Risk and Reliability of Offshore Structures
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Module - 01
Lecture - 12
Sampling estimates

Welcome friends to the online course title Risk and Reliability of Offshore structures.

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We are talking about lectures on Module 1. The focus on Module 1 as we already know is probability in possible reasoning and introducing reliability in terms of probabilistic estimates. In this lectures, which is Lecture 12 we are going to talk about the Sampling Estimates. We already said the accuracy of reliability or probability of failure essentially depends on how you carefully choose the sampling density function h of x.

So, in the last lecture we discussed about the importance of choosing h of x, if h of x which is the sampling density function is not carefully chosen one can land up in a (Refer Time: 01:32) method of developing a simulation which may not result in here close are accuracy of estimate of probability of fail. So, h of x also depends upon the random variables which essentially form the sampling of the whole exercise. Let us now focus on the importance of sampling estimates. We already said our focus is to improve the accuracy of failure estimate, so obviously if you really wanted to improve the accuracy of the failure estimate of a probability of failure one should focus on how to improve the efficiency of the simulation method.

We have discussed couple of methods in the last lecture. Let us discuss one more method by which an improvement is slightly shown. Let us say an improved technique an improved approach of simulation is graphically shown here try to plot this.

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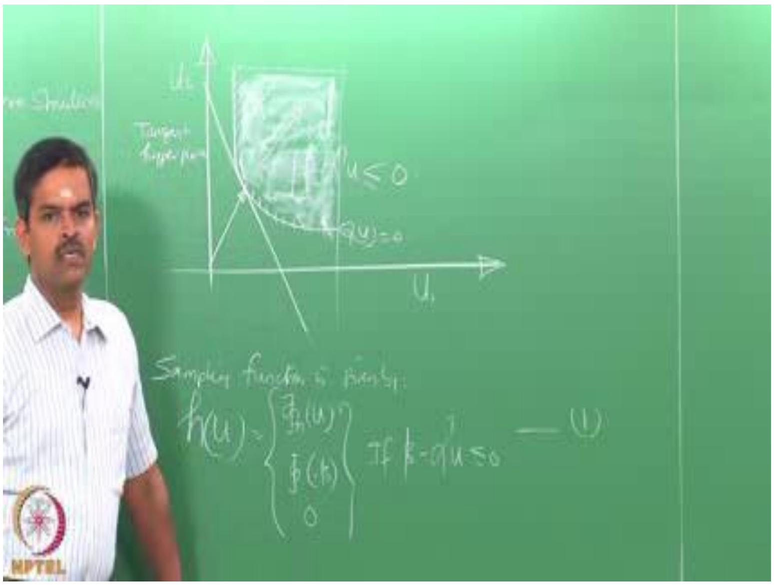

Let us say this is u 1 and this is u 2. We already said to improve the efficiency; so this is my failure envelop which I call as g u as 0. I draw a tangent hyper plane to select the design point, so this helps me now to locate the design point which can be the region of alpha and of course the value beta minus alpha transpose u should be less than or equal to 0.

So, in this estimate a sampling function is given by h of u, phi n, phi of beta 0 if beta minus transpose less than 0. Let me calls this equation number 1. In this approach you will notice that it is very important to estimate u, such that a specific condition should be satisfied.

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So, it is necessary to estimate u u such that the following condition is satisfied, so the condition is u u 1 beta minus transpose u should be less than or equal to 0 equation number 2. Now let us see how to estimate this u, so there are different steps involved in this. Let us say step number 1; simulate the standard normal variate u i, where i is 1, 2, 3 till n minus 1. In step number 2; simulate the clipped normal variable z, which can be generator from the probability density function given by f of z z is 2 by root 2 pi exponential minus z square by 2 where z is less than or equal to 0.

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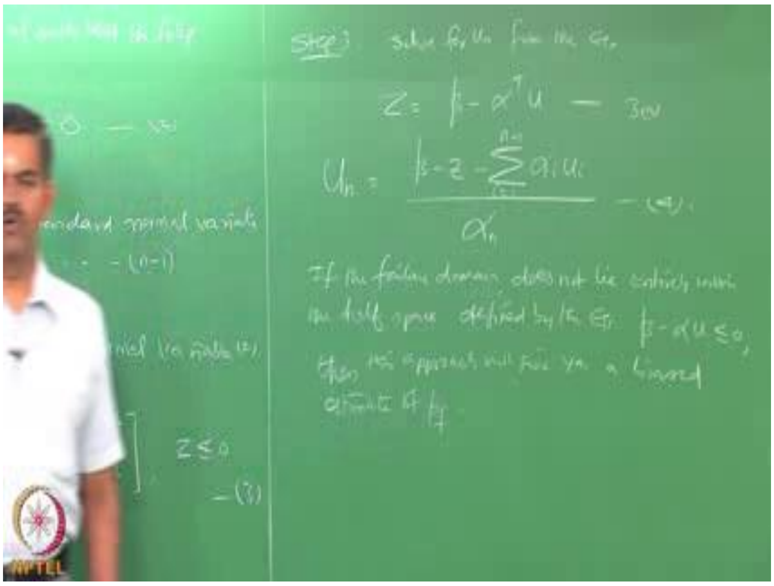

Now solve for u n from the equation z equals, which already we have here equals beta minus alpha; equation number let us say 3 a. On the other hand u n will be given by beta minus z minus summation i equals 1 n minus 1 alpha i u i by alpha, is number 4.

Interestingly, if the failure domain does not lie within the half space or entirely within the half space defined by the equation beta minus alpha u less than 0, then this approach will also give you biased estimate of probability failure. So, I have to be careful about this. Now let us try to plot a typical failure domain generate from this method.

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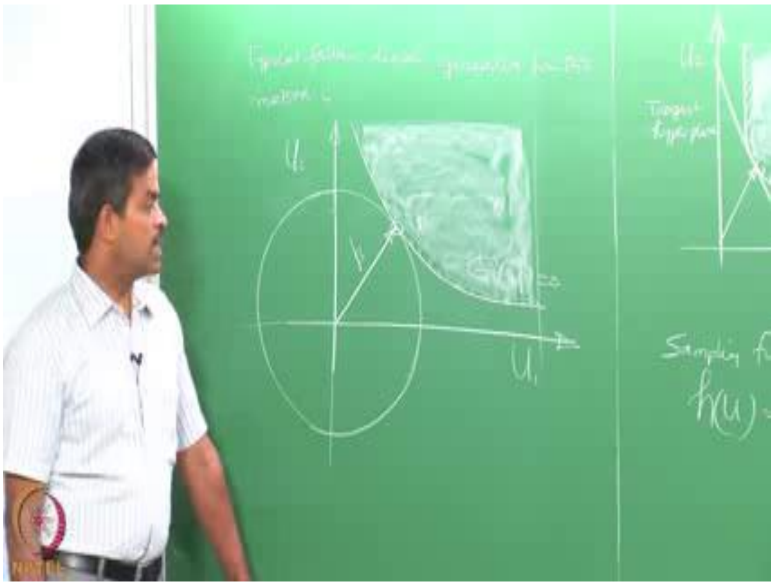

A typical failure domain generated from this method is shown in this figure. So, that is the design point and of course this will remain tangent to the design point at that value and that is my failure envelop. As usual this is u 1 and this is u 2, and this is beta the reliability index and that is my design point u dash. Of course, envelop shows it is g of u equal to 0.

So, that is the failure envelop which has been improved compared to the earlier failure envelop which you had using tangent hyper play. So, all these methods are only in improvement on how to actually define the sampling density function properly so that can land up in a closer estimate of probability of failure.

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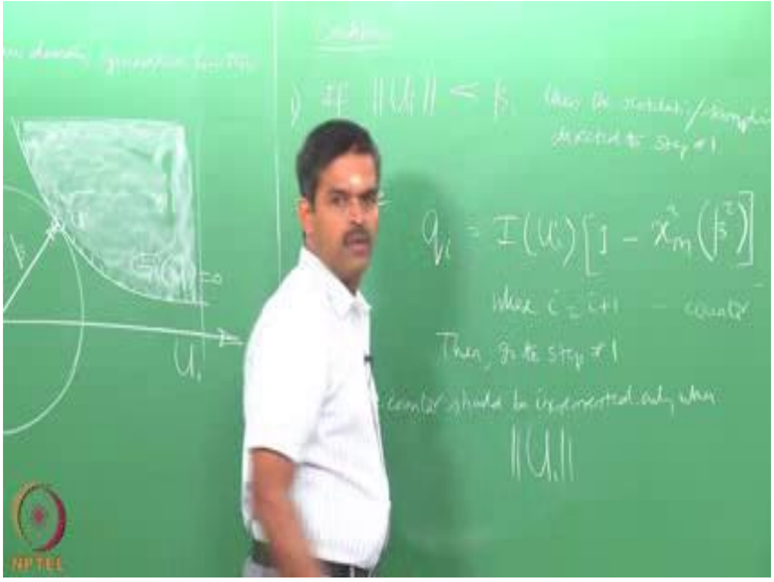

Suppose for example, the certain conditions if u i is less than beta then the simulation or the sampling will be directed to step number 1. Step number 1, simulate the random variable again. Else, if this condition is not true then q i is given by the intensity estimator 1 minus x square m beta square of the value, equation number 5. Where, i is i plus 1 nothing but the counter for the simulation.

For every increment then go to step number 1 to proceed with a simulation. It is important to know the counter will be incremented only under specific condition; the counter should be incremented only when is greater than an equal to beta.

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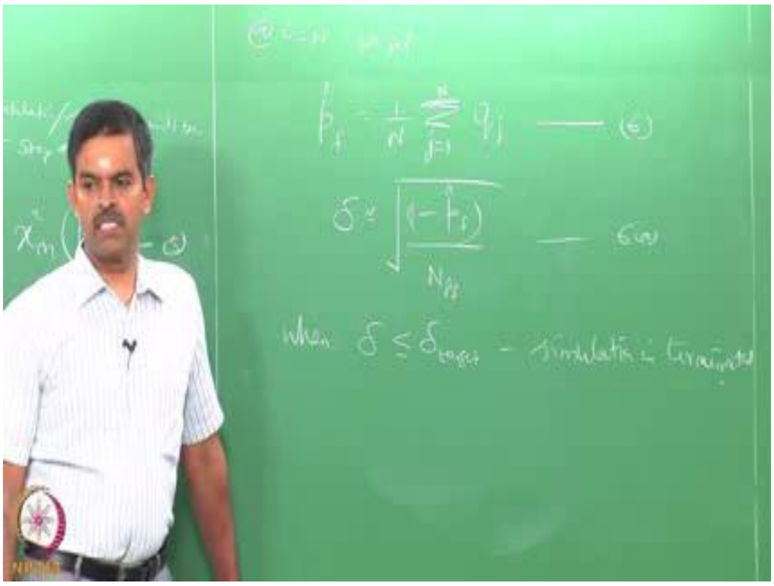

At i equals N we get probability estimate as 1 by n summation of j equals 1 to n q j, equation number 6. And, delta function will be approximately equal to 1 minus probability of failure by N probability of failure that delta is used to set the target to either stop or to continue the simulation, equations 6. As we understand when delta is less than delta target simulation is terminated. It means that prescribe maximum number of iteration simulations is reached. It is also adequate if the sampling is done with a normal density at each design point.

So what you mean to understand is, at the every design point if the normal density is being used for developing a sampling statistics then the accuracy of probability of estimate of failure will be better, let us look this diagrammatically. Now whole difficulty is locating the design point. We are trying to get the design point in such a manner that my probability of failure estimate is as accurate as possible and the points of sampling fall within the failure domain.

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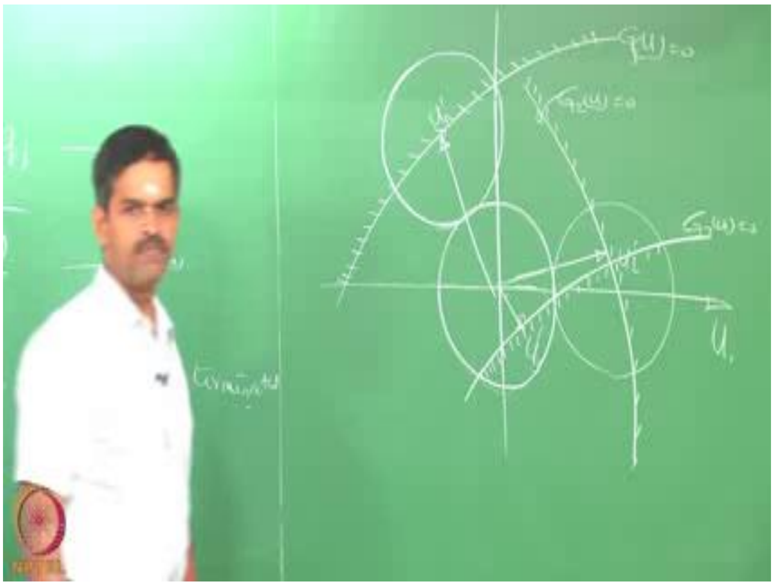

Now the point is if I have got many number of design points let us say, I have a design point at this specific location which becomes my failure envelop and the design point at this location is normal. I call this has let us say u 2 i, G 2 0. Similarly I have one more design point, where have one more failure envelop which is G 3 u set to 0 and I have a design point u 3 prime. One more failure envelops which is G 1 u 0, and again I have a design point which I call u 1 prime.

So you have got multiple design points now; u 2 prime which is normal at this point to this failure envelop, u 3 prime which is normal at this design point to this failure envelop, and u 1 prime which is normal to this failure envelop which is G 1 u 0 at u 1 prime. So, one should be able to really know now if I got multiply designs points like this how to actually able to get a good sampling, because one can always land up an either u 1 u 2 or u 3 a depending upon what is a failure envelop and the failure envelop is governed by the choice of your sampling density function h of x. In this case if you have multiple points.

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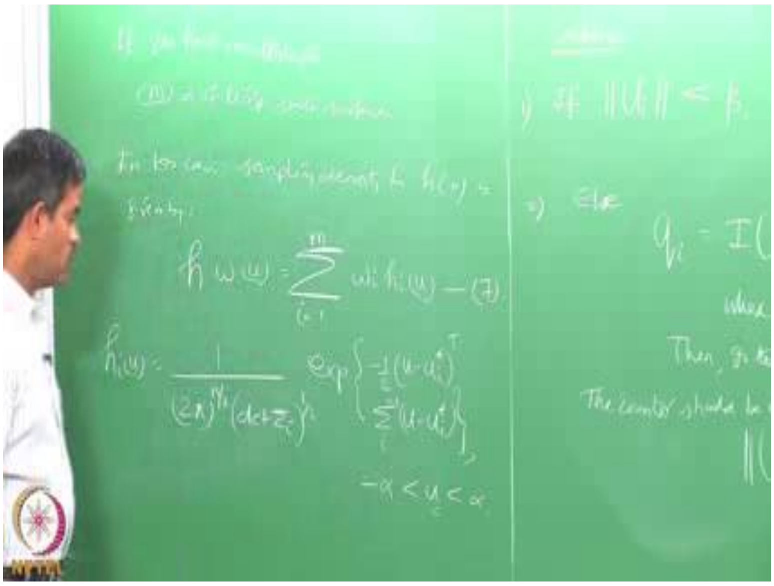

If you have multiple design points you get m number of limit stage surfaces. Was every failure surface is limited surface. In that case the sampling density function h of x is given by a weighted function I call it as h w u, which is going to be the summation of i equals 1 to m because there are m failure envelops or m failure limits state surfaces. Which is now multiplied by w i h i of u, call this has equation number 7.

Where h i of u which is the sampling density function is given by this equation which is, minus half of u minus u i star transpose multiplied by i to minus 1 u minus u i star. The condition is minus alpha less than u, less than alpha is a range within which the sampling value of the density function will be chosen, so equation 8. One can easily see here the above set of equations 7 and 8 gives multi-variate normal pdf with mean vector u i star.

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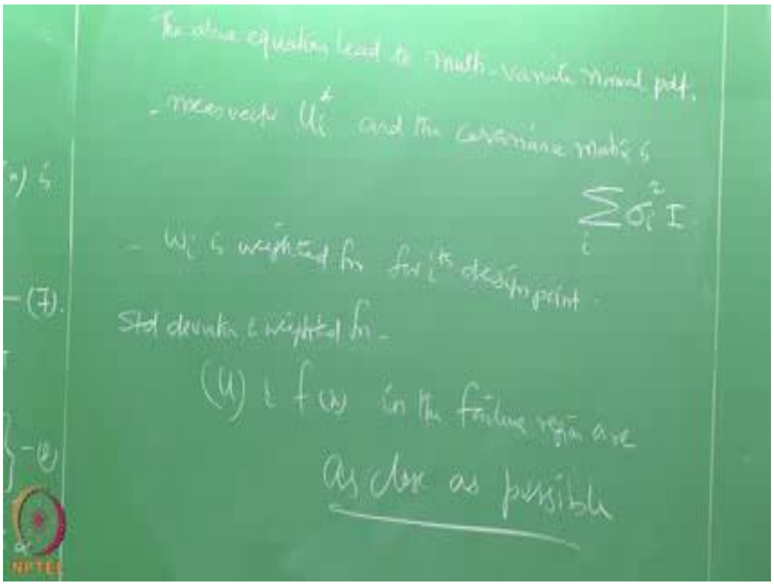

The above equations lead to multi-variate normal pdf. The mean vector of this will be given by u i star and the covariance matrix is I sigma I square. In the whole argument w i is called the Weighted Function. Since, i represent a specific count so it is a weighted function for the ith design point. Now as you see in this figure we have many design points u 1, u 2, u 3, so for every value of the design point that are weighted function of this. So, w i is actually the weighted function for ith design point which have generator.

A standard deviation and the weights of this particular equation will be chosen such that, now the standard deviation and the weighted function because that is now the problem should be chosen such that u and f of x, in the failure region or as close as possible. So, one can choose this (Refer Time: 25:13) error and try to get the proximity as far as possible between the chosen points u and the tough of f x. One of the major advantages of this method is that it gives you an unbiased estimate even though this method may not be very efficient as compare to other approaches. The essentially reason for this is approximately half of the sample points fall within the same domain of this method.

However, if the failure region is not identified prior to the analysis this method will indicate the unsuitability with the critical deviation of the coefficient of variation from that other standard value. So again I am warning that you are not able to converge or your sampling points will not lie to the maximum in the failure domain. Having understood this let us move on to another argument call Statistical Theories of Extreme Values.

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Friends as we know now and I have learnt; in reliability estimate parallel systems can also be estimated by use of certain limit distributions known as the extreme value distributions. One of the areas of application is to determine the strength of the material under worse combination of loads. Let us say for example, maximum wind gust etcetera. Now let us consider example to understand this particular problem. Let us assume a system s which consist of n components; this system consist of n components. The system can be an offshore plat form; components can be members of the offshore plat form like, pontoons columns etcetera.

Now each member or each part of the component or each functionality of the given system may have a different time life time period. So, let the life time of these components be T 1, T 2, T 3, there n components therefore n time periods. Now these life time values are assumed to be independent I can give an example let us say; T 1 corresponds to life period of a drilling derrick, T 2 corresponds to life time period of a living quarters or the block module of a living quarters etcetera.

So obviously, the life time period of each one of these components which form system may not be depend on each other. So these life times of components are assumed to be independent and identically distributed with reliability function r of t. So, let r of t is the reliability function with which the life time estimates of components or distributed. Please understand these components are independent of each other. So, all of them put together because they all form a system I am not interested in knowing the failure of a component of course, but I am looking over all the failure of a system. I am looking for probability of failure of the given system.

We all know the probability of failure a given system is dependent closely on the failure components I agree, but I am not interested in only knowing the probability of failure of a specific component but I am looking for the influence of the failure of this component on the overall failure of the given system s. Therefore, let T s system being the life time of the system.

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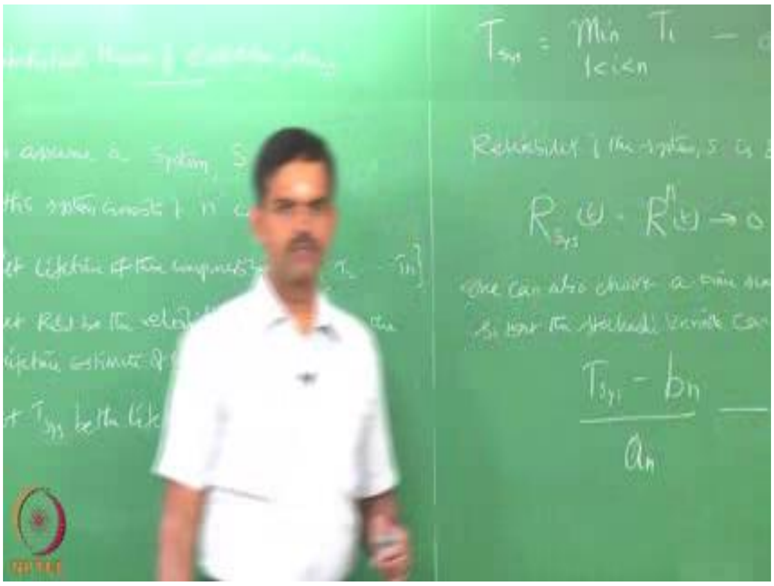

So, T sys is actually given by the minimum 1 less than i less than n of T i. I am also interested in knowing the reliability to the whole system. So, reliability of the system s is given by as function of time which will be R n of t, if n tends to infinity. In general system reliability can be estimated within a given time scale, by choosing 2 sequences such that the sequence of stochastic variables is expressed for a given a function. So, one can also choose here time scale of 2 sequences so that the stochastic variable can be expressed as T sys minus b n by a n.

Now we will move on to understanding of the sampling statistics, extreme value statistical theories, selection of sampling density function, selection of choice of random variable, choice of simulation methods, and try to apply them slowly on different variables which are going to be responsible for estimating reliability of an offshore structure. So, let us start with environmental loads talk about how you will model the environmental loads.

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The environmental loads act on offshore structures can be classified into different manner. So the classification of these forces; one permanent loads or dead loads, these are called as P class loads. Two; operating or live loads, they are called as L class loads. Third; deformation forces called as D class loads. Fourth; environmental loads including earthquake loads called as E class loads. And lastly accidental loads called as A class loads.

So, environmental loads acting on offshore structures can be classified into 5 as given in the black board now. Permanent loads of course include time dependent loads as well. For example; gravity loads such as weight of the structure, weight of the permanent installations, and fall under this category. Why this class of loads can be estimated by the higher accuracy characteristic values used address the associated uncertainties. Even though one can believe that P class loads can be estimated with greater accuracy.

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However to account for uncertainties in their estimates one uses characteristic value. Next comes the L class loads which are live loads. Live loads are generally associated with nature and the operational features or operational mode of the plant and equipments of the top side. Therefore, one can also choose carefully the characteristic values for live loads which depend upon the type of structure under consideration, so depends on type of structure under your consideration.

The next comes a deformation class loads which is called D class loads. D class loads are actually caused by the imposed deformation on the structure which arises from large displacement. We are talking about complaint type structures which actually result in large displacements because your design for large displacements that is how we alleviate the loads acting upon them. So, these loads may also arise form deformations caused by the temperature deference, differential settlement of the foundation, settlement of support systems that arise in case of structures like; article of the articulated towers, triseratops etcetera.

Because triseratop we know the deck rests on the ball joints, therefore depending upon the m 5 characteristics of the ball joint the top side or the top side settlement can be different and that can also cause large deformation displacements to the given system. Therefore, one has to be very carefully choosing the characteristic value prescribed the design. So, characteristic values prescribed in the design course should be carefully chosen it should be the maximum possible value. You must choose the maximum possible value, because this value will be associated with a very high degree of uncertainty. So of course, is supported by different a such as lin 1967, lin and cai 2004. So, they advocate that one should choose the maximum possible value of the characteristic values prescribed the resign force is or the resign course so that they can be associated take care of the highest possible uncertainties.

Now environmental loads arise from the ocean environment that include wave loads, wind loads, current, the earth quakes, ice loads, etcetera. Each one of them is different events they have different periods. The mean written period is of course used the account for the characteristic value of this class have loads, because you come to environmental loads they include different varieties of loads coming from wave, wind, etcetera each one of them will have a different period. I am interested know the mean period of this entire class of load which is rather very difficult to estimate.

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So, environmental loads or E class loads consists of loads from waves, wind, the earthquake, current, etcetera. Each event can have different periods. I am looking for the mean period of E class loads; it is rather be difficult to find out this. The mean period of course is used, the account for the characteristic value of this class of loads. The last one is the accidental class of loads a class loads which are experienced with the platform from different conditions like, collision, drop objects, explosion and phi. The characteristic value associated with a class loads should be carefully selected depending upon the subject matter of investigation, operational manner and experience and technology involved in safety and discuss suspends.

Therefore, all these kinds of environmental loads or all these kind of class of loads acting on offshore structure can be accounted for representing their degree of uncertainties by using appropriate characteristic values. Obviously, one can easily come to a conclusion that a single characteristic value cannot represent the variation uncertainty on different class of loads. However, as advocate a different such as one should use the maximum possible combination of characteristic value because this is useful in accounting for or representing highest degree of uncertainty which are phase by these class of loads in estimating. Now, one can ask me a question how this will affect the reliability study.

Ladies and gentleman's important for us to know that environmental loads or these classes of loads one of the important input which going to cause failure to my system s. So, if I am not able to estimate the input loads in a proper manner or I am not able to account the uncertainties associated with these loads my estimate of probability of failure of the system s may not be accurate. So, one should have enough knowledge and carefully choose characteristic values associated with each class of loads as advocated with the design course. So, a variety of uncertainties arise in all class of loads characteristic values of course are used the account for these uncertainties as given by Maddox and Wildenstein, 1975.

In this lecture we start at introducing you to understand the importance of sampling density function. We also said something about the sampling theories of extreme values. We are now moving towards understanding the estimates of input which are going to govern the probability of failure of offshore structures. We started the discussing environmental loads, we will take you forward to understand this once we complete the discussion we will take you back the reliability theories and then we will apply these knowledge of possible reasoning and probability theory to that of reliability estimates in the 2nd Module.

Thank you very much.