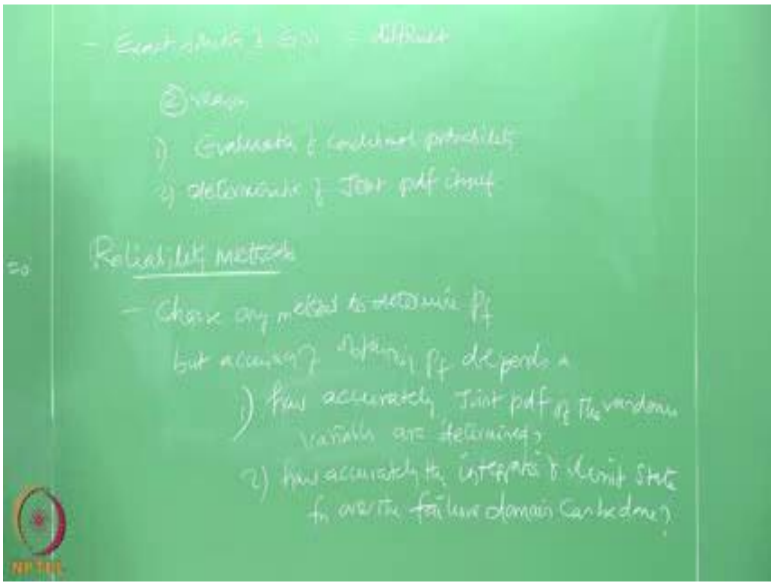

Let us say f of x is the joint probability density function of x. The condition probability is defined by the terms within the parenthesis which is by the stochastic analysis of the structure for random loading y of t.

Now, there is a problem with these equations 2, the exact solution of equation 2 to determine out crossing analysis is difficult, why because there are 2 reasons for this because of 2 reasons, the first reason could be evaluation of condition probability and second could be determination of joint probability density function itself. In fact, determining the joint probability density function is also one of the reasons for not being able to obtain the exact integration of the equation 2.

So, the failure domain of this is what to see in the screen. Now, which is being also shown to you earlier that is failure domain, which is seen for a time variant reliability problem. Having said this, let us revisit back the reliability methods in the focus of time variant and time in variant problems or in addition to that in the focus of uncertainties arising from varies at as discussed early.

So, let us talk about reliability method again, but with a different prospective this time. There are many reliability methods or there are many methods of finding probability of failure of structures involving the functions of random variables accuracy of all these methods depends on 2 critical issues. So, one can choose any method to determine the probability of failure, but accuracy of obtaining probability of failure depends on how accurately the joint probability distribution functions of random variables are determined, that is first. Second could be how accurately the integration of the limit state function over the failure domain can be evaluated. So, 2 critical factors will govern the accuracy of the method which is a reliability method.

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In addition based on the methods of estimate of joint probability distribution functions they can also be classified either as analytical or numerical. So, one can even say analytical method or numerical method of reliability based on the pattern or based on the method by which we are estimating the joint probability density function. So, one of such methods is very commonly applied on the offshore industry is first order moment method.

Let us say FOSM, in this method a first order taylor series approximation of the limit state function is used and hence the name first order, only second moment f statistics of the random values to obtain the probability failure that is why the method is called first order second moment. So, first order approximation second moment therefore, first order second moment method

If 2 random variables are used to derive this method, the limit state function for 2 random variables r and s is given by z is equal to r minus s, assuming that r and s are statistically independent.

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If they are normally distributed then z is also normally distributed. Therefore, mean of z can be said as mean of r minus mean of s variance can be said as variance of r variance of z. Now, the probability of failure is given by probability of z taking negative value that is probability. Now, as we said earlier if z is a normal variant, if we assume that r and s are normally distributed then z will also be normally distributed in that case probability of failure, can be easily given by a close form solution which is going to be a phi function of minus mu z by sigma z, where phi is a cumulative distribution function of the standard normal variant.

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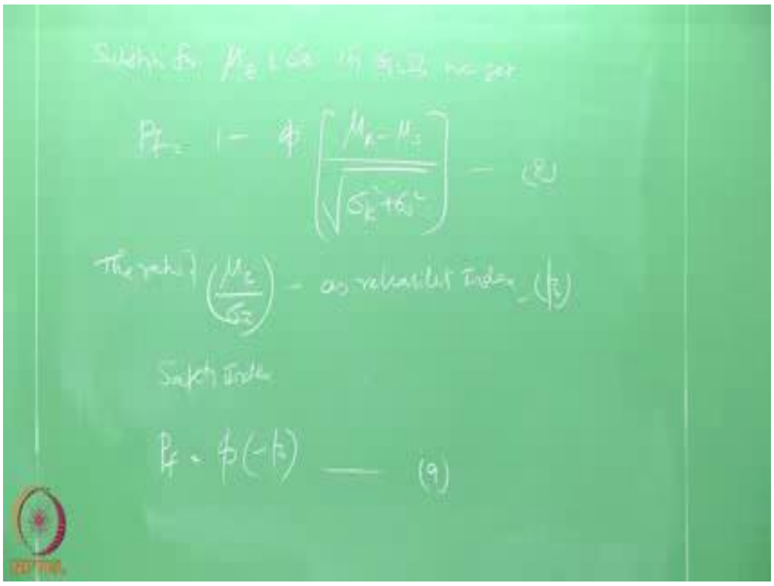

Now, substituting for mu z and sigma z because we need this in equation 7, we get probability of failure is 1 minus phi of mu r minus mu s by r square equation number 8. Now, in this equation the ratio of mu z by sigma z is called as reliability index indicated by beta which is otherwise called as safety index also in that case probability of failure is then given by phi of minus beta, let us say if the variables are log normally distributed then the limit state function is given by z of r by s, where z is the normal variable and probability of failure is given by 1 minus phi of l n of mu r by mu s root of 1 plus 1 plus divided by root of ln.

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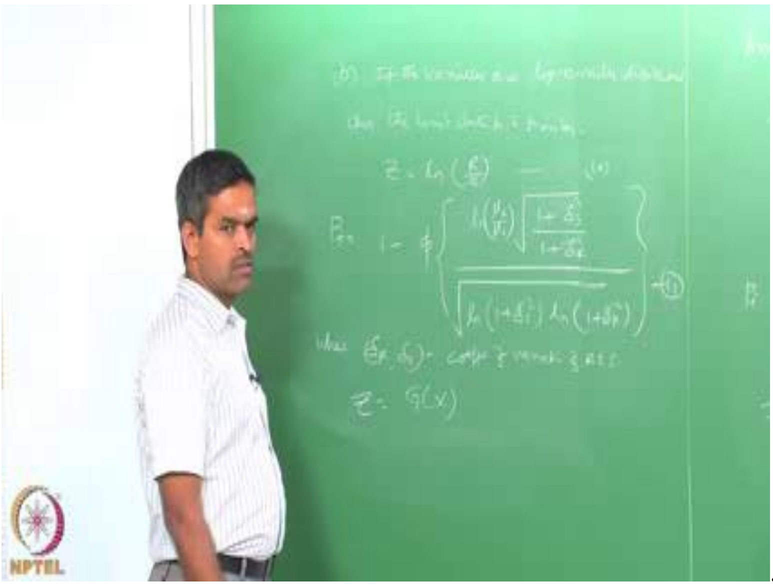

These are sigma, there are deltas please make a change here deltas where delta r delta s are coefficient of variations of r and s. Now, the above formulation may generate and can be even generalized to any random variables because we have taken here 2 random variables r and s, can generalize it for n number of random variables denoted by a vector x. In that case the performance function can be simply g of x. So, the taylor series expansion of the performance function about the mean values can be given which can be studied for FOSM which we will see in the next lecture.

Friends, I hope you understand in this lecture, we have covered the important aspects of reliability methods. What are the constrains which govern the accuracy of the reliability method? How the choice of probability distribution function can also lead to accounting various uncertainties in different levels in the analysis? We started of the different methods of reliability, we started with the first method which is first order second moment method, we took 2 cases; one is if the variables are normally distributed if the variables are log normally distributed. How can you estimate the performance function and how can you find out the probability of failure of this function using these 2 normally variants or log normally variants, we will continue with this in the next lecture.

Thank you very much.