Dynamics of Ocean Structures

Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 09
Damped – forced vibration

So, in the last lecture we discussed about the undamped forced vibration, where we have taken the forcing function as p naught sin omega t, where p naught is the amplitude of the vibration, and omega is the frequency of the forcing function.

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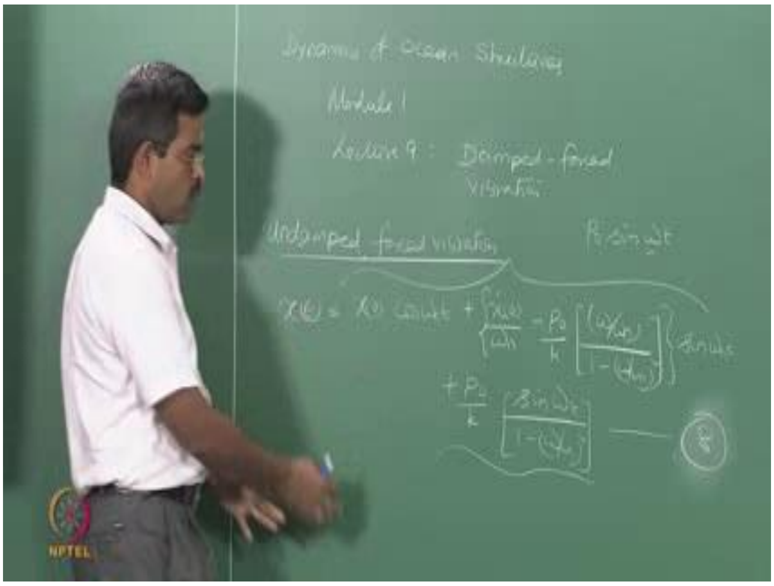

Then we load the equation of motion, solved the second order on the differential equation, found out the complimentary function in particular integral, eliminated the constants of integration as a and b, depending upon the initial conditions x naught and x dot naught at t is equal to 0, and we found out that the solution has got two parts; one is what is called the transient response, other is what is called the steady state response, and we already said the stand in response not generally important.

Steady state is important, because this will always exist, does not depend on the initial conditions given to the problem, and this exceeds at the forcing frequency, and therefore, it is important. However even in steady state response there is a component of omega and present here, but the (Refer Time: 1:20) ratio. So, there is in the actually effect much as you see here in contribution. So, having said this, let us pick up only this term which is important to us; that is x of t b only the steady state response and see what happens further.

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We know that k is the stiffness of the spring express, generally a Newton per meter in SI unit, it means it is actually the force, for one meter or unique meter displacement. Suppose the force is p 0, and the stiffness is scaled what I get will be, let us say Newton per Newton per meter, I will actually get the displacement. So, I can call this as static displacement, why displacement, because it is amplitude by the stiffness of the spring, and why static, because there is no dynamic component present is always constant, and I call this as x s t, s t stands for static. Now, I rewrite this equation x of t by x s t is given as 1 by of sin omega t, I call this as equation number nine. Now depending upon the value of omega with respect to omega n, the characteristic of this responsible change, I will show you how. Let us pick up this equation.

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Let us say x t by x s t is 1 by 1 minus omega by omega n of whole square of some function which is sin omega t for omega less than omega n, let say. This value will be a fraction; therefore, the whole function will be positive. It means the response and the forcing function will be in phase, what does it mean. If you give the force to a spring mass system to move it to the right, the body will move to the right they will be in phase.

Now, for omega greater than omega n; that is any frequency content which is larger than the, actually frequency of the system; obviously, you will see this function, or this term will become higher than one, it means this will be negative. On the other hand the response function will not be in phase with the force. So, I should say, the response and the forcing function will be out of phase, what it means. You give a force to the right, but the system will try to move to the left, they will be out of phase. Now, the same equation as got two terminologies which is of different nature, distinctive different, I can unify them in a single statement, let us see how.

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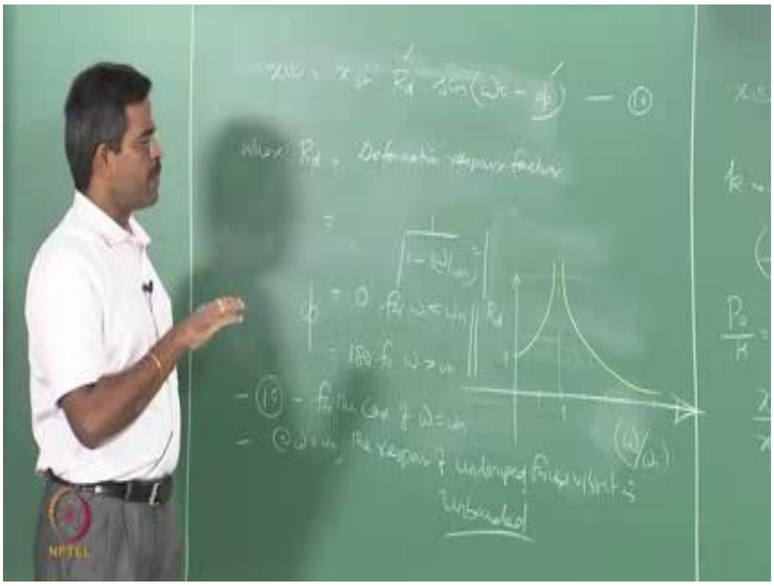

So, I say x t is x static r d sin omega t minus phi. I introduce a new term here r d, and of course, a phase triangle phi, which will account for the significant change in the behavior in a single expression. I call this equation number ten, where r d is called deformation response factor, which will be equal to mod of 1 by 1 minus omega b omega n square I take a mod.

Therefore, the sin is not protected from this equation. I have taken the mod of this; the sin is not protected; now sin is out from the equation. Now, the sin has got an actually come from phi; therefore, I say phi will be equal to 0 and 180 for omega less than omega n for omega greater than omega n. Now I can plot this, where I will try to plot this ratio omega phi omega n versus r d, this is equation for r d, if omega by omega and 0, we will not this going to start from one. Let us have some divisions on this, just to show a symbolically what are the values. Let us pick up that this number is one, it means omega is equal to omega n

Now, as you approach this value in this function this will become infinity. So, as you approach this domain, will become infinity; as you go away from the domain it will converge, you can plot it and see. So, I can write a very important inference from this. The inference is equation ten does not qualify for the case of omega equals omega n. You cannot use equation ten for this; that is first inference. The second inference is at omega equals omega n, the response of undamped forcing forced vibration, is unbounded. It goes to infinity actually, we do not know where it is ending, it is unbounded. Now, you want to capture this, I want to capture this, I cannot use the same equation procedure, because the rule for particular integral, when omega equals omega n cannot be applied as we have done in the last derivation, never slightly modify this, we will do that now, but I want to define the very important function here the function is, what is called resonance.

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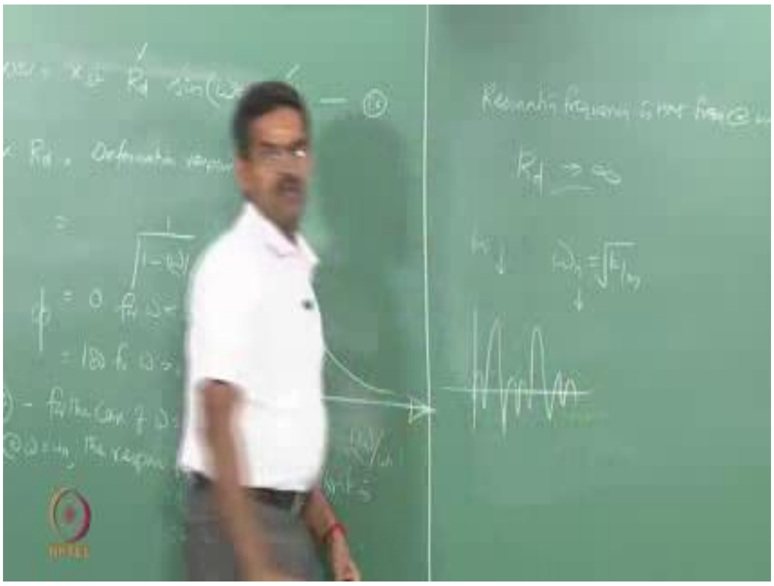

Resonating frequency is that frequency at which the deformation response factor tends to become infinity. You have to define frequency resonance like this. You should not say resonance is natural frequency equal to forcing frequency; no it is that frequency at which the deformation resonance factor tends to become infinity. Now, there is a very specific meaning why it is a tend to become infinity, because you can still controls this, it is not become infinity, it will never become infinity you can control, to control this you must bring down the response.

So, you have to introduce damping this number one, even if you do not introduce damping, let us say you do not introduce damping at all, when they r d tends to become infinity, when it grows when the response grows it will damage the structure. Once the structure is damaged the stiffness of the structure will be released the structure will become flexible, because it is broken once the stiffness is decreased the structure become flexible omega n, which is nothing, but root of k by m will change when omega n changes the behavior of the system will be out from this band.

So, resonance does not existing in offshore structure, actually there is no constant of resonance in offshore structures at all. If you read or write or hear from somebody, kindly challenge them that he does not understand anything about resonance, it is because resonance will indicate a damage to the structure the moment. The structure is damaged is flexibility goes down structure looses the string; that is called material degradation or strength degradation. Once the strength degrades omega n shifts one omega n shifts the condition of one; that is omega with respect to omega n does not stay for longer time; therefore, the system is designed in such a manner that the system will come out of resonance automatically you do not have the anything.

One classical example is, complain structures, that is why they are complain; that is way they have flexible number one number two omega is not a unique value in offshore input, omega is a series of waves coming. So, only for one such omega, it will be resonating. There are one million omega is coming, there are one million omega is coming, so resonance does not matter actually. So, it is not have concept to be actually bothered, but still I want to capture this and show that at resonance actually what happens mathematically, because mathematically I cannot solve this, and I cannot give equation here with the equation 10, and 9 I cannot give.

So, let us picked up that particular segment and see what happens here when omega equals omega n even an undamped system, when damping is introduced what happens, because bounded in undamped it becomes unbounded, but let us see what happens. Actually I want to capture this, because really infinity it is having some value, what is that equation. We can capture that is call dynamic amplification factor, it will amplify. Let us see how, how does it amplify, how much it amplifies and see. So, we want the equation for that any questions here for resonance, are actually not botheration, mathematically, yes, because you cannot capture this equation.

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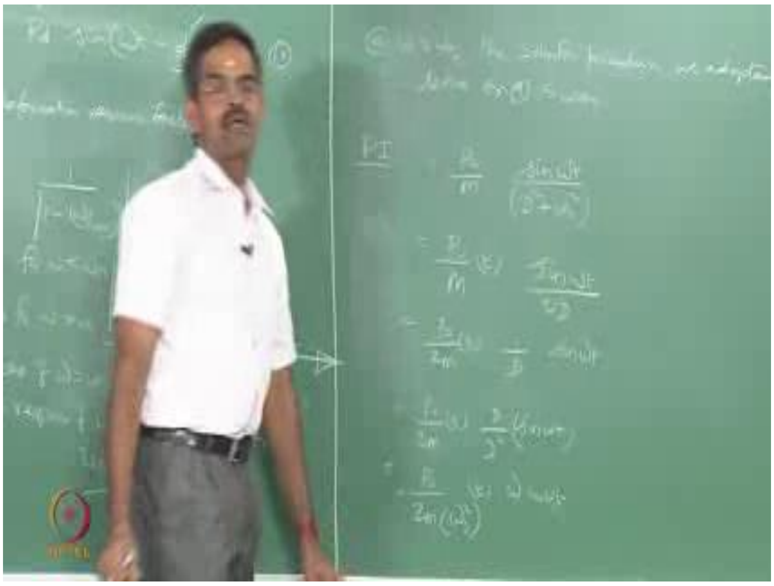

Now, at omega equals omega n the solution procedure we adopted to derive equation nine is wrong. So, let us take only the p I part, particular integral part, because that is where botheration is. The compliment function will not change, only the p I will change. So, I read the p I again back which is p 0 by m sin omega t by d square plus omega n, this p i. As per rule generally, to find the particular integral, I must put d square has minus omega square as long, as omega and omega are not equal the denominators in become 0, there was no problem in the earlier equation.

Now, it is becoming 0 because omega is equal omega and now. Now in that case I should say p 0 by m a variable multiplied, then I should say sin omega t by 2 d; that is how we generally do. So, let us say p 0 by 2 m t 1 by d sin omega t, either integrate r p 0 by 2 m t d by d square sin omega t, it is easy to understand. Put d square again as minus omega and square p 0 by 2 m omega n square t omega cos omega t, I put omega r or omega n it does not matter, because they are equal now in my argument.

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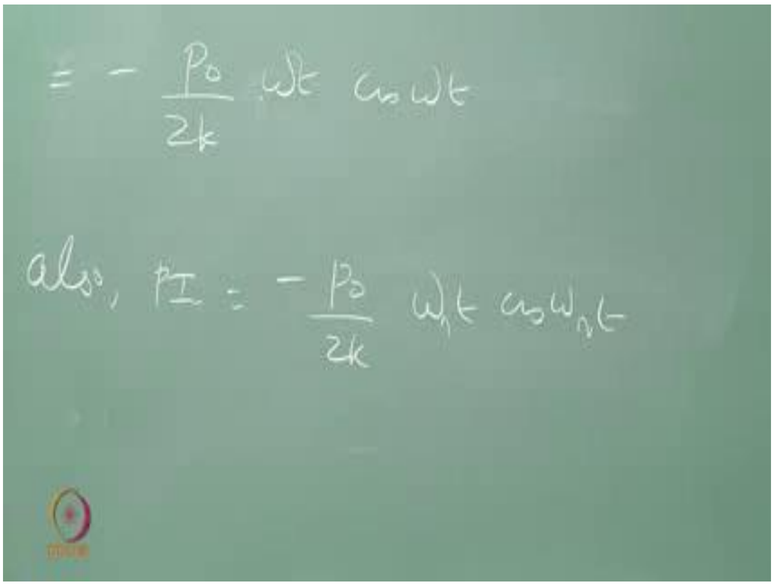

So, omega square is k by m. So, I get minus p 0 by 2 k t or let say omega t cos. I can also say p I is minus p 0 by two k omega n t cos omega n t is going to be the same actually, there is no main difference between these two, because we are looking at a case where omega's omega. Now let us look at whole x of t. So, I will call this equation number eleven.

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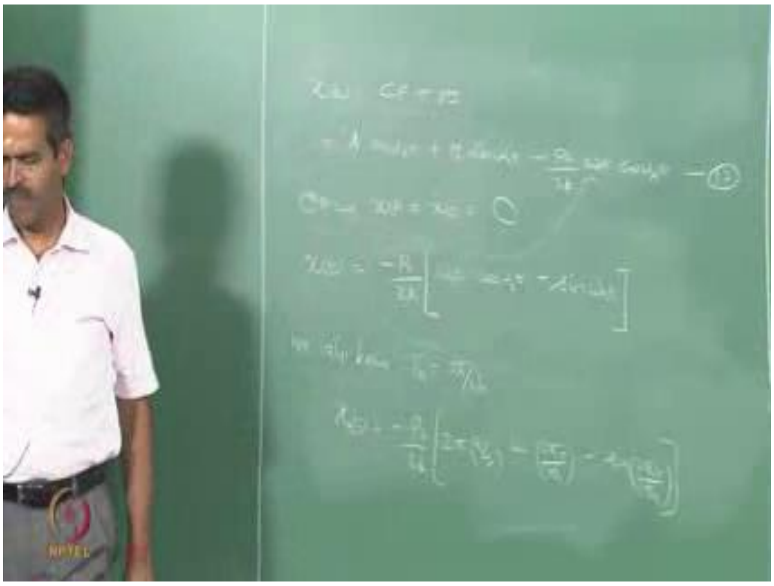

Now whole x of t will have the complimentary function plus particular integral, which is a cos omega n t plus b sin omega m t minus p naught by 2 k omega n t cos omega that is me equation for the response of undamped forced vibration at resonance. Now, let say at t is equal to 0, x 0 is equal to x dot 0 is equal to 0. I have not given either initial displacement or initial velocity, it is set to 0. So, can you find out a and b and tell you what is x of t. So, I get x of t as minus p 0 by 2 k omega n t cos omega n t minus sin omega n t. This term anyways there, and the one term comes from here, after substituting the initial conditions you will get this. Now we also know t n is 2 pi by omega m; therefore, we replace omega n by 2 pi a by t n. So, let say x of t now is minus p naught by 2 k 2 pi t by t n cos 2 pi t by t n minus sin 2 pi t by t, we already know that p 0 by k is call x static.

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So, let us rewrite this equation as the response ratio that is x t by x static s minus half that is how, it stays here, of 2 pi t by t n cos 2 pi t by t n minus sin 2 pi t by n. Now I want to plot this. So, the variable here if t by t n, I start giving different values. So, let us say vary t by t n to plot x of t versus x of t. So, let us quickly open a small table t by t n, the cos function value, only cos function value, the sin function value and the final value. Let us quickly understand starting from 0 half one three by two, and this is simple numbers. So, you will quickly fill up these values, you try to plot them, and I will show you the plot here, give me these values. So, this will become 0 minus pi by 2 pi sorry plus pi by 2 minus pi plus 3 pi by 2; that is how you will get you can check.

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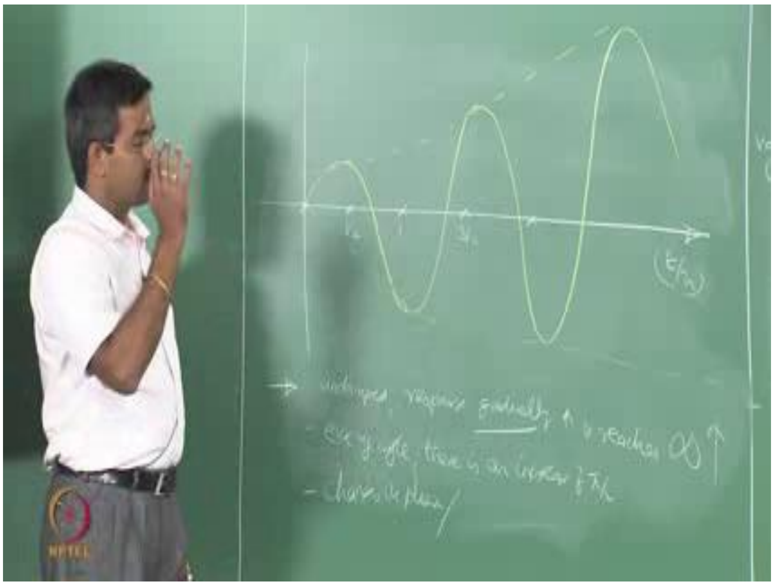

So, I will try to plot this t by t n a different value, so 0, half, 1, one and half, 2 and, so one half, one three by two and square. We already said that x 0 and x dot is 0 0. It has to start from 0 it is two from here also, it will be 0 and the first values plus pi by 2, the second value is minus pi, the third value is 3 pi by 2. So, there are some observations from this figure, the observations are the following, if the response or if the system is undamped the response gradually increases, and reaches the infinity. We go to infinity every cycle, there is an increase of pi by 2 0 pi by 2 pi by 2 pi pi 3 pi by 2 3 pi by 2 2 pi every cycle, there is an increase of pi by 2 see here. Thirdly, the response has an increase monotonously it changes, it is phase that is sin what does it mean. It imposes cyclic response positive negative positive negative.

So, there is a cyclic reverse in force, is also happening right, and we all know that a gradual build up which reaches infinity slowly steadily, but cyclic, the introduce fatigue. It is a problem for the system though are the infinity may not reach instantaneously; that is a very important point, even at resonance, even though system is not damped resonance will not reach immediately; that is the very important catch resonance build up will take time. So, because it is gradually increasing even at resonance, when omega equals omega n when the system is undamped still the growth is gradual, only the resonance does not shoot immediately. Now, one can easily understand what is the period for which a resonating wave of omega equals omega n will act on the structure; the typical wave periods in a given system.

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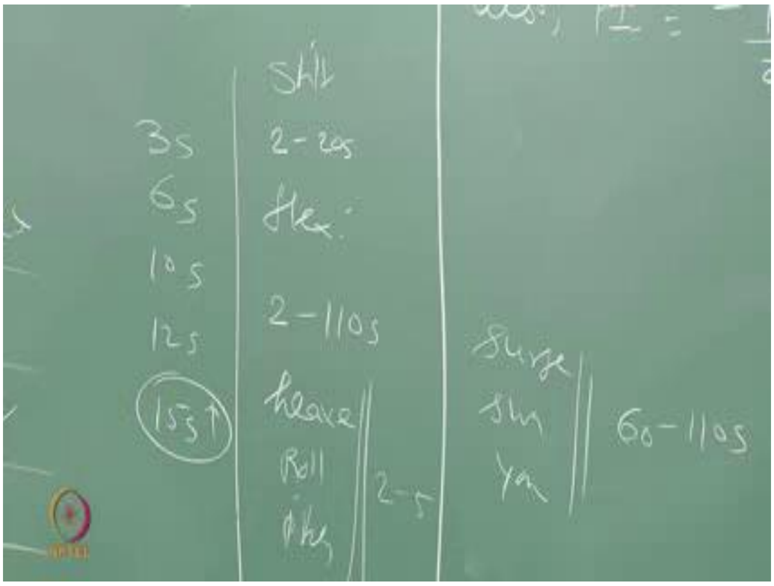

However in a given sea state that is, varying from 3 seconds, 6 seconds, 10 seconds, 12 seconds, 5 seconds is very high, a very abnormal waves. And what are the typical periods of my structural system. My structural system periods are if it is stiff, it varies anywhere from 2 to 20 seconds. If it flexible or compliment it varies anywhere from 2 to 110 seconds also. For example, a TLP, the stiff degrees like heap roll pitch vary anywhere from 2 to 5 seconds, and the soft degrees like surge, sway, and yaw will be very anywhere from 60 to 110 seconds, where there is no possibility of a wave of this period, to have counter action of omega equals omega and never happen.

If at all the wave has resonate with the system, it resonate only where the stiff degree. This stiff degree is already stiffed and enough will not respond. This is the design by how complain structures are evolved. So, therefore, we did not have to bother about even omega equal's omega n. We have taken care of that particular behavior in the design itself like a TLP, there is no problem, but if omega becomes omega n, if the system is undamped, if the growth is gradual which in infinity which is causing cyclic stresses or cyclic kind of behavior, this will impose in heap degree a heavy fatigue.

Heap degree is directly connected to tethers; this may result in pull out of tethers. Of course in case of stiff structures like jacket, gravity based structures etcetera, they are directly resting in this wave periods, and they are very serious in effective. There is because of this reason, in the new generation platforms people have designed the platform such a manner, that by design itself they do not enter into frequency of resonance domain at all.

Now these are the typical wave periods for which structures have designed in offshore. So, we do not know to really bother about that, but unfortunately even in t l p, we are certain domains of degrees of freedom which are stiff, they may resonate it. We have to take care of that, this is about undamped system. Now let us argue, put this argument in a damped system and see what happens. By the undamped system he said the response gets unbounded, it is infinitely growing like a bell, grows like a bell, it is infinity, gradually; any questions here? So, let us now extend this discussion for damped forced vibration and see what are.

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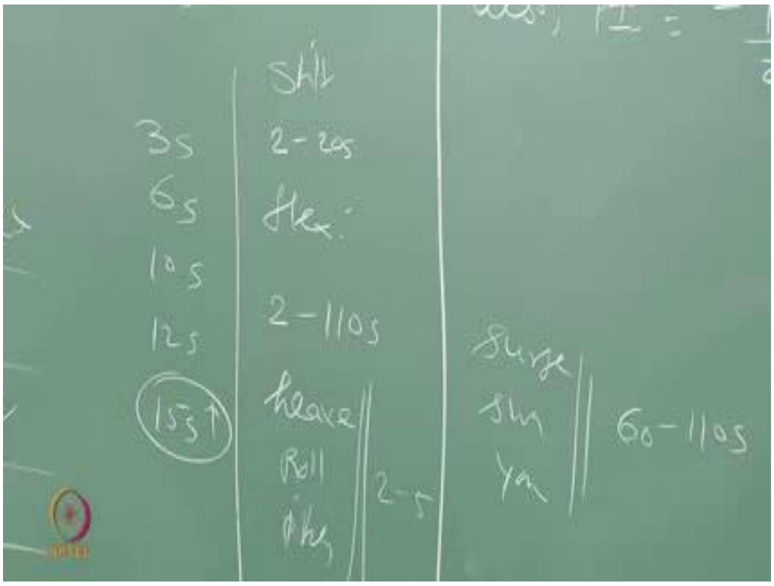

We already know the essential characteristics of dynamic system or the following mass, which is the inertia of component, stiffness which is a restoration component, damping with energy dissipation component, and a f of t this is external agency. We already said for the dynamic analysis to be prescribed for a given system, we must have sufficient mass representing the system, and the system should be designed to a very centering capability; therefore, restoration should be present. Therefore, no system can exist or can exist without mass in k. Only assumption we have freedom is, it would have c or not to have c, either have to f of t or not have f of t. Now in my case all are going to be present, because damped present force capability presence.

So, I have a full system now. So, the full system mathematically will look like this. You draw this carefully one part is here, and the other part is here, kindly do not connect these two. This is connected to the system is connected to the support. So, they came on dashing, that is why it is called a dash pot. So, I can write equation of motion for this from the Newton's law, we already know that. I am using viscous damping. We already know why we are using viscous damping. I am taking a very simple sinusoidal function, just for demonstration. We will solve other problems later in the second module; the RHA will be from the spectrum directly we will solve that. But now we will, for demonstration we will take up this sin omega t.

So, can we please do the solution for this complimentary function in a particular integral, can give me a x of t, because we have already done them in a many number of times. You should be able to give me a x of t. You re write this equation with omega n inside and so on and so forth and you want to form a auxiliary equation, then get a particular about complimentary function. Then let us at least give me the complimentary function first, what is x of t complimentary function, which already we know, we have already done this, e to the power of minus zeta omega n t a cos omega d t plus b sin omega d t, where omega d is call the damped to vibration frequency, where omega d is omega n 1 minus zeta square and z which is the ratio of damping, a critical damping, we have already know this. So, compliment of fraction is known to us. Let us talk about the particular integral, the second part of the solution.

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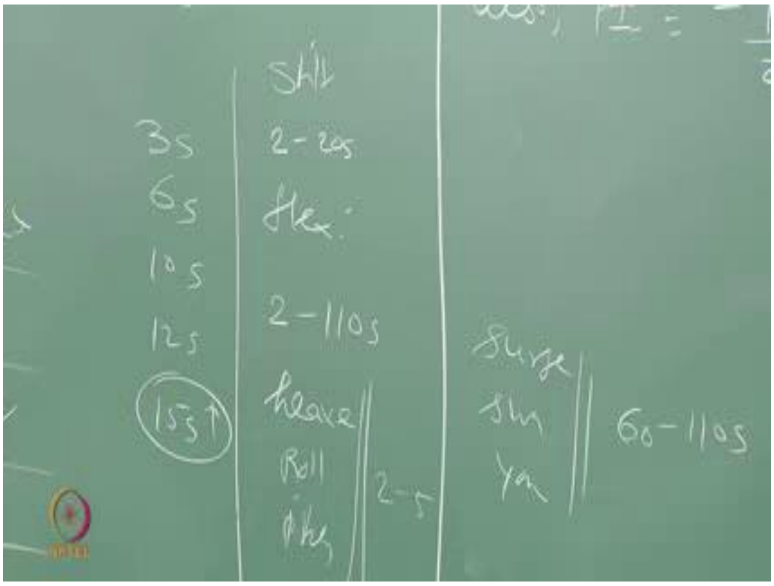

So, which is p naught sin omega t by m of d square plus c by m of d plus omega n square n. So, substitute d as minus of omega square, because it is a sinusoidal function. So, substituting d square as minus omega square p naught by 1 sin omega t by minus omega square plus c by m d plus omega n square. Let us take omega and square out p 0 by m 1 by omega n square. So, c by m is 2 zeta omega n. So, c by m is 2 zeta omega n. So, p naught by m omega square; c by m is two zeta omega n of d.

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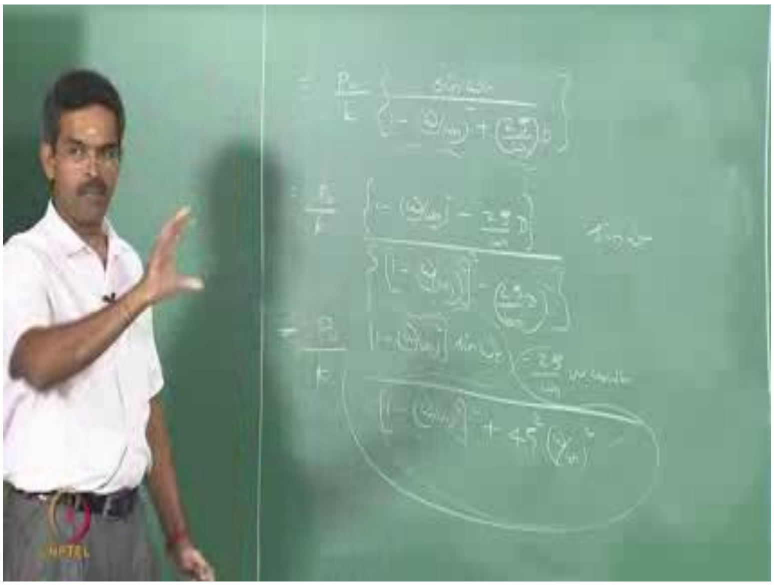

So, that is going to be p naught omega and k by m. So, by k sin omega t 1 minus omega by omega n in the whole square plus 2 zeta by omega n of d. So, consider these as one term, these are another term a plus b, multiply a conjugate here. So, let us say that p 0 by k 1 minus omega by omega n square minus 2 zeta omega n d of sin omega t by. So, I am getting this conjugate here, I get a square minus b square and so on.

So, let us substitute here, one minus omega by omega n by whole square, the whole square minus 2 zeta omega n d the whole square. Now sin omega t, let me multiply with this. Now I will take this term multiply sin omega d separately, I have d here therefore, I will differentiate this, I will get a d square here let substitute omega and minus omega square here, but omega square in this the minus will minus will become positive, I will manipulate this equation, and write like this.

You please simply this and get me what are the equation you will get. So, one minus omega by omega n whole square of sin omega t minus 2 zeta omega n omega cos omega t divided by 1 minus omega by omega in the whole square of whole square 4 zeta square omega, and square I put plus will here, because d square is minus omega square that minus in this minus will become positive. So, 4 zeta square omega by omega n whole square, is that ok. So, I have two terms specifically here; one is of course, p 0 became you (Refer Time: 36:33) static. I will take it to the left hand side; one term is this, the other term is this total, I will write them separately.

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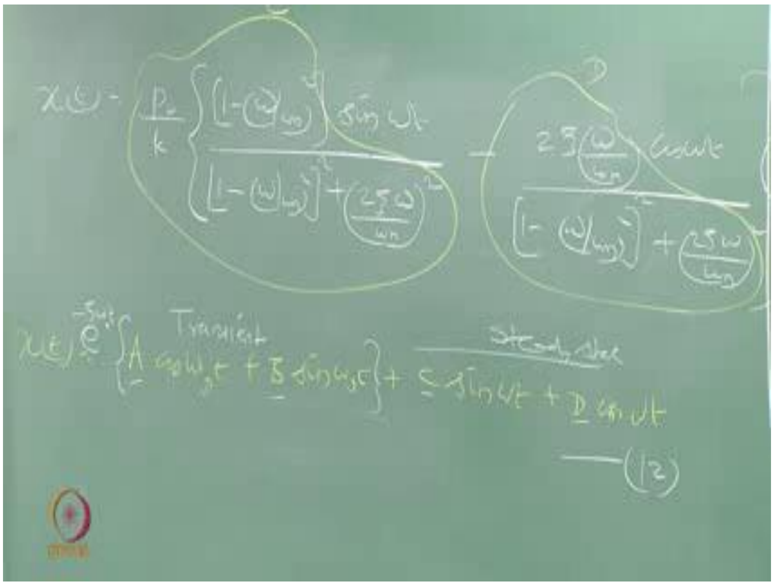

So, x of t is p 0 by k 1 minus omega by omega n whole square of sin omega t by 2 zeta omega by omega n the whole square. This is one and the same, minus 2 zeta omega by omega n of cos omega t by. You may ask me a why I am rating it separately, there is a reason for this, I will come to that. One can easily see from here, I moved this argument to the left hand side, it becomes x by static which is my factor. So, one component is a multiplier of sin other component is a multiplier of cosine.

I can express this graphically in an argon diagram, let me do that. So, for the time being I will call this as c, and of course, p naught by k of this as d, and my x of t is a cos omega d t plus b sin omega d t plus c sin omega t plus d cos omega, t then really I can write like this. Of course, there is a multiplier e to the power of minus zeta omega n. So, one can easily identify these are with initial conditions related to the initial condition of x 0 and x naught; therefore, this is transient, and they have no initial conditions, there is no x 0 and x naught 0 here. So, a steady state that is what we get this equation.

So, let me call this is equation number twelve, may be thirteen or twelve, because eleven is here, may be twelve or thirteen. Our argument does not stop here, because our argument is at resonance what happens to my system when it is damped, at resonance what happen to my system when it is undamped we captured, when we are able to capture that particular segment. So, here I must substitute in my p I that omega is equal to omega n and see what happens, that I will see in the next lecture this where we are interested, damped forced function we have an equation now, but here there is no resonance, there is no omega equal to omega m, because there are two omega separate forcing frequency and natural frequency, because omega d is a function of natural frequency in terms of zeta, we already know that, they are distinct

Now, I want to club these two and write an equation and see at resonance when it is damped, when it become unbounded I want to know, because at resonance where is undamped it is unbounded. Now here what is happening I would like to know, and if I introduced damping what actually happens to the system. is it response, is controlled, is it becoming 0, is it becoming negative, I will like to know this, mathematically. Physically we know when introducing damping they will no resonance infinitely coming up, it will controlled, because damping will decay the response we know that, but I want to see mathematically how it is coming, we will discuss in next lecture.