Dynamics of Ocean Structures
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Lecture – 08
Damped - Undamped forced vibration

So, in the last lecture we discussed about the damped free vibration model, where we say there are two kinds of damping; one is the coulomb damping which based on the frictional forces, or the viscous damping which is proportional to the velocity. We already said why we follow viscous damping for ocean structures.

Today, this lecture will compare and show you, why the coulomb damping is comparably inferior, with respect to that of viscous damping. So, let us complete this discussion. So, we have this x of t equation seven which we discuss in the last class. We found out from the complimentary function, for a given equation of motion, which is second on in a differential equation. We found out that it is a function of initial displacement of velocity. It depends on omega d, which is damped vibration frequency, which is mixture of cosine and sin function which you can try to plot.

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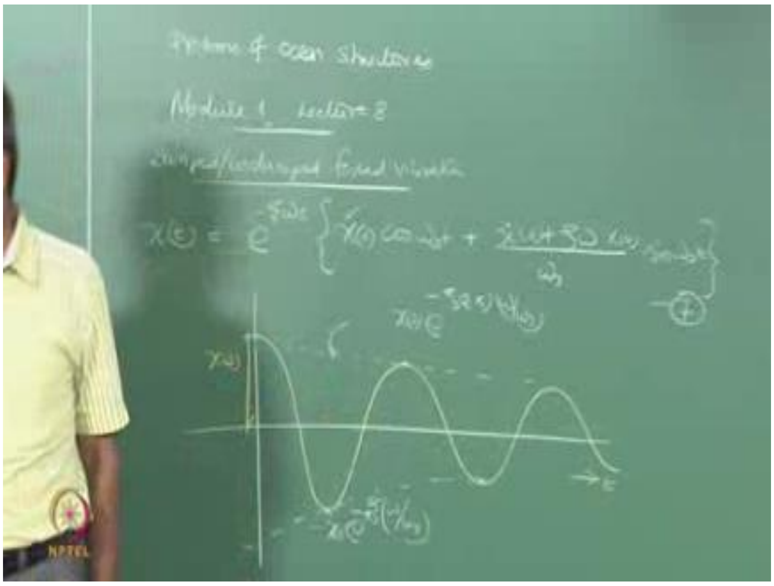

Now, the whole equation parenthesis multiplied by an exponential d k function, because this e power minus zeta. So, if we try to plot this for different values of t. Now the question is what value of t should I take? I must try to take a value of t in such a manner that the cosine and sin function will change their face. So; obviously, cosine sin function will change the phase at every pi interval, I started with 0, I said pi omega d. So, omega d will cancel only pi will remain, then 2 pi be omega d and so forth we got a plot like this, which is time. let us say this is my initial displacement which we know, which I am taking as x naught may be, let us say it would be like this, we are using this way. So, x naught and x naught naught, as the initial displacement given to the system, and we know these values, we know this value, we know this value, we know this value. Let us write down these values for our understanding. This value was minus x 0 e to the power of minus zeta omega b omega d, let us say pi. Let us says this value is positive x 0 e to the power of minus zeta 2 pi omega b omega d and so on.

So, if I call this as x 0 x minus 1 x 1 x minus 2 x 2 and so on so forth, because I say minus 1 is negative value plus 1 is positive value. We will always see that in everywhere the x naught the original displacement, is getting multiply by a d k function, and it ensures mathematically that the response will come to 0. We can also say the d k is exponential by connecting the top or the bottom, because I can also give a minus x naught, it will be an exponential d k function. We can always say it is a d k, because its negative, it is exponential because e power. So, it is an exponential d k and we all know mathematics the exponential d k is much faster in d k compare to linear function. We will show that the coulomb damping, compare to viscous damping is much more inferior, because the d k is going to be linear; whereas it is non-linear here, and it is faster, as he said in the beginning that we are interested in finding out the zeta value. Now one may ask me a question without knowing the zeta value how this was plotted.

This is actually response obtained from the system using a sensor, you may keep an accelerometer, or you can keep a displacement transducer on a moving body, give initial displacement to the body, the body will be keep on moving, acquired the data and plot it like this. So, let us say this is an exponentially obtained plot, but my interest is not this. My interest is to get zeta, or at what zeta this will become zero. My interest to get zeta which is the damping ratio, so let us see out to get zeta. This is where we stopped yesterday and this is how we plotted. I hope nobody has any questions to understanding this; we are able to get this. Now let us try to find out zeta. Now I will pick up two positive responses, two positive. I can also pick up two negative, let us pick up two positive responses.

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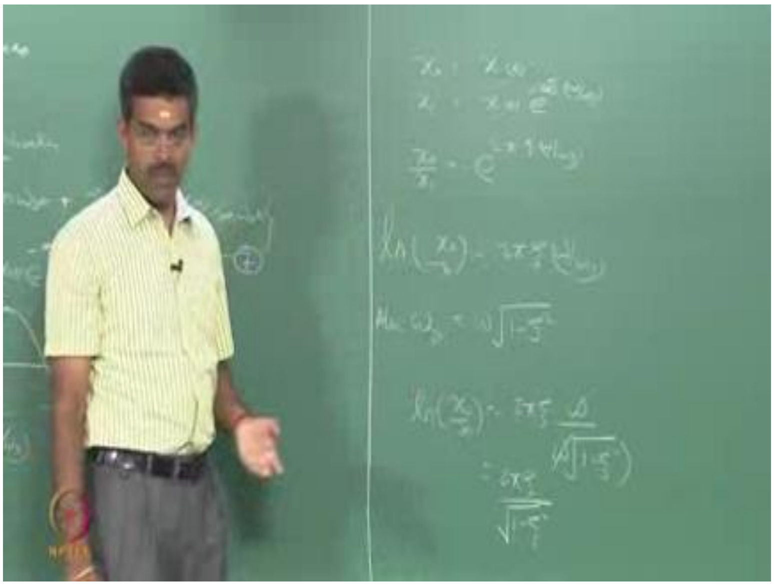

Let us say the first response is x naught; that is what I am picking up from here. The second positive response I call this as x 1, which is nothing, but an multiplier of x naught with e power of zeta omega by omega d of 2 pi, or let us write it here minus 2 pi z; that is the value here. Let me find a ratio of this; that is x 0 by x1. Now what I am trying to do here is the ratio of successive positive displacement; the ratio of successive positive displacement, why I am not doing at the successive displacement? Because successive displacement as a phase of change; one is positive, one is negative. So, the amplitude of this will be actually sum of these two.

So, I am not interested in that. I want to know what would be in a given cycle, because we all know the cycle starts from here, completes and ends here. So, in a given cycle what is the d k? So, I was lo at the, either the positive jumps or d k or the negatives jumps on the d k, I lo at the positive response - I lo at the ratio. I can also the x 1 by x naught why I am doing this, because we all know that is going to be positive number, no more than zero, because x naught is more than x 1 why x 1 is multiplier of x naught to the d k. When you do this have an advantage this negative goes to the numerator. So, it becomes simply e to the power of 2 pi zeta omega by omega d; that is an advantage why do this

Now, I can take logarithm on both sides. I can take napierian logarithm, which is natural logarithm on both. when we do that, it simply becomes 2 pi zeta omega by omega, because log e to the base is one. And we also know omega d, also omega d is omega root of 1 minus zeta square, substitute back here, I will get the natural logarithm of, is going to be 2 pi zeta omega 2 pi zeta by root of 1 minus zeta square.

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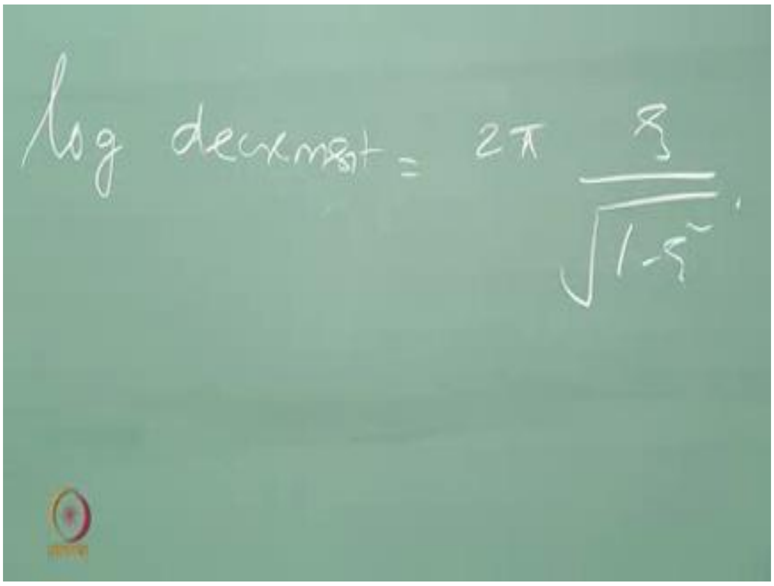

Now, the left hand side I call as logarithm decrement, why. this decrement because x 1 is less than x naught. So, decrement logarithm because this log value here. So, I will the simply way say this as; the logarithmic decrement, is simply 2 pi zeta by 1 minus zeta square.

Now, let us lo at this equation, if you really wanted to know the zeta value here, I must know the logarithm decrement of this. Now if you are slightly intelligent you can know; sir I have this plot obtained from the system directly; therefore, I know these values basically from the plot. Therefore, the left hand side of this equation is know to me actually. I can straight way find zeta from this. Now the question comes, do I have to compare only the successive positive? Can we compare this positive with nth positive, this negative with nth negative, if I do so what would be the significance of this in this equation.

The first question comes how this equation can be generalized, because this equation is only giving a decrement of successive positive and negative. So, I must bring this equation in general form. I can pick up any positive amplitude with that of the positive amplitude, and include that value variable in the equation; that is number one. Number two will it be different if I pick up, this peak with an nth peak and find zeta. This peak with successive peak and find zeta, will it differ, because that is the ambiguous question now. If it is so then how long should I take the value? Should I take the value till reaches zero, then all this ambiguous questions as to be answered? So, for answering these questions, this equation must be generalized. Is it clear? I want to know the generalized equation.

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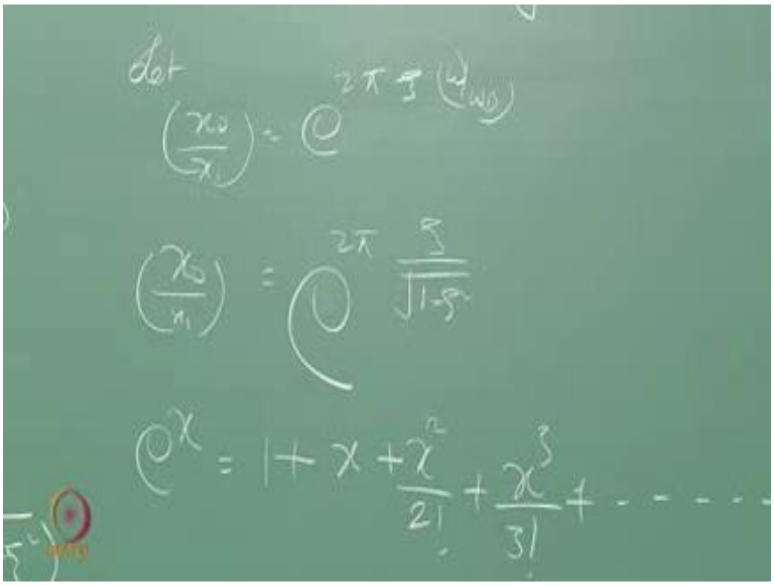

Now, let us pick up a generalize equation, but before that let us try to see. Let x 0 by x 1 b e power 2 pi zeta omega by omega we know this, or e 2 pi zeta 1 minus zeta. Now let us quickly get into some exponential series, if we have e power x, how do we expand this; let x is the variable there. Let me write it very clearly this is e power, there is no multiplier here, and all are exponential power. So, if I do this, 1 plus x x square by factorial 2, this is how it goes. Let me use this series of expansion here, where x is this argument, now expand this.

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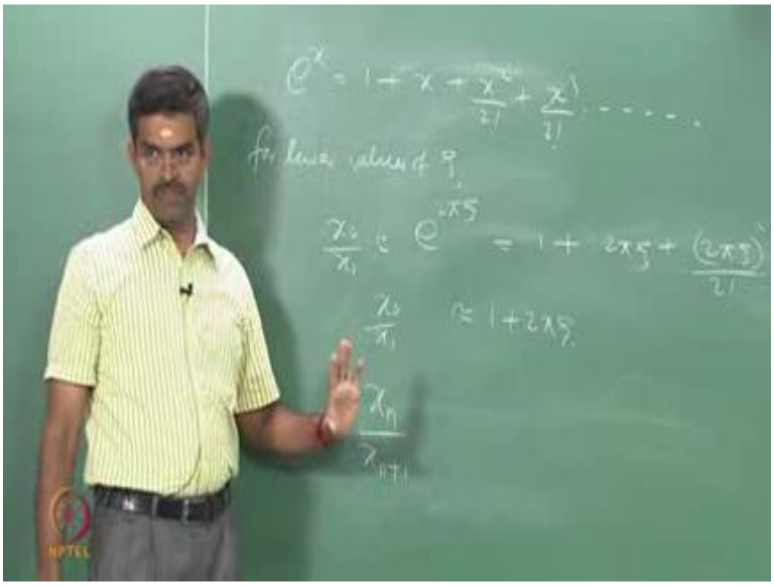

So, we know e power x is 1 plus x square by 2 factorial x cube by 3 factorial etcetera. Now, for lower values of damper damping zeta, because zeta is the damping ratio; we already say it is 2 percent or 5 percent which is 0.02 or 0.05, they are very low. In that case the denominator will tend to become unity.

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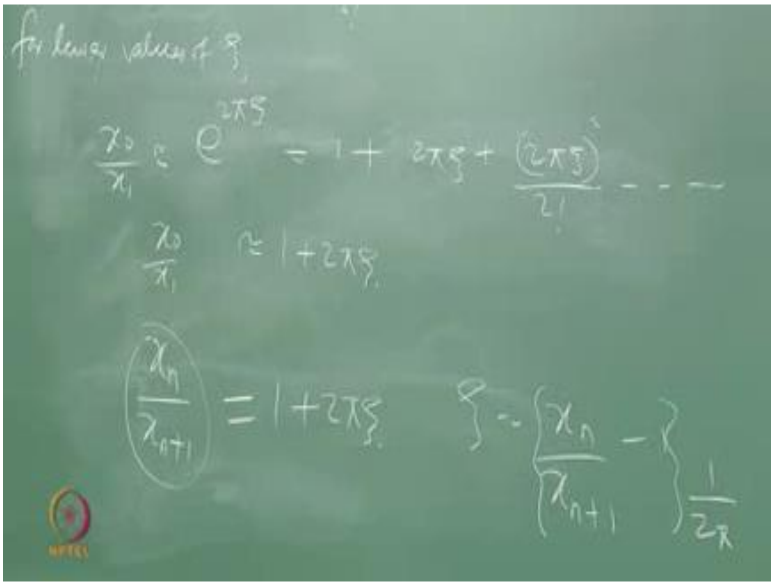

So, I can always approximate x naught by x 1 as e power 2 pi zeta. I can approximate this. Therefore, the exponential function - x series can be 1 plus 2 pi zeta plus 2 pi zeta whole square by 2 factorial and so on. Now for lower value zeta, the higher powers of zeta can be neglected, because the value of zeta is very low; therefore, this can be still approximated as 1 plus 2 pi zeta.

Now I replace this equation on the left hand side as x n by x n plus 1. Please understand here I am only still taking the ratio of successive amplitudes. I started with x 0 went to x 1, I can also start at x n and go to n plus 1, still this argument is as same qualitatively as this; therefore, I write this as 1 plus 2 pi zeta, because argument is to find out zeta now it is very easy. If I know this value I can easily find zeta as, I wanted zeta I will get zeta now. Now again the answer is, if I do not want to lo at the successive amplitudes, I want to lo at mth amplitude, how we generalize this equation get zeta. Now let us draw a curve and pick up the mth amplitude, and generalize this equation quickly.

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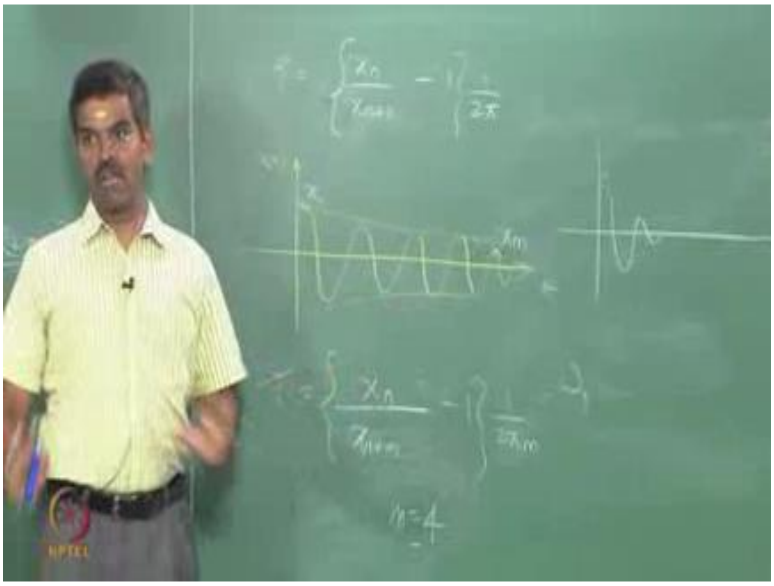

So, write the equation here zeta is x n x n plus 1 minus 1 of 1 by 2 pi. Let us say any plot is like this, which is x sorry which is t which is response here which is x of t, it goes like this. Please believe me that they are exponential decaying. Now I have this value with me, let me call this as x 1. I have this value with me which x m. So, I must say zeta as a generalized form as x 1 by x 1 plus m, or let us say x n you can start from anywhere, let it be x n this x m x n n plus m minus 1 1 by 2 pi of m. Now in this example what be the number m. So, lead this peak start counting from the successive positive peaks; 1 2 3 4. So, m in this example will be four. So, x 1 by x 5 minus 1 1 by 2 pi m 4 will give the zeta, is very interesting for you to know that, whether you picked up the successive amplitudes or pick up mth amplitude, zeta will not change significantly, do not change

But it is very difficult for you to get a system which is having a d k with a very long cycle. This indicates system is highly elastic. Generally in offshore structures systems being very stiff and rigid, you will not even get more than two cycle, because they are very stiff, they will come to response dead end of the response very quickly, because offshore structures are designed, to come to response which is called a re centering capability, as quick as possible. So, you cannot expect a large number of cycles for to come to response there, it will come quickly therefore, you will have no opportunity except using the successive peaks, in finding out zeta.

Therefore it is very clear and evident in the literature, that even if you use zeta with mth peaks decaying qualitatively, zeta will not change significantly; that is a property of the exponential d k actually, mathematically not change much. So, we can find out this, simply this peak is known, this peak is known, I get the ratio. Now I simply subtract this and use equation, and find the value of zeta in percentage. This will be a number multiplied by hundred you get zeta. So, this how zeta is plot. Now with this zeta if you plot, you must get back either this equation, or this figure or this figure from the original function which is equation seven; that is how it is done, any doubt here. So, we discussed damped free vibration system.

Now, one can ask me a question if the vibration is free, where the damping source is coming from. Damping source can come from hydro dynamic, it can come from air. This also the system is enable to design to a re centering configuration therefore, damping is present in material, in structural member, in environment, there are very sources. So, you will not be able to get the damping of difference sources separately from this equation. We will give you the overall damping of the structure. If you really wanted to know from this what are the component of hydro dynamic, what is the component of material, what is the component of material degradation, and member degradation you will never get, that you will have to separate analysis for this, we can do that, but it this equation is highly generalized, which we are interested in. We wanted to actually know what is that ratio which brings the damping or structure to rest, we are interested in that.

Within that what is the contribution of different elements will be of intrinsic research interest, but in general dynamics we are interested to know, is it becoming into zero, yes, how many cycles known, what is the value, we know the value. So, we are answering all the questions almost, which are unknown in the given equation by zeta is 1 of the unknown in the given equation, because if you do not know zeta you cannot find omega d. Omega d is a function of z actually, you cannot find this. So, you do not know what is the damp vibration frequency at which system is resonating or oscillating, we will not know, so we need zeta.

So, what generally people do is, they do not conduct experiments to get zeta, they assume zeta, which is recommended in the structural systems as 2 to 5 percent. You can take any value, somebody ask why have we taken 2.5, you can give a justification, provided you can conduct an experiment, and show zeta close to 3 I have taken 2.5 and so on so forth.

Remember it is a structural characteristic which is derived from an experiment. Therefore, this will have all errors including the experimental boundary conditions; experimental conducts, scale value, material property, because concrete can be damped very quickly, where as elastic cannot be and so on so forth. Be very careful about making the statement when you give a research proposal. This will include all in 1, you cannot find separately from this equation. There are methods which we will not touch in this course. We stop here now for the moment, and then will move on to next topic. So, far we have discussed about damp, forced, I mean free vibration. Now if the force vibration is present what would be the change, mathematically what would be the change?

So, before we answer this, let us try to ask why coulomb damping is not effective in offshore structures. We have seen the viscous damping is exponential decaying therefore, it is effective. Let us talk about coulomb damping; coulomb damping is a damping developed between two surfaces, because of friction, a typical plot of coulomb damping will lo like this.

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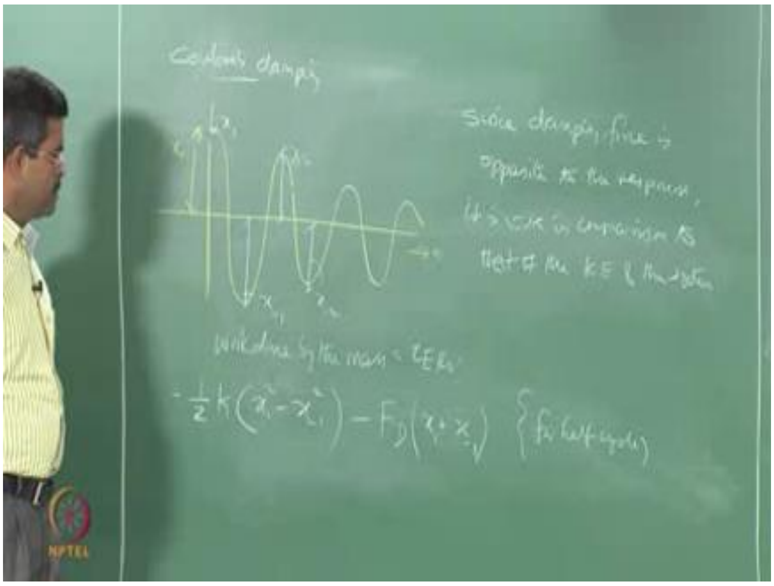

Now, since damping force is opposite to the response generally, there are negative damping also. Generally the damping force will be opposite to the research, because it has to bring down the response to the equilibrium state or neutral value. So, it is negative in comparison to that of the kinetic energy of the system Now let us see what are the total energy present in the system, in a frictional damping like this. We all know that the work done by the mass, should amount to zero. So, let us say what is the work done by this mass; 1 is half k b square which is the kinetic energy, where we do not have velocity, initial velocity is zero. So, we do not have the kinetic energy, because velocity is zero we have a restoring force which k x square k is spring stiffness. Therefore, I write here as x 1 minus x minus 1 square; that is if I call this as x 1, I will call this or this value let us say as x minus 1 I call this value as x 2 I call this value as x minus 2 and so on so forth for the legend.

This is the restoring force minus the damping force, which is arriving from the damping which is negative, because it is opposite to the response. Now this will be the total displacement in a half cycle, I am doing only the half cycle. So, I should say x 1 minus of x 1 minus of minus so it becomes plus, doing for the total amplitude for a half cycle. why it is half cycle, I am using only x 1 and x minus 1. I am not using x 1 and x 2, I used only x minus 1, so its half cycle, which will be of course, set to zero. Now we simplify this, which is given as half k which will become half.

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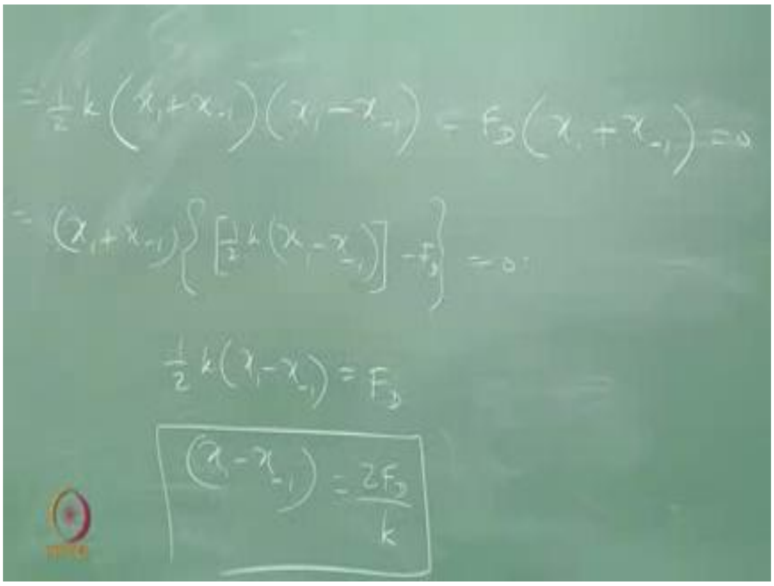

Now; obviously, this cannot be set to zero, because you have a value. So, I will set this to zero. So, x 1 minus x minus 1 is twice f d by k which is linear for half a cycle.

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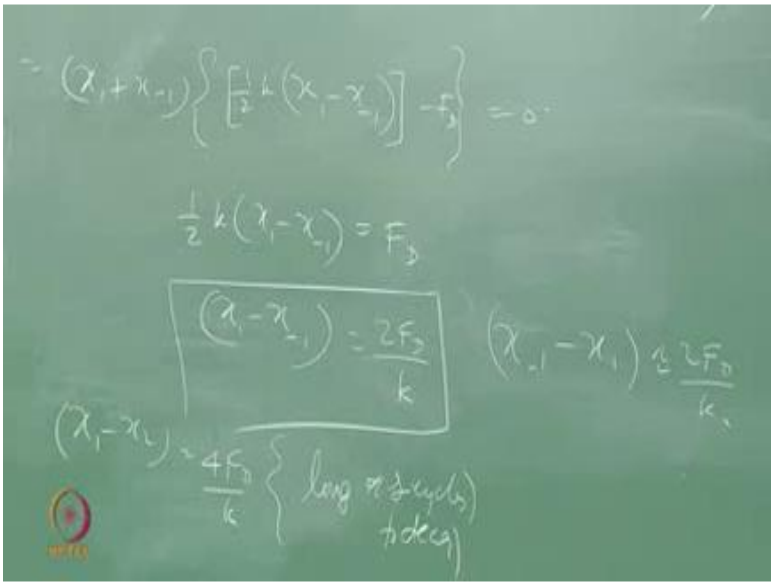

Now, I can extend this as x minus 1, minus of x 1 that is the second half, is also going to be similarly 2 f d by k. Therefore, x 1 minus x 2 this decay which is this value, before f d by k, 2 f d by k decay here, and further 2 f d by decay here. So, therefore, if you really wanted to know that this should be set to zero, you must do that many numbers of cycles to become this zero, because the decay between the successive cycles is fixed as a linear function. So, this will take a long number of cycles to decay. It is because of this reason coulomb damping or frictional base damping, is not admitted in offshore structures.

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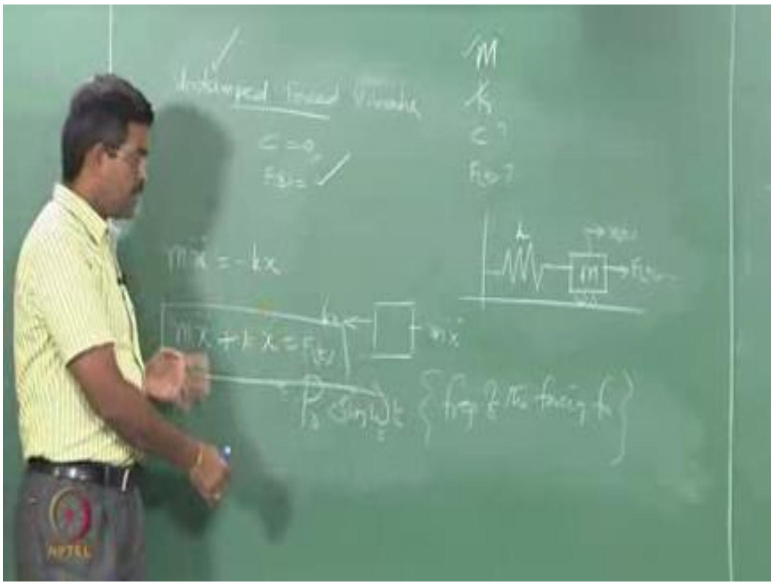

Let us move on to un-damped forced vibration. We already know the essential characteristics of m k c and f of t. The system without m and k is not possible. There always (Refer Time: 28:18) in every system, because system should have a restoring force that is the re centering capability in the design. System should have a mass. If the system does not have a mass, even though the acceleration is very large initial force is low we do not have to do dynamic analysis. So, mass and k are always present in a system, if you really want to dynamic analysis. C may or may not be there; f of t may or may not be there. In this case we say undamped force. So, c is set to 0 f of t is present. So, let us draw the system, single degree freedom system, idealized spring mass model, there is no dash pot, because c is 0.

So, undamped I can draw Newton's motions for this which k x which is m x double dot. So, m x double dot will be equal to minus k x, which should be force into acceleration.

So, m x double dot plus k x is 0, but I have a force additional, but in this case is going to be some f of t. I will take this function as some amplitude with some frequency, where omega is called frequency of the forcing function. So, now, this has got two solutions the right hand side is not zero. Therefore, you got two answers here for this problem; one is the complimentary function, other is the particular integral, let us do both for this and try to sum them up and see how the solution los like.

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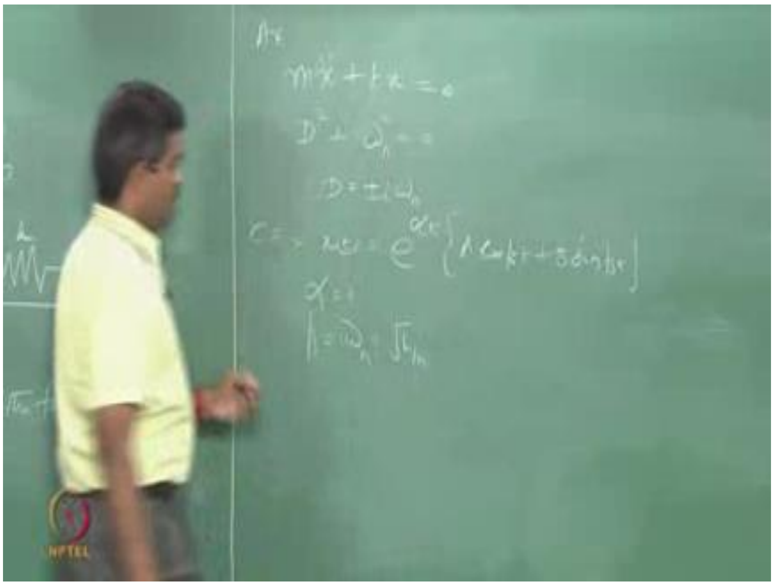

So, let us say m x double dot plus k x is 0; that is my auxiliary equation I say d square plus omega square, because k by m is omega of x 0. So, d is plus or minus omega i. So, complimentary function can be x of t, which is e power alpha t a cos beta t plus b sin beta t, where alpha is 0 in this case beta is omega. Now to clarify, there are two omegas here; one is the omega which is coming from the system, with a natural frequency which is nothing, but root of k by n which is appearing from here. There is another omega which is the forcing functional frequency. We have to differentiate these two for our nomenclature. I will substitute all these values as omega n; n indicates natural frequency which is the system property, omega indicates forcing frequency which is not a system property. So, just to differentiate in a mathematical, otherwise you will be confused.

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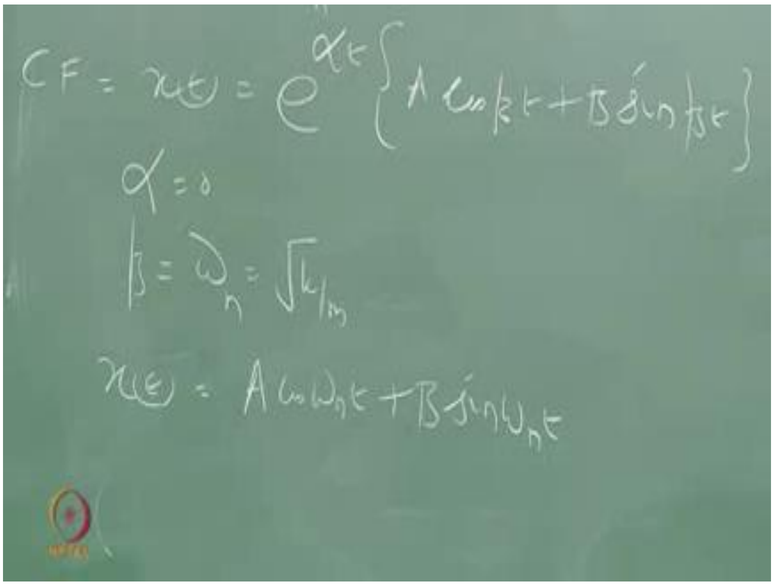

Now, I write complimentary function x of t is a cos omega n t plus b sin omega n t. There are two unknowns a and b I can evaluate them as initial condition.

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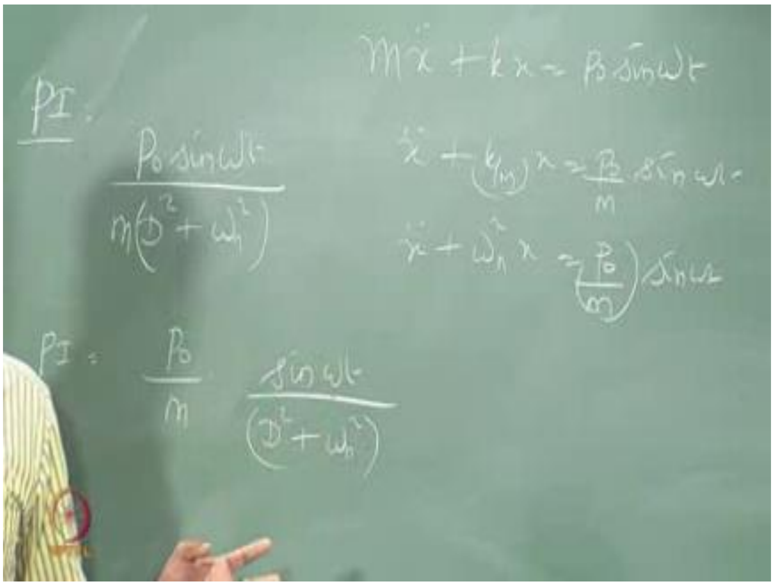

Now, let us go to the second part of the solution which is the particular integral. So, particular integral with the second part of the solution, the right hand is p 0 sin omega t by d square plus omega n square; that is what I have in the equation, and you will appreciate that, the equation of motion is m x double dot plus k x is p naught sin omega t, I divide this by m I get a double dot plus k by m of x is p 0 by m of sin omega t. I say this is x double dot plus omega square x is p naught by m sin omega t. So, I should not write p naught here, I must say p naught by m, because m divided here. So, let us say p I is going to be p naught by m sin omega t; that is the argument d square plus omega square.

We all know to solve this, because if I have a function which is here, what I should do is, I must say d square as minus omega square as long as the denominator does not become zero I can solve this, is or not. If it become zero you differentiate the numerator and keep on doing it until the denominator should not become zero; that is the rule, to solve a particular integral. I am going to do the same thing here p naught by m sin omega t by minus omega square plus omega n square.

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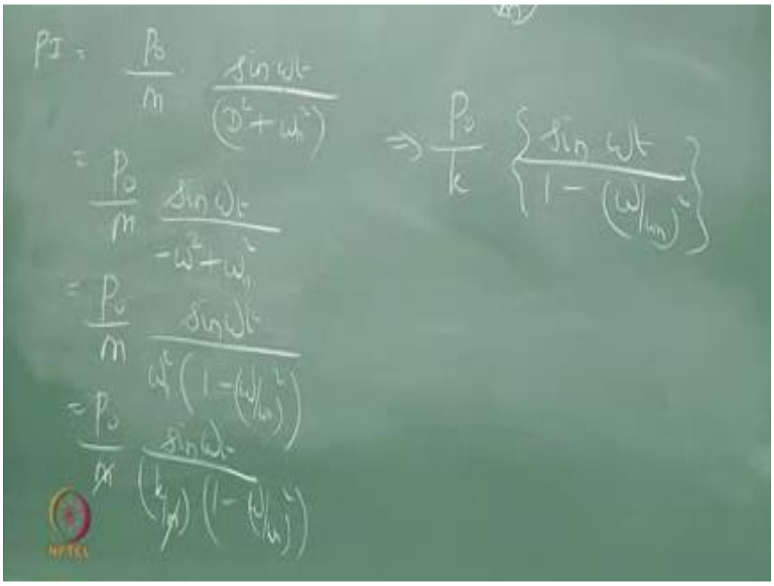

Now, we really do not know whether it becomes zero or not, because we do not know the values of omega and omega n, we can leave it as it is. I re-write this slightly in a different form which I say p naught by m sin omega t, I will take omega n square out 1 minus omega by omega n whole square. Omega and square is k by m we already know that, its k by m is omega n square. So, I substitute here p naught by m sin omega t k by m 1 minus omega by omega n square, I go away it becomes p naught by k sin omega t by 1 minus omega by omega n the whole square.

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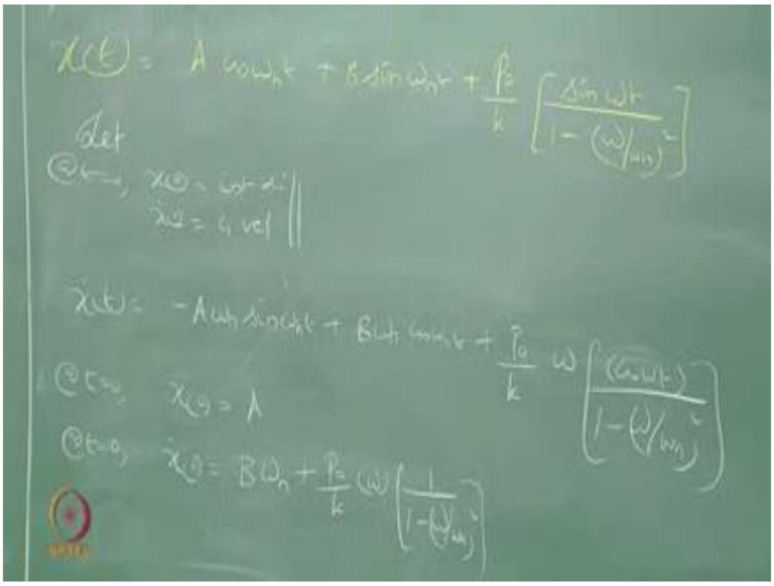

So, therefore, my x of t I expand this and write it here, my x of t is going to be which is the response of the function, which is undamped forced vibration will be a cos omega n t plus b sin omega n t plus p naught by k sin omega t by 1 minus omega by omega n square. I want to eliminate a and b as usual. So, at t is equal to 0 x 0 is the initial displacement and x dot 0 is the initial velocity let. So, let us differentiate this x dot of t minus a omega n sin omega n t plus b omega n cos omega n t plus p naught by k omega cos omega t 1 minus omega by omega n square.

So, let us substitute this condition in these two equations. So, at t is equal to 0 this term will become simply these two terms will go away, because it is sin function? So, I will get x 0 as a. Now at t is equal to 0 substituting in the second equation this term will go away. So, will become x dot 0 as b omega n plus p naught by k omega 1 minus omega y omega n the whole square; therefore, I can easily find b as x naught 0 by the whole argument minus of this and so on.

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Let us say I have the value of b with me. Let me write down the full function. So, x of t after getting a and b will be x 0 cos omega n t plus x dot 0 by omega n. I was splitting these two terms separately; minus p 0 by k, there is a omega and multiplier here, so when you take it this way this will get divided here. So, I should say omega by omega n, because this omega n we get divided here, because this a sum here divided by 1 minus omega by omega n square of sin omega n t plus p 0 by k omega cos omega t by 1 minus omega by omega n the whole square sin omega t, even here also sin omega, sorry it should be sin omega p 0 by k sin omega t by 1 minus omega by n square, actually picked up this term from the velocity. So, let us check x naught cos omega n t x naught by 0 omega n minus p naught by k. The argument of sin omega n t is for the whole; because that is b sin omega n t b is a value we got from here plus p naught by k sin omega t so on.

Let us discuss about this equation, I call the equation of eight. There are some interesting discussions on eight, let us see what is that. The equation eight has got two terms, why, there are three terms, but I will put them to some two terms, because one term is function of natural frequency, the other term is not a function of natural frequency. You may say sir this term also has omega n here; that is what ratio, but ultimately the multiply is only omega.

So, two terms one term are the first term, is a function of natural frequency of the system, the second term is not a function of natural frequency of the system; that is first difference or first observation I have. The second observation, now first term depends on the initial conditions of x naught and x dot naught, the second term does not a initial condition dependence, so dependence of initial condition. Suppose I say initially x naught and x dot naught are 0, they will not exist, but this term will always exist; the one which is dependent on initial condition and a function of frequency natural system is call transient vibration.

The one which does not depend on natural frequency of system, and is not dependence on initial condition is called steady state. Now the response has got two components; transient response and steady state response. Now, one may ask me question why this response is called steady state. This response called steady state, because this response will always exist in the given system. This will only diminish when there is no p naught, where p naught is the amplitude of the forcing frequency or the forcing function. So, if you do not have a forced vibration, this term will not be there, which was not there in equation six and equation four respectively. It is here because you have got a forced vibration given to the system which is (Refer Time: 43:22) which is always present; therefore, this response will always represent that is why it is called steady state.

Now, interestingly in this equation eight, if the forcing frequency becomes matched with the natural frequency, the function will be in infinity, but I cannot handle that in the mathematical solution here, because lo at p I sin omega t by b square plus omega n square is the p I if I say this is omega n. I cannot solve the p I in the same form as I solved in this equation. Therefore, the solution drawn for omega not equal omega n which is given by equation eight, cannot be substituted by simply saying this is omega n this becomes one infinity and so on: no, cannot be done, because the procedure for solving a particular integral when these two are equal, is different from this, mathematically we cannot do that.

Therefore, to get back to the same equation where (Refer Time: 44:41) set in, I lo at this whole concept slightly in a different form call dynamic amplification factor; that is how the in structural dynamics the whole concept of frequency is expressed as a ratio of beta, where beta is the ratio of the frequencies. Let us see what happens beta becomes one form the same equation. Strictly speaking if beta becomes one the equation should not give me an answer, because it will become infinity, steady state response will become infinity, is it not. Should not give an answer right, let us see how.

Do you have any questions here? So, in this lecture we are discussed about two things; one we have compared coulomb damping with that of the frictional damping or viscous damping, and we understood why coulomb damping or frictional based damping model are not effective for offshore structures though, there are deception of energy or damping arising, because of friction of between two particles or two members in any joint in offshore structure, because the decay is linear take long number of cycles to decay. Whereas, in instantaneous response re centering capability is required in offshore structure; therefore, in reality when they are designed like that, we must adopt a damping model which is proportionate to the real behavior, which is coming from discuss damping, number one. Number two we also understood from this lecture how to get zeta if I have an exponential plot of a decay of response, how to get zeta for n number of peaks.

The third case you already understood, in a given forcing function available in a given system though damping may not be present undamped system. There are two kinds of classically explicit response coming out from the total response; one is a transient response, one is a steady state. Transient has got specific characteristics; steady state has got specific characteristics. Steady state will always be present that is why it is called steady state, transient will not. Now, one may ask me a question here is a transient response not important, why we actually handle this, can we only deal alone with steady state. The answer is no, because in offshore structure unfortunately talk about fluid structure interaction, the transient responses like springing and ringing, play a very major role to cause instability.

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So, transient response also becomes important in offshore structures; friends' offshore structure is the only one classical system, which is where transient response is considered to be important. There is no other system except offshore structural system, where transient response is taken as a steady in dynamics. People generally ignore this. Here it is important, because springing ringing will talk about this in module two, I will tell you how this becomes important, which can cause instability to your platform, even though transient steady state response may not be important. So, very important we cannot ignore this. So, generally people neglect this as it is not important and so on and so forth that is the case in offshore structures. So, we have learned this summary now, we will close the lecture now. If you have any doubts kindly post it to announcement form in NPTEL.

Thank you.