Dynamics of Ocean Structures
Prof. Srinivasan Chandrasekaran
Department of Ocean Engineering
Indian Institute of Technology, Madras

Lecture – 07
Vibration of SDOF Systems

In the last lecture we discussed about the undamped free variation system. We already said the essential characteristics of a given system could be the mass, could be the restoring force, apostrophe stiffness matrix and the resistance offered by damping C and the external agency which is F of t, out of which we already said M and K must be present finding system, C and F of t may not be present, so if C is not present it was considered to be undamped system.

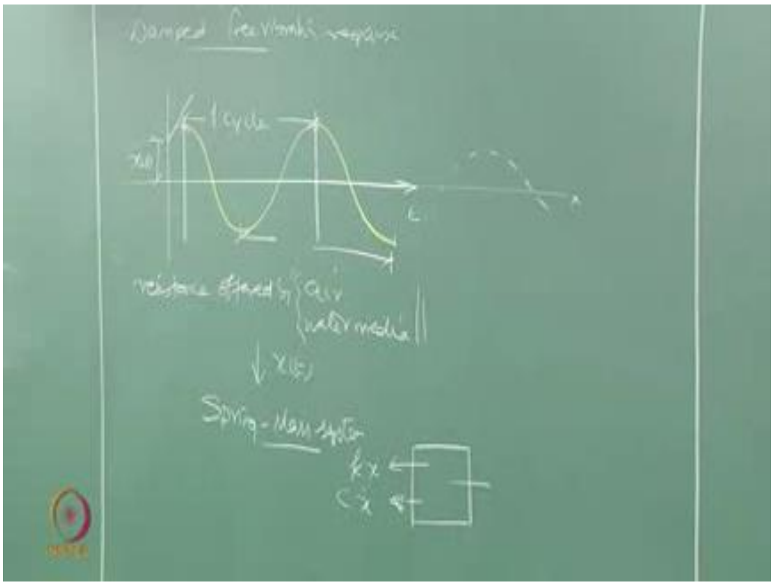
(Refer Slide Time: 00:19)

In the last lecture we discussed about, so this was the result what we had in the last lecture - where x 0 and x 0 by the initial conditions x 0 is a displacement at t is equal to 0 and x 0 is the velocity at t is equal to 0, which were the initial conditions which enabled us to find out the constants c 1 and c 2 in the solution of ordinary differential equation which we solved and got the response. This was a response which we got in the last lecture.

(Refer Slide Time: 01:41)

Then move on to one step further, we said now let us say mass be present, K be present, C be present, but F of t be absent.

(Refer Slide Time: 01:54)

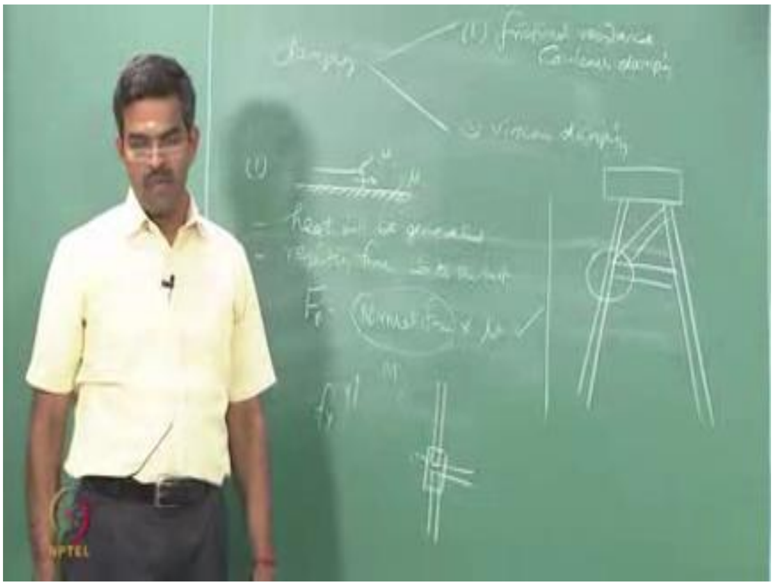


So, we will talk about Damped Free vibration Response. Now let us try to plot this and you will see and as I would say this is my x 0. So, it has got an initial slope. So, x dot 0 is a derivative of that is also present is not 0, and you will notice interesting phenomena in this response that this response is not actually decaying at all. The amplitude will be constant it will be continued for infinite number of cycles, but in reality it is not so. Any resistance offered by air let us say by water media, in case of ocean structures, will try to reduce this response at least after n number of cycles - one cycles is from one peak to the next positive peak this is one cycle. It can also be from one negative peak to the next negative peak this is also one side, anyway.

So, the amplitude will not remain constant; will be keeping on decaying and obviously, the response come to 0. So, there is resistance offered by the media which is not captured in this equation because we have not included that resistance which is offered by the media. Now, we will capture this in this model. So, my equation of motion obviously for a simple spring mass system (Refer Time: 04:23) diagram, let us say this is my mass in the acceleration, the restoring forces is kx and one more restoring component because this is the resistance component offered which will be cx star.

Now the question is what are the source of damping, how many types of damping are there. Damping can come from two sources – one, it can be from the frictional resistance which is called as coulomb damping, the other one is coming from the hydro dynamic damping which is viscous damping.

(Refer Slide Time: 05:02)



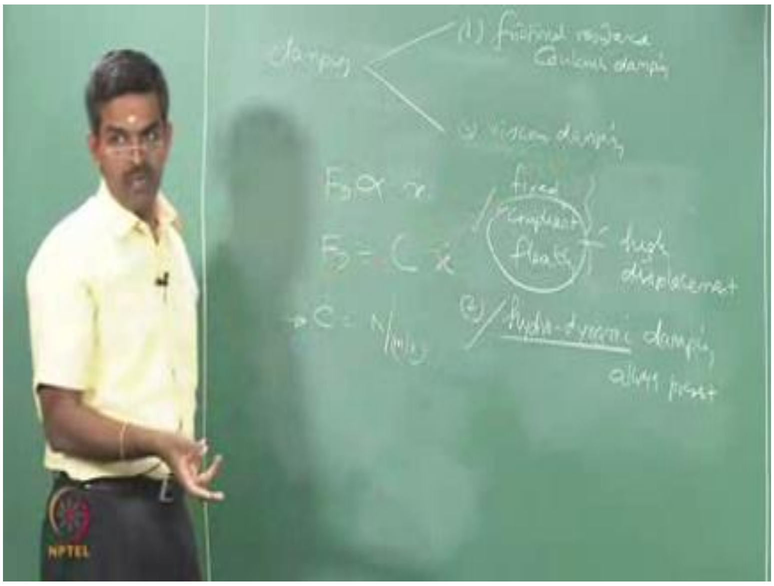
Let us talk about the first case which is the frictional resistance. We know that any two body or let us say one body is moving against another surface, heat will be generated and the resisting force will be normal to the surface, so that can be, the resisting force can be the normal force multiplied by coefficient of friction. This is valid in our case because the normal force, which is nothing the inertia or the mass is very high therefore, this distance force will be very high.

The only difficulty in this case is the dissipation of energy or the dissipation of the response or the decay of the response in this case will be very slow. So, once we derive the equation of motion for this and solve this, I will show you a comparison between the viscous damping part and the coulomb damping part, and I will show you why in offshore structures people follow viscous damping model rather than the coulomb damping model.

So, valid example - now where are we apply, interestingly I have a top side detail, maybe I have a template structure, may be these are the lacing and battens; let us look at this connection, I have one member, I have another member which is connected with an y joint let us say, now these can be either welded or bolted etcetera. So, there are two material involved now. You will all agree that the material strength or the characteristic grade of steel used in this connection is far higher, compare to the characteristic grade of steel used for the members because joints or connection are subjected to high stress concentration factor in offshore structures.

When two different material of different yield strength grades or attached or connected when they rub against each other they will offer a resistance. So, this is a very valid example model applied in offshore structures where they have connections, but in a new generation structures these connections are essentially avoided. Let us take for example a TLP there is no rigid connection in a TLP of this order. So, therefore, this model or this proposal of using coulomb damping for expressing (Refer Time: 08:38) response may not become valid for all kinds of structural models then, what is a valid model? The second model applied or available is viscous damping.

(Refer Slide Time: 08:56)



In this case, the damping force is proportional to the velocity. One can ask me a fundamental question here, why the damping force proportion to velocity will be always present in an offshore structure. We all know whether it is fixed or compliant, considering these two they will have very high displacement that is why they are called compliant systems, compliant means movement.

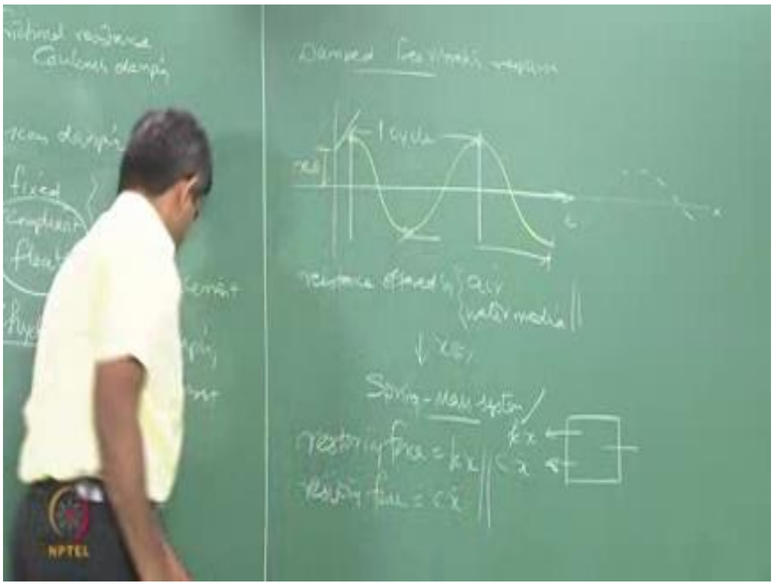
When they have high displacement they will have high velocity, when they have high velocity the damping force proportion to the velocity will be very much present in the system. So, it is a very valid reason why viscous damping can be applied to ocean structures, this is first reason. Now these damping forces remove the constant proportionality equal to some constant C of x star, we call this C as a damping coefficient. The damping coefficient now will have force of newton per meter per second because this force expressive newton velocity is meter per second. That is why we say the damping coefficient C will have a unit in SI as Newton per meter per second.

So, the one reason is our new generation systems are designed to high displacement they will result in high velocity therefore, dissipating force or the decay component of response will be significantly present because velocity of the structure will be significantly there. So, this qualifies application of viscous damping in offshore structures.

The second reason, offshore structures generally placed in a media which offers also damping which we call hydro dynamic damping and we all know that offshore structures without water does not exist. So, this will be always present. So, even if the structure is not moving the water particle around the structure will move which will cause dissipation. So, this becomes a second reason why, viscous damping is comfortably recognized and recommended to be applied for ocean structures. Now compare to these two reasons with respect to other one, we generally prefer to have viscous damping as a dissipating force or as a resisting force in the system compare to the top coulomb damping.

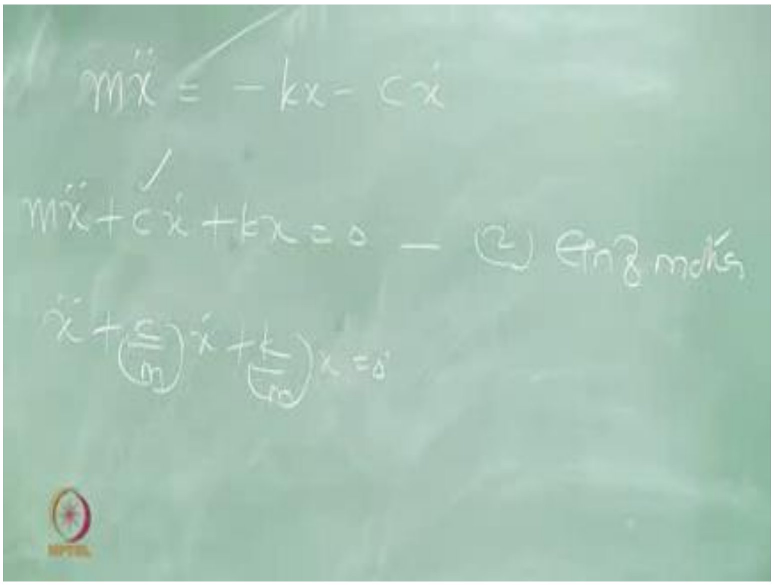
So, therefore, in my system the C should be proportional to x dot. So, there are two forces now - one is the restoring force offered by the spring stiffness.

(Refer Slide Time: 12:08)



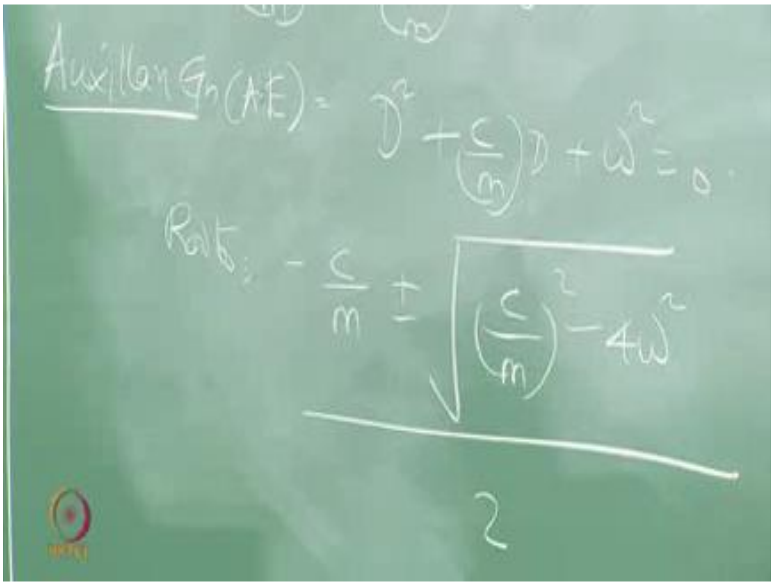
So, now we can write equation of motion simply mx double dot which is the force into acceleration Newton's law should be given the equal to minus kx minus cx dot; this negative indicates that the inertia force applied this way and these two are restoring.

(Refer Slide Time: 12:34)



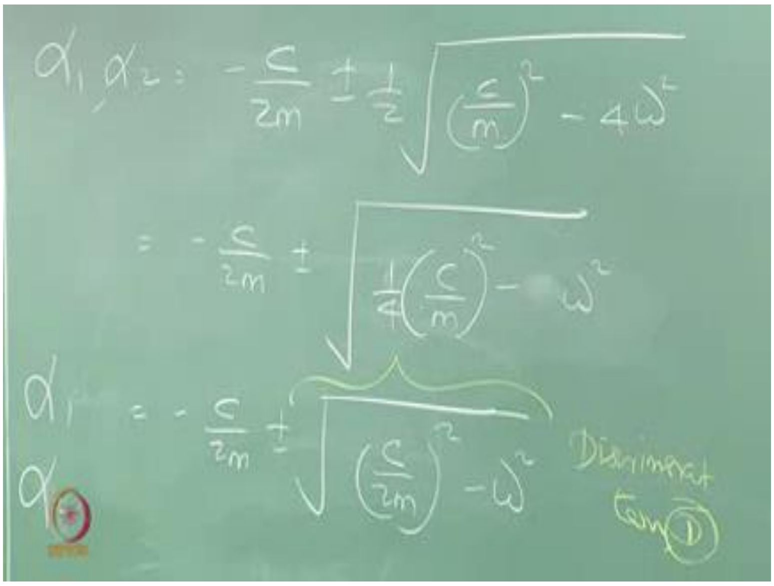
They are trying to bring it back the system to normal that is what we call as recentering capability there is a minus sign applied here. So, my equation motion becomes m x double dot plus cx dot plus kx is 0 is my equation of motion. This equation of motion is qualifying presence of damping term therefore, this is damp free vibration because right hand side has got no external excitation applied to the system, it is 0. So, let us rewrite this equation x double dot plus c by m of x dot plus k by m of x as 0.

(Refer Slide Time: 13:37)



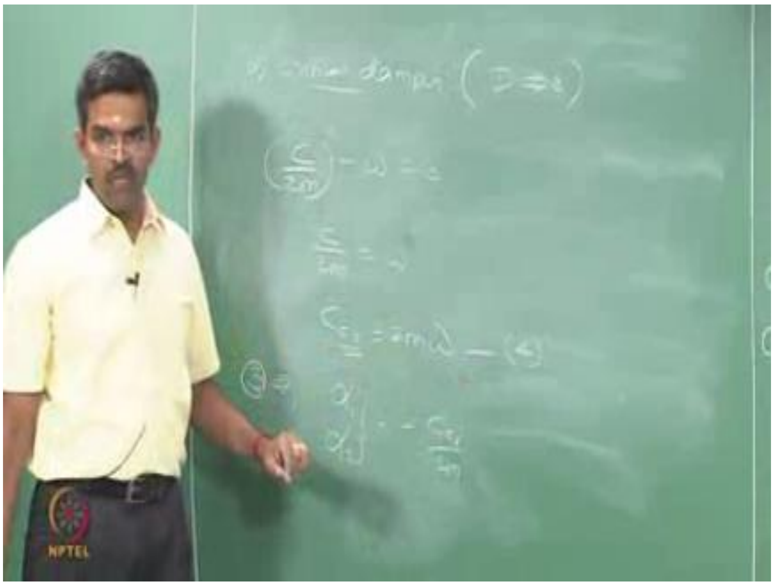
So, the classical auxiliary equation d square plus c by m of d plus omega square is 0. So, as a second order ordinary differential equation therefore, the solution should be the roots of this equation, which will give me a complementary function will be.

(Refer Slide Time: 14:24)



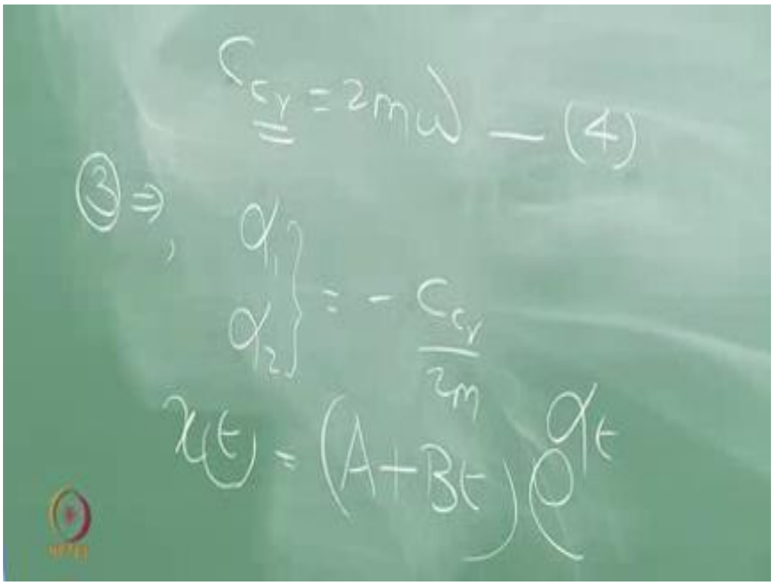
If I say these are my roots, so they are real and distinct therefore, I can easily write my complementary function for this. But now I will pick up this argument, which is the discriminant term D and try to qualify the statement depending upon the qualification of this term. For example, this term can 0 this term can be positive this term can be negative depending upon the qualification of this term the roots will vary for example, if the discriminant term is 0 the roots are real and equal we have a different complementary function. If the discriminant is negative, the roots will be imaginary you have a different complementary function and so on so forth.

(Refer Slide Time: 16:24)



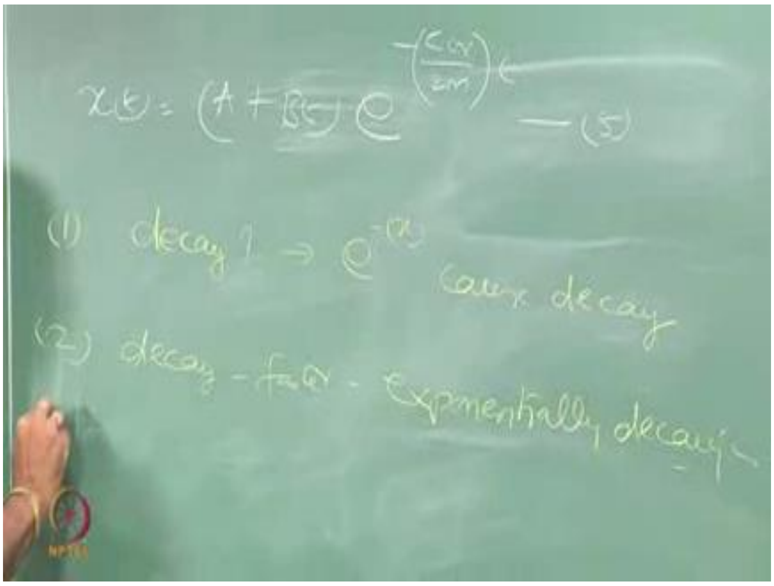
They are now classified in structural engineering as three different terminologies - one is what we call critical damping. Mathematically it is nothing but the discriminant is set to 0 that is this term is set to 0, mathematical that is called a critical damping. The moment I say this, so c by 2m the whole square minus omega square set to 0. So, c by 2m is set to omega square sorry omega. So, I should say this is C c r is 2m omega. I am using c r indicating that it is critical damping. I will call this is equation 3, I will call this is equation 4.

(Refer Slide Time: 17:51)



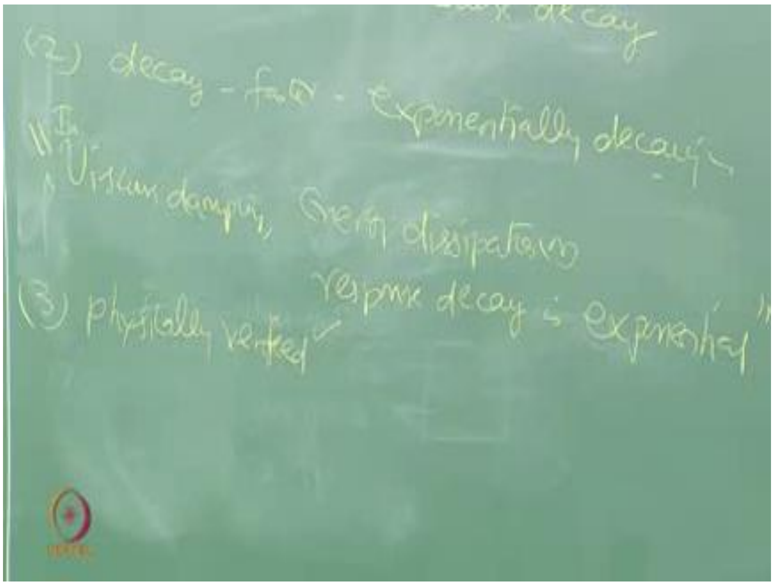
So, equation three will be now modified as alpha 1 and alpha 2 both will be equal to minus C c r by 2m, I am just replacing c with the suffix c r indicating that it is a critical damping solution. Therefore, x of t for both the roots real and equal we know it is A plus BT of e to the power of alpha t, where a and b are the constants which can be evaluated depending upon the initial condition, so x of t can be simply a plus BT of e to the power of minus C c r by 2m - equation 5.

(Refer Slide Time: 18:11)



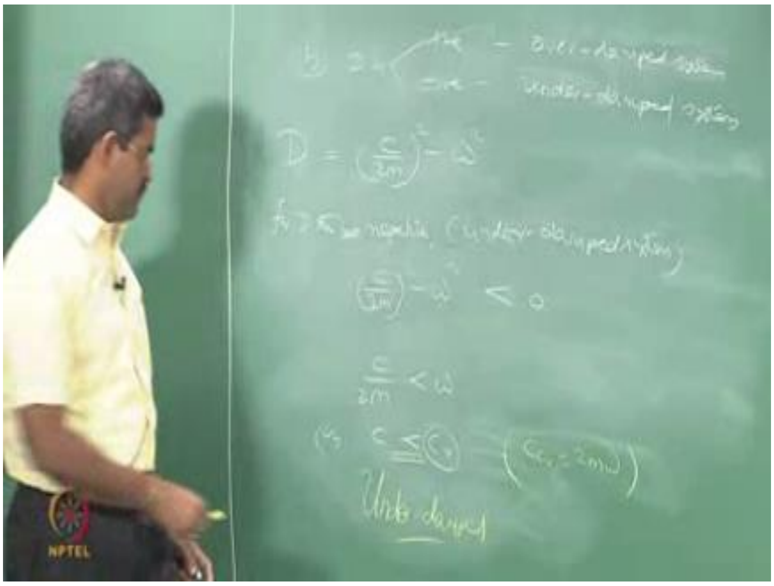
Now, let us look at this equation and try to quantify or let us say make some observations. Now, the first observation here what we make is this response is going to decay, how? Because this term as an exponential decay term, because mathematically e power minus variable will cause decay. The second observation is this decay will be faster because it is exponentially decaying.

(Refer Slide Time: 19:18)



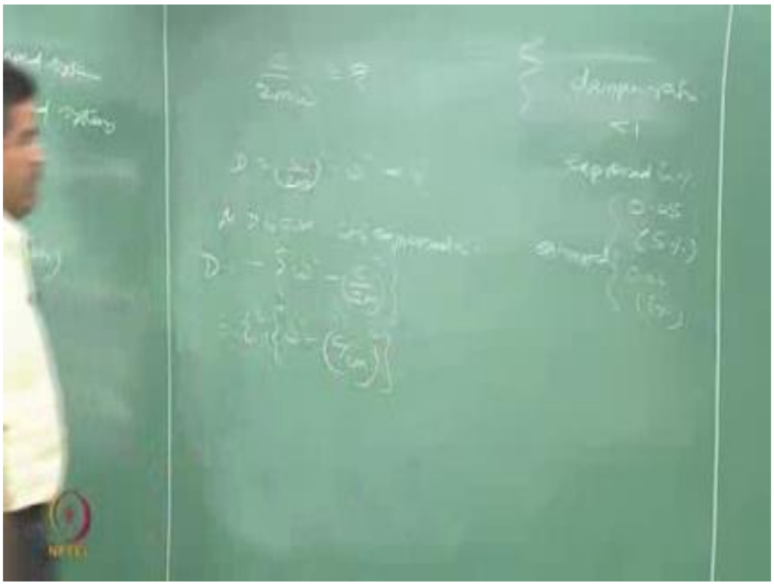
So, the observation is if you use viscous damping energy dissipation or response decay is exponential. Then one will ask a question inquisitively, if it is coulomb damping how the dissipation will look like. Let us try to answer that slightly later. But take it granted it is not exponentially decaying, it is going to linear decay which is also physically examined - have a platform or a moving system or a vibrating system in a body which is filled with water; when the water is vibrating around the structure you will see that the motion the structure will be dissipated faster, so physically this is verified; this is verified, it suit's us.

(Refer Slide Time: 20:52)



This is only one part of the story where the discriminant is set to 0, the second part of the story where the discriminant is either positive or negative. In case discriminant is negative this is called under damped system, in case it is called over damped system. Let us write down the equation for discriminant again D c by 2m square minus omega square that is a D term. For D to be negative that is under damped system, c by 2m square minus omega square is less than 0, why? Because C c r, we already know this we wrote (Refer Time: 22:23).

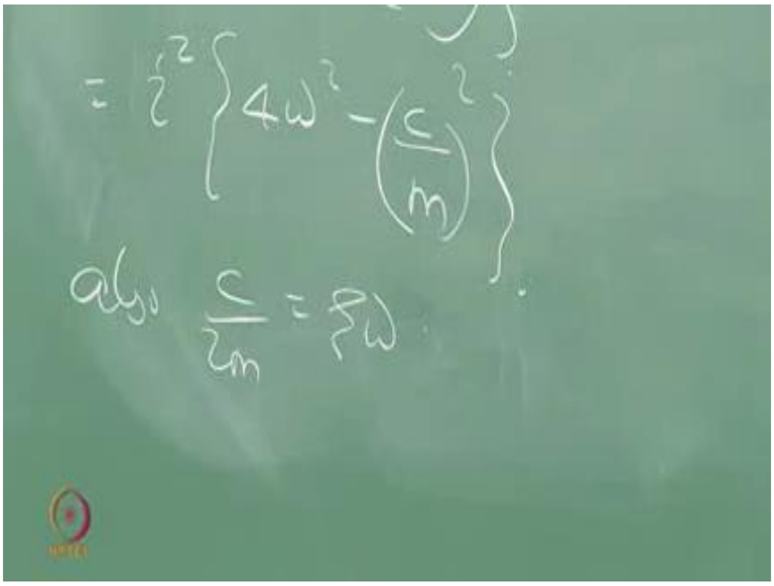
(Refer Slide Time: 23:01)



So, C is less than c r that is why it is called under damped system. So, C by 2m omega is expressed as zeta is called damping ratio, always less than 1, usually expressed in percentage - 1 means 100 percent. How can you say it is less than 1? We already know that C is less than c r, in offshore structures generally is taken as 0.05 or 5 percent.

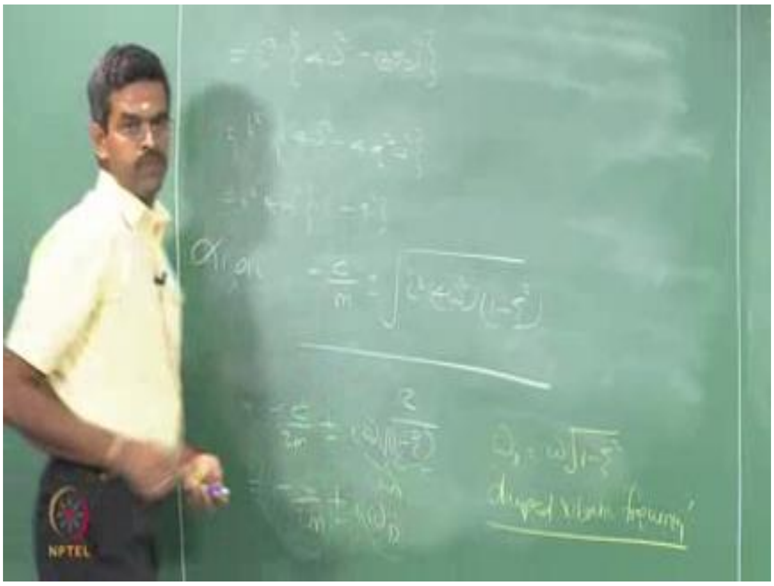
In steel structures it is usually taken as 0.02 or 2 percent that is usual value. Now one may ask me a question, how this can be estimated. The only way to estimate damping coefficient is by experiments, damping coefficients cannot be estimated analytically; you can estimate zeta only by experiments I will show you how. So, let us come to this argument D c by 2m the whole square minus omega square less than 0. So, it is negative.

(Refer Slide Time: 25:53)



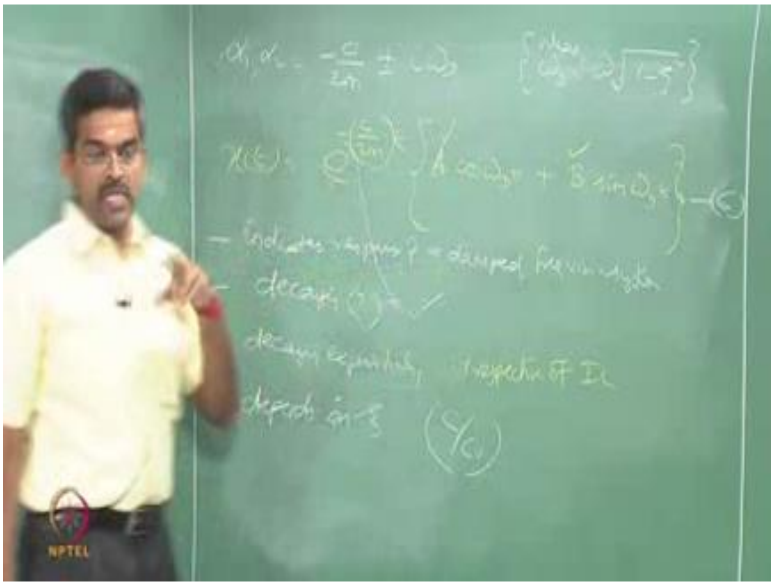
This 1 by 4 factor outside we can remove that, and we should introduce it here in the multiplier.

(Refer Slide Time: 26:32)



So, we already know C by 2m also C by 2m is zeta omega therefore, this can be written as i square 4 omega square 2 zeta omega the whole square, i square 4 omega square minus 4 zeta square omega square, i square 4 omega square 1 minus zeta square. Now we can write the roots - alpha 1 alpha 2 as - that is how this 4 is gone in. So, this will give me now minus c by 2m plus or minus i omega 1 minus zeta square. I call this term as omega d where omega d is omega 1 minus zeta square called as vibration frequency, so minus c by 2m plus or minus i omega d. So, real part imaginary part. So, I can write the complementary function.

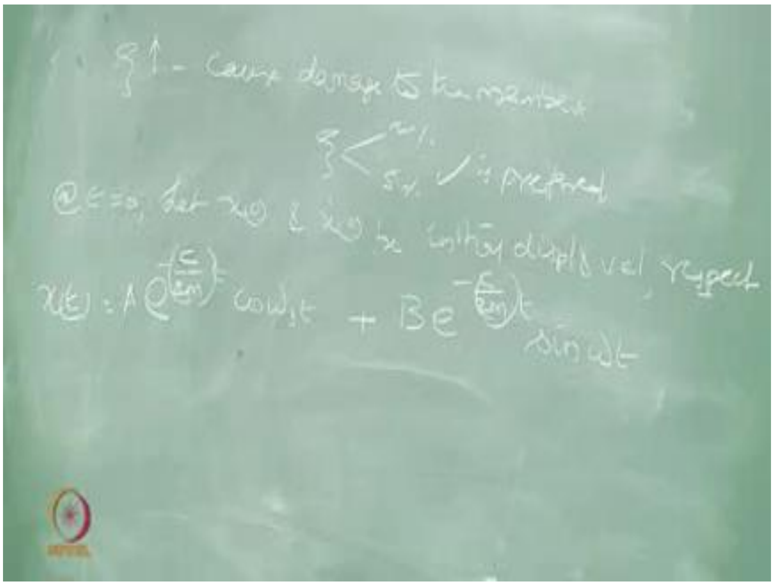
(Refer Slide Time: 28:34)



We can also write some observations from this result. This is going to be equation number 6, some observations on equation 6. Equation 6 indicates response of a damped free vibration system, it also shows when the response is decaying how there is e power minus term present here. The response is decaying exponentially irrespective of initial conditions because initial conditions is now going to give me the value of A and B whatever may be these values it is still decay exponentially because there is a multiplier on this. So, this will show that the response will come to 0. How many cycles that we will see, but it is going to become 0 because there is decay.

The third observation we have is, the response also depends on damping. The amount of damping you give, which is nothing but your ratio. Friends there are two ways to bring down the response – one, the exponential decay which is related to the damping coefficient and the mass of the system, the second is give a very high damping, still it will come down. The second option is not preferred in offshore structures because damping of high value will cause damage.

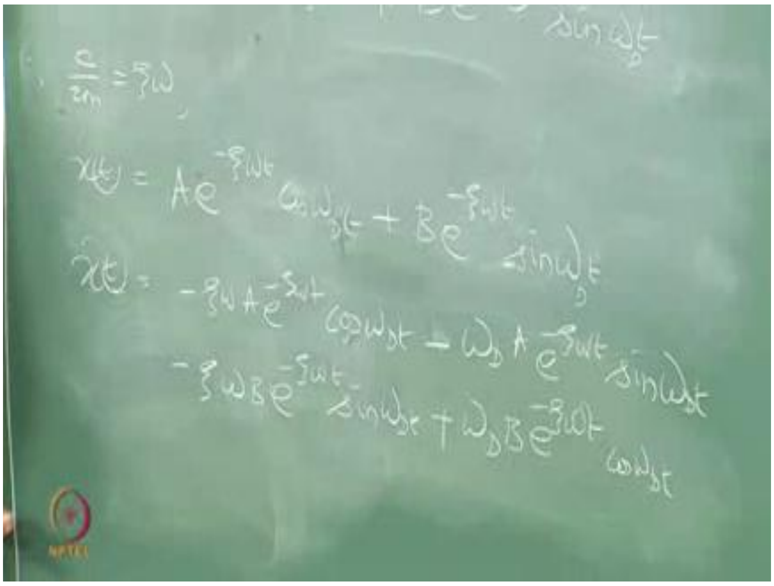
(Refer Slide Time: 31:46)



So, therefore, a lower value of 2 percent, 5 percent is preferred. So, this resembles a complete model of understanding that my offshore platform after specific cycles of number of cycles in time scale should come to 0, or recentering position which satisfies the equation and the response x of t shows mathematically it will decay and lie down. Now to plot this first I would like to evaluate A and B then for different ratios along the time scale I want to mathematically and graphically show you how this will decay. Now mathematically you are you are able to express the equation here, but once you plot the values and say for every cycles you know how much the decay which we call as the logarithm decrement, mathematically it will be very interesting for you to know how this depends on z.

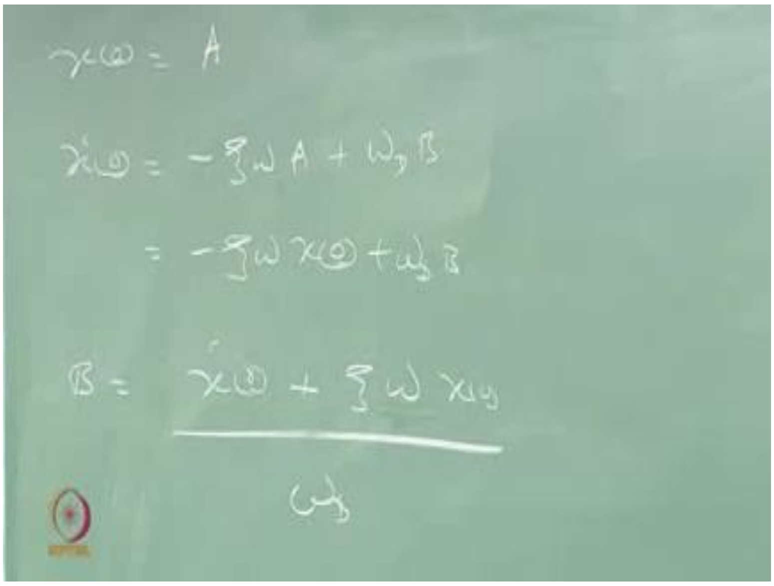
So, first let us evaluate A and B, I want initial conditions. So, at t equal to 0 let x 0 and x dot 0 be initial displacement and velocity respectively – so, x of t.

(Refer Slide Time: 33:54)



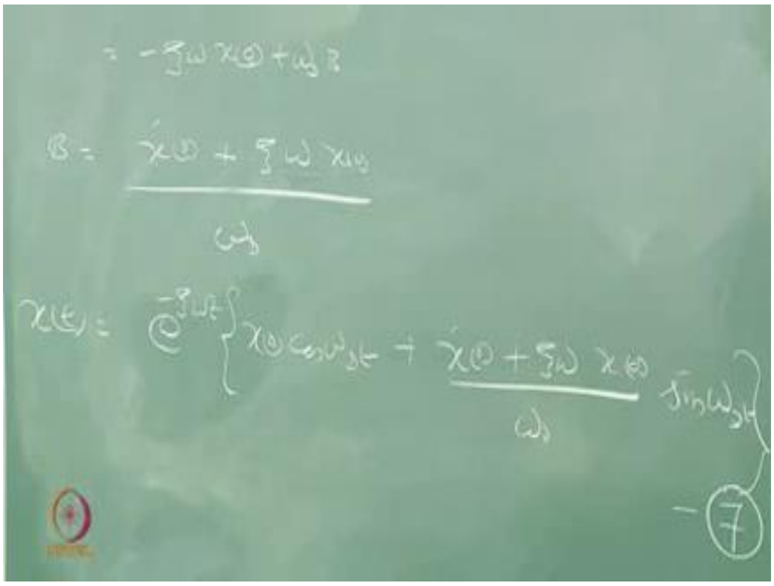
Please make a correction here this is omega d, damped frequency. So, let us differentiate this; that is my differential. Let us now substitute the initial conditions, where x 0 and x dot 0 are the displacement velocity at t is equal to 0.

(Refer Slide Time: 35:49)



So, let us say x 0 is this equation I am picking up - t is 0 this term goes away because this is a sin term this becomes unity, so it is a minus unity. So, x dot 0 now (Refer Time: 36:12) the second equation t becomes 0 therefore, the sin terms will go away, the cos terms will become unity and exponential powers will become all 1. So, minus zeta omega A plus omega d B, which is minus zeta omega A is x naught plus omega d B which gives me B is x dot 0 plus zeta omega x 0 by omega t.

(Refer Slide Time: 37:09)

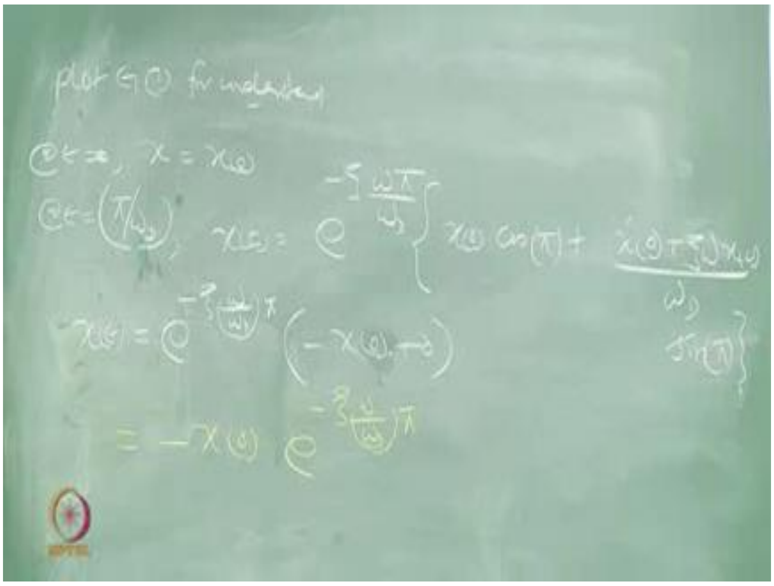


So, I get the full solution now which is x of t as x 0 e to the power of minus zeta omega t cos omega d t plus x dot 0 plus zeta omega x 0 by omega d sin omega d t. That is a multiplier here of e zeta omega t, what I will do is slightly rewrite this equation. Now it is alright, e omega t is in both case available, so I will picked it up and x naught is what is a cos omega d t and B is available here which I wrote.

Equation 7, now it is rather difficult for us to realise from equation seven actually how the decay is happening, we all know that it is a mathematically decaying because there is a multiplier e power minus zeta omega t available despite the initial conditions of x naught and x dot naught this will decay, there is no doubt and there is an omega d multiplier also omega d is nothing but omega one minus zeta square any higher value of zeta, this value will lower and lower.

So, it will cause damping, it will cause decay no doubt. I want to plot this, now the variable for plotting is only t I must select the value of t intelligently in such a manner that this function can be easily recognisable and can be plotted.

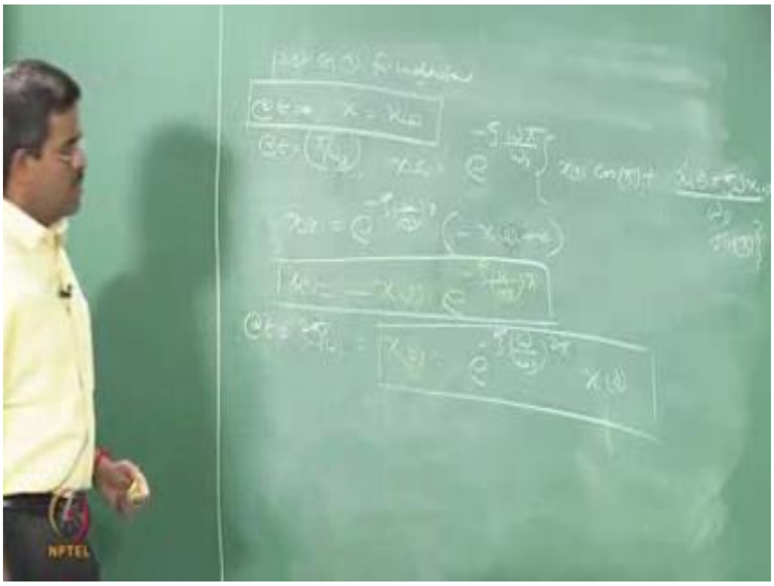
(Refer Slide Time: 39:31)



So, let us say I want to plot equation 7 for understanding of course, if you just write this equation give the variable and excel it, it will plot automatically for different values of t. Now, let us see here how manually we can do that. We know that at t is equal to 0 x is x 0 which you will get from this equation automatically, let us say t is equal to 0 this becomes 1, this becomes x 0, this terms any way goes away because there is a multiplier of sin, we will get back this condition right.

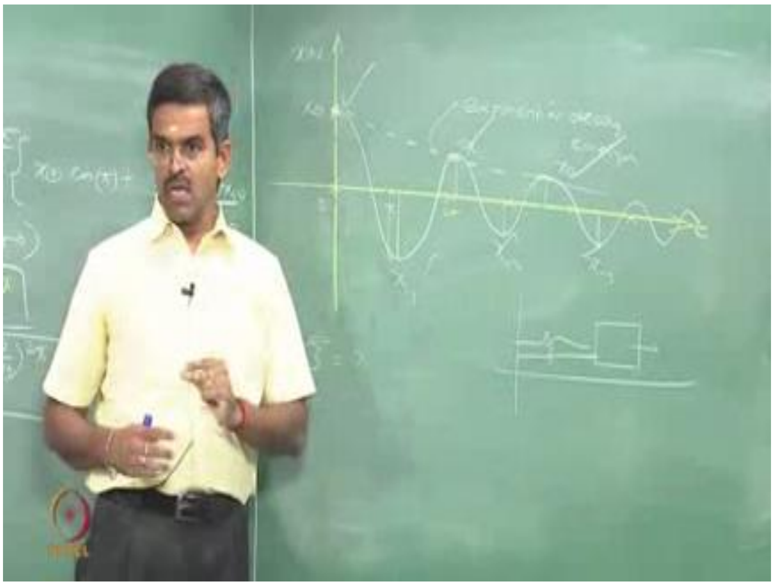
Now, at t equals pi b omega d, this is omega suffix d this is damped vibration frequency it is not omega d, pi b omega d. Now when I use this here pi b omega d possibly this omega d and omega d in t he variable argument will be cancelled, I get cos and sin as multipliers of pi which will be easy for me to recognise in the mathematical function. Now can you give me the value for this? If you expand substituting here you will see that this will become cos pi, this will becomes sin pi and these values are very well establish in the trigonometric functions we know that. It is easy to find out the value of x of t. Can you give me the value? Because, this will become negative so one can say that this is going to be minus x 0, e to the power of minus zeta omega by omega d of pi.

(Refer Slide Time: 42:27)



So, next value at t is equal to 2 pi by omega t. Now, one can ask me a question why I am selecting interval of pi, we know that for a sin cosine function, let us say a typical sin function 0 pi by 2 pi 3 pi by 2, 1 2 pi. So, it is actually changing it is sign that an interval of pi. So, I am making that interval here to see how it is actually decaying. So, let us take the next value as 2 pi 0 pi 2 pi obviously 3 pi by 2, 4 pi by 2 etcetera like that I keep on do that. Now let us see what happens x of t; that is what I will get. So, these are the values I am getting. Let us quickly plot this 3 and see what happens.

(Refer Slide Time: 44:03)



The general equation 7 we already have in your printed page we try to plot against t at every point of our interest this is 0, this is pi, this is 2 pi and so on, at 0 I had a positive initial displacement, positive initial displacement this is accept, equation 7.

There is some value; I do not know this value, so no displacement given to the system. I think you will able to answer this question, why do we have to give a no displacement to the system. The set initial vibration to the system we need to give this. So, we have given this value, it can be negative also does not matter. So, at pi it was negative. So, we started some positive at pi became negative.

Remember this value or this ordinate or this value in the graph will be much lower than this. Let us mark it accordingly, if this is this way this is slightly, why? That is a multiplier of e power minus; similarly when you go to 2 pi it becomes positive of x naught some multiplier, it is positive right. So, there is a cycle change and remember this value will be much lower than this and this is lower than this. So, that is a decay happening. So, if I try to plot - come to 0. So, if you join the positive amplitudes of all you will see that is an envelope generated this is what we call, it is an exponential it is not a linear line it is an exponential decay.

Now, one can ask me a question interestingly how to estimate zeta from this. Now take a spring mass system with or without dash plot set a vibration to this it will come to 0 plot an accelerometer or a displacement transducer try to capture this decrease in response experimentally, you get this plot automatically from the experiment.

Now I note down this peaks I call this as x 0, I call this is x 1, I call this is x 2 I can call this as x of minus 1 x of minus 2 x of minus 3 and so on. So, this negative indicates their lower envelope positive indicates their upper envelope. And I am not plotting this from equation 7, I am plotting this from an experiment which directly obtained from the instrumentation, may be accelerometer may be transducer. So, I get this curve. From this curve I can always read these values whatever may be the number right. So, it means these amplitudes be positive or negative it known to me by magnitude I know them. Once knowing this how to get zeta, that is our question - because we are interested in knowing at what zeta this decay occurred, we did not know that because we did not model that zeta. Remember, very interestingly if you do not know zeta you cannot plot this envelope from equation 7, see we cannot plot understand the point, this is where the catch is.

If you do not know zeta value you cannot use equation 7 to plot this envelope, you have to substitute some value for zeta. Now my idea is obtain zeta after the envelope is obtained directly from the experiment, what is that zeta which has caused this decay - we will see the next class.

So, any questions here any questions. So, let us quickly summarise what we have seen we have taken a damped free vibration system, we derived equation of motion from Newton's law and from the free body diagram wrote the equation of motion, solved it as second order ordinary differential equation, substituted initial conditions and found out the constants and qualified x of t for two modes of damping of course, in this lecture we focused viscous damping.

In the next lecture we will take up coulomb damping, compare these two and see why viscous damping is effective and recommended in offshore structures. For sure we have concluded that yes, if I have a damping ratio for any value of zeta this is exponential decaying which is required in the design as a form requirement of offshore structures what we call as recentering capability of a given system, it must come to 0. Interestingly understand you did not start from 0, you start from x 0, but system will come to 0 - this is where the problem is.

You need not have to start from 0; system will come back to 0 that is recentering. Recentering is not where you started system will come back, no, system will come back to state of equilibrium, this is disturbance of equilibrium; clear this is what we have done.