Dynamics of Ocean Structures

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Lecture – 06
Equations of motion

Last lecture we discussed about the characteristic of dynamics systems, and we said that, a mass is important because inertia force is generated from the mass proportional acceleration. We need the restoring component for re-centering purposes. We have also represent the frictional component, and of course, an external force is required, to cause vibration or to set the motion in place.

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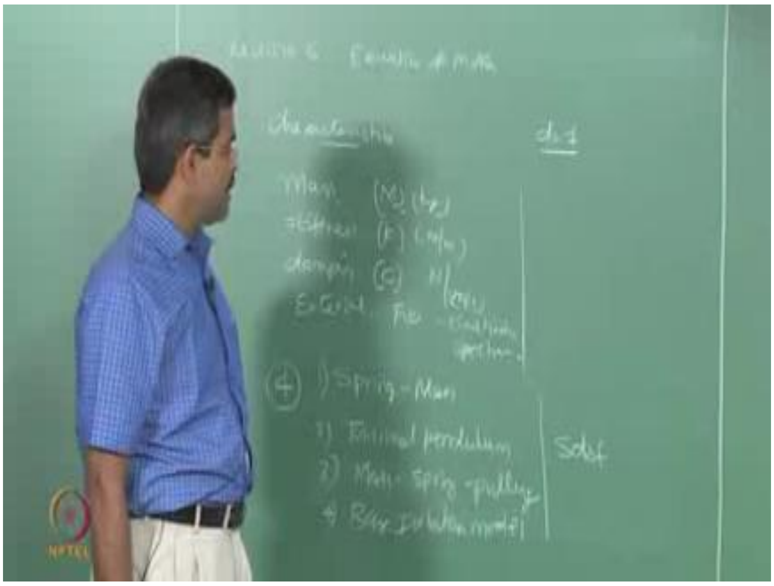

So, mass expressed in KG, stiffness expressed in newton per meter, damping, expressed in newton per meter per second, and external force F of T is a time history, or a spectrum.

Now, the question comes there are 4 ways by which you can mathematically model a single degree freedom system, one is a simple Spring Mass system, other can be a Torsional Pendulum, the third one can be a Mass Spring Pulley system, the forth one can be a base isolation model. Wherein, in all these cases there is only one independent displacement degree of freedom, which makes the system, as single degree of freedom system. So, degree of freedom is the term related, to individual, number of individual displacements. It is not the point where the mass is concentrated, we already said we lump mass at this point, because we are measuring, or we interest in measuring the inertia force. Therefore, it is always better to lump the mass at the point, where your displacement is to be measured. Therefore, we say, that more or less the point where the mass is lumped, is almost same method of degree of freedom, but the definition is not that, it is independent displacement coordinates required, to express initial component in a given system.

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Now, today we will talk about how to mathematically model this. The moment I say mathematically modeling, I must try to get out, or find the mean, to find the response of the system. As we all know the given system is dynamic in nature, the response will also be function of T, and we all know if we differentiate the spectro time, it becomes velocity. If I differentiate further a spectro time, it becomes acceleration. So, these are the 3 set of responses I generally require, or acquire, from a given system if the system remains dynamic in nature, because the force excited is dynamic in nature.

There are 4 ways I can always represent a given system, single degree, mathematically.

There are 5 ways I can write equation of motion. Now to solve, the mathematical model, or the analytical model using equations, I must express the whole behavior, in terms of an equation, which represents the displacement, of the body with respect to time. We call this as equation of motion. Motion terms related to displacement, and I am going to write an equation. Obliviously we know, the displacement going to be function of time, therefore, it is going to be a differential variable, in time, therefore, the equation what you attempt right will be, then differential equation, ok?

Now, we already said it cannot be partial differential equation, because though mass may vary along it is length, but we have lumped them at a specific point. We are discretizing the mass, and we are not considering the variation of mass, along the length therefore, note two variables or consider at a time we will consider only one variable that is a time domain variable. So, I will get simply ordinary differential equation. So, there are standard procedures available mathematics to solve, and ODE which will quickly see, once we write the equation of motion. There are 5 ways I which, by which, I can write the equation of motion.

Let us start from a simply mechanism. Let us say the first one is simply harmonic motion, we generally study in physics. Of course, the method of writing equation of motion is applicable to all the 4 models. We can write the equation of motion to any one of them, by all these 5 methods. We will take up one simple example as Spring Mass system and deliberate it. In subsequent example, we will take up partial pendulum; we will take a Mass Spring Pulley system. We will try to demonstrate, different method of writing equation of motion to any one of them, as and when we encounter the problem.

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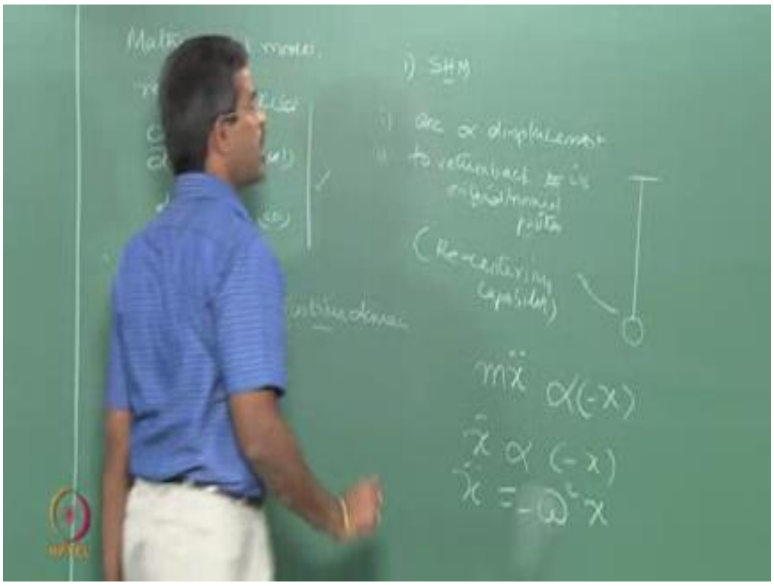

Now, simple Harmonic Motion has got 2 characteristics. If we recollect back, the first characteristic, a simple Harmonic Motion is having is that, the acceleration is always proportional to displacement. The second, body will be always redirected, towards the original position; example, a simple pendulum. You take a pendulum, suspend it a ball. Try to oscillate the ball, the ball will always have a tendency to come back to, it is neutral position, after n number of oscillation what we call them as period of oscillation. So, the tendency is, to return back, to return back, to it is original or normal position. Technically and structurally, this particular characteristic is called Re-centering Capability.

This is one of the important characteristic a structure must have, when it is subjected to vibrations. You must allow the system, or the system must observe the forces, and try to attempt, to come back to Re-centering Capability. Otherwise a structure will impose, or get encounter a permanent displacement, which is undesirable. Today’s structure is location GP as A. The structure does not have a Re-centering Capability. Tomorrow the location of the structure will be GP as B, which I do not want. I want the structure to remain at A, irrespective of the force acting on the structure.

So, Re-centering Capability is very important. This is exactly the same expressed in physics, saying that, the body subjected to oscillation, the body will always have a tendency, to come back to it is normal position. That is these 2 are basic algorithm since (Refer Time: 07:31) motion. I will use them here, to generate the equation motion. Let us see how, let us say MX double dot that is the acceleration force is always proportional to displacement. Now to implement this second condition, I should say, it is proportional to minus of X. Minus because, it is coming back. It is always directed in the opposite manner. If I try to pull the ball to the left, the ball will move to the right, if I try to push the ball to the right, the ball will move to the left, there the tendency of the ball, to always oppose the motion, so minus.

So, in simple terms acceleration proportional to displacement, acceleration is equal to some constant. Let us say this constant is omega square of X.

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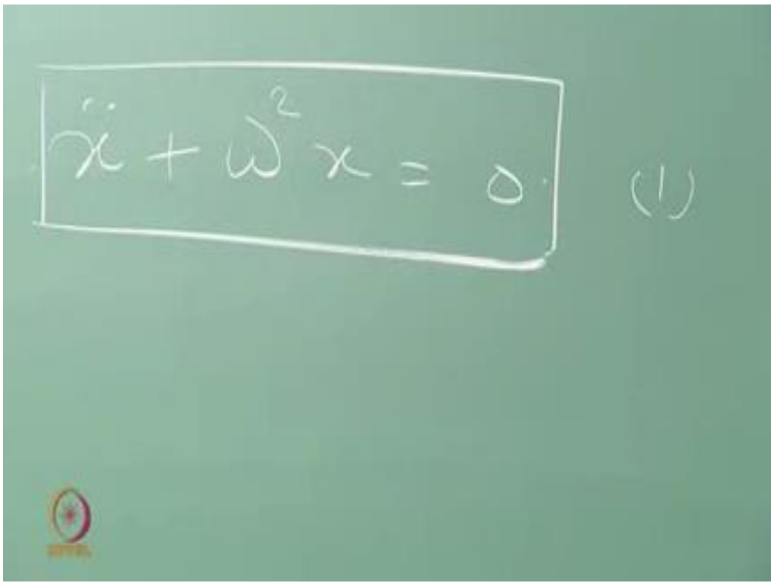

So, I should say X double dot, plus omega square X is zero. So, this becomes my, where as in this specific example, omega square is taken as constant of proportionality. There is no other meaning for this. But it has got a specific meaning, as I elaborate it further. This is nothing, but proportionality constant, that is all, because this proportional and putting a constant, I am equating it, and I bring it rearranging the term, and I get this equation of motion. MX double, x double dot, plus omega square X is zero, is the equation of motion, as derived by applying the principles of simple Harmonic Motion, to vibrating body, which is nothing but a, Spring Mass System, which is the first category what we have.

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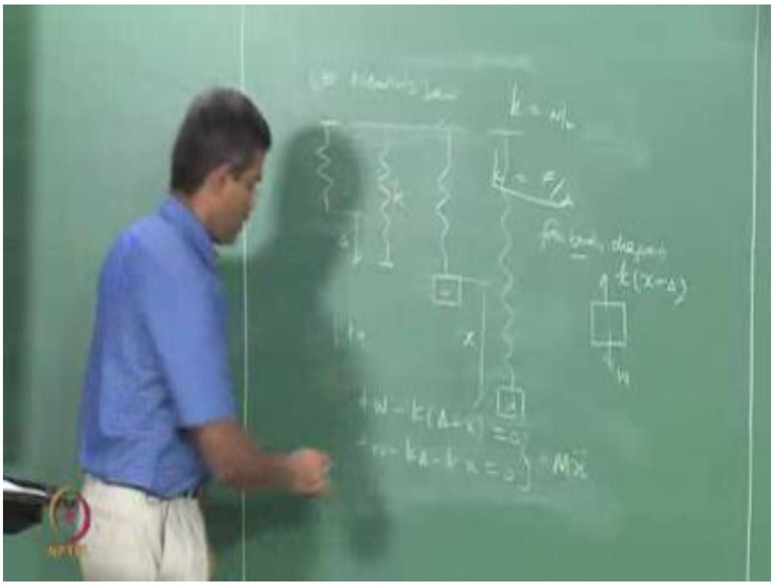

Now, we will apply, Newton’s law, to write the equation of motion. That is the second method by which I can write equation of motion, Newton’s law. I will take up a simple system, lets say it is a having a spring. When I suspend the spring, and leave it free, because the spring will have it is (Refer Time: 09:47), you will see the spring will elongate. Let us say that elongation is delta. Now what is the force responsible for this elongation? Because elongation is always caused only by a force; you have a body, the body is displaced by it is length, you need to apply a force to displace it by a length, or to contract the force your body, you have to apply a force to it is opposite direction. There is force require, to cause displacement.

Now, we know that the spring has got a spring constant K. What does it mean? K is expressed in Newton per meter. It is, for example, for one meter displacement, lets say X Newton force is required, to displace or to cause a displacement of one meter. So, if I want to really find the force responsible, to cause this deflection then I can easily say that force going to be K delta. K is the stiffness of the spring, and delta is the displacement.

So, the force is k delta, because stiffness is force by displacement will give me the force which is causing this displacement. Now, at this level, I suspend the weight W. Now this weight will again cause further extension which I now call as X. That is the displacement.

Now I draw a free body diagram. I pick up this; write down all the forces on that. So, this is the body. The force acting now here is W, this is acting now, the force opposing this will be, stiffness times of displacement and delta, is it not? So, now, there is a force valance. So, let us apply that all downwards forces positive. Write force equilibrium in equation, plus W minis of K of delta plus X is set to zero. So, plus W minus K delta minus KX is zero. And that should be equal to MX double dot that is my 4, Newton's law. So, we know already this W, which was caused by the force, because extension spring will be canceled.

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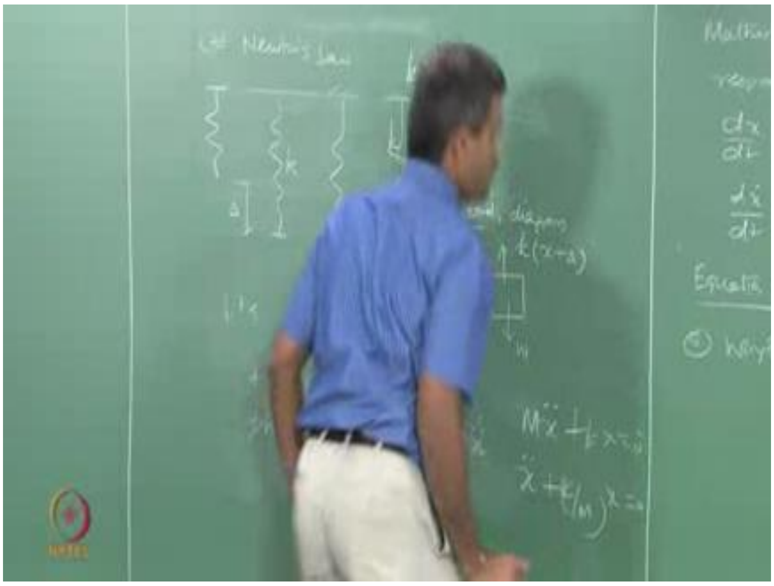

Now, I say MX double dot plus KX is equal to 0. So, X double dot plus K by MX is 0 if I say K by M is omega square.

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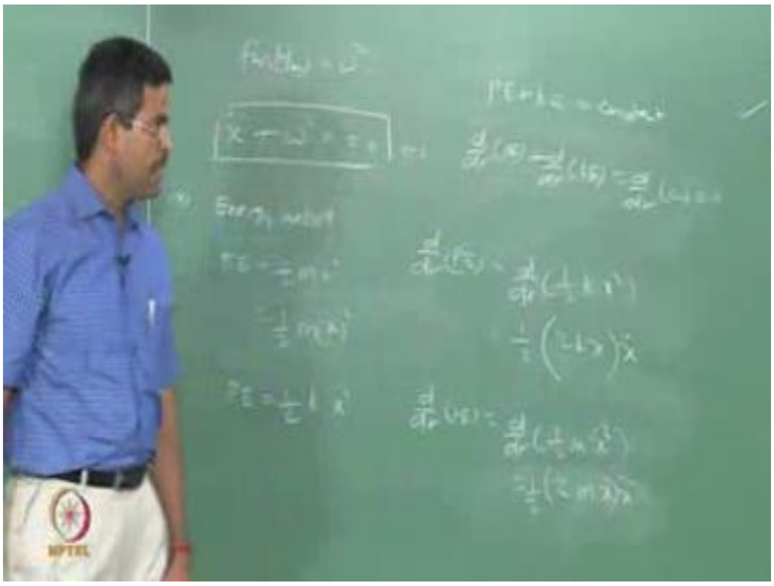

For K by M if expressed as omega square, X double dot plus omega square will be 0 is as same as, the equation of motion what we got from, simple Harmonic Motion. The third method, by which we can write equation of motion, is Energy Method. Let us say kinetic energy can be expressed as half MV square, we already know it is half MX dot square because X dot is the velocity. Potential energy is half KX square for a Spring System. Sum of energy in a given system has to remain constant. Law of conservation of energy, so I am going to straight to do it a different manner, I say, differential of this, plus differential of this, is differential of this, which will be 0. So, let us say differential of this. So, D by DT of potential energy, which is D by DT of half KX square which is half 2 KX, x dot - D by DT of kinetic energy, which is D by DT of half MX dot square, which will be half twice MX dot X double dot. Let us rearrange here, half 2 KX, X dot plus half 2 MX dot X double dot should be 0.

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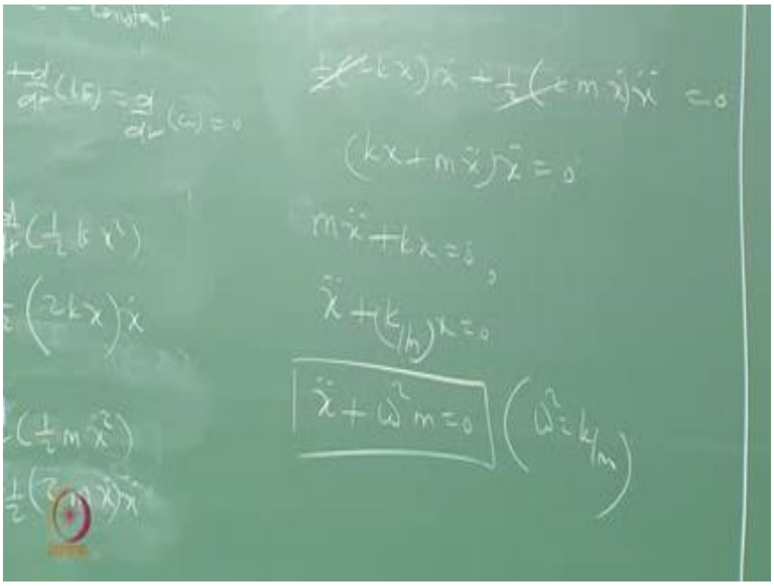

So, simplify X dot is common. So, KX plus MX double dot, of X dot is 0; Obviously, X dot cannot be set to 0, because if X dot is set to 0, there is no vibration. Therefore, KX of M X double dot, plus KX is set to 0, which implies that, X double dot, plus K by M of X is set to 0, X double dot plus omega square of set to 0, where omega square is K by M. Equation of motion is again same as we got from the previous two examples. So, in this case, energy method says that the sum of potential and kinetic energy should remain constant. I have taken expression for kinetic energy separately, PE separately, differentiated them, and equated here, and arranges them, and I got the equation of motion which is as same as, Newton's law or simple Harmonic Motion.

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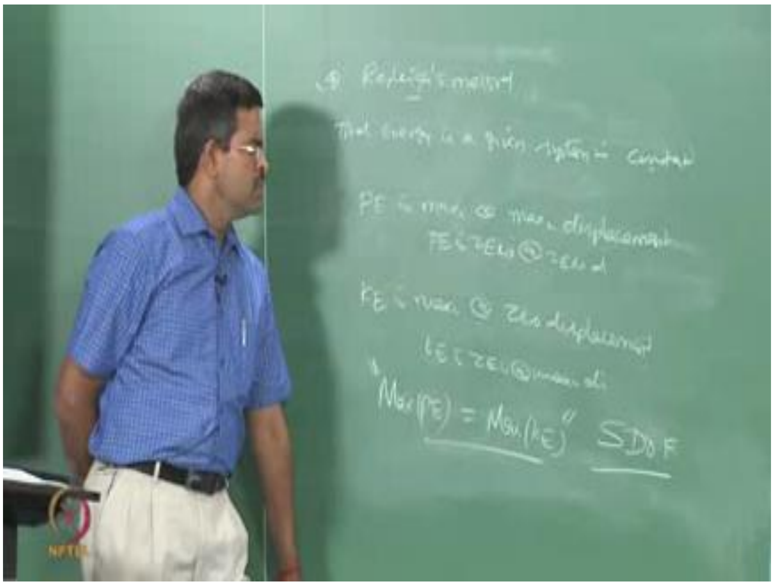

The forth method is again, an energy principle, which is called Rayleigh’s method. This method says, the total energy in a given system, is constant. We all know that, potential energy is maximum, at maximum displacement. Kinetic energy is maximum, at 0 displacements. Whereas, potential energy is 0, at 0 displacement, and kinetic energy is 0, at maximum displacement. So, at any given point of time I can say, maximum potential energy will be equal to maximum kinetic energy, in a given system. Use this principle, to write equation of motion. This is applicable only for single degree freedom system, for multi degree, you cannot apply this. So, there is no serious difference, in terms of writing equation of motion, from Rayleigh’s method, compared to energy method. Energy method accounts for both the energy system present in that, and saying the total energy is constant. Here it looks only for the maximum value of that. And equate them. So, fundamentally by principle, both of them address the same concept, but this is easy, to operate a single degree, and this remains as a constant.

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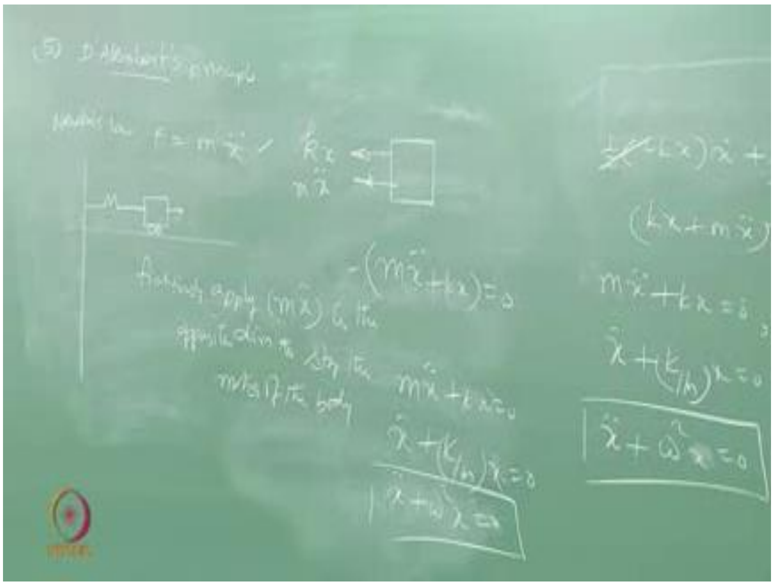

The fifth method, by which we write equation of motion, is called D Alembert's principle. It is a principle given by a scientist by name D Alemberts. We all know that Newton's law, states that, force is mass into acceleration. We all know this. When a spring mass system, is made to move in a direction from left to right, by a force, to stop the displacement of this body, you can fictitiously apply MX double dot in the opposite direction, to stop the motion of the body.

On the other hand, if you have a body, where the restoring force in the body, is also offered by KX, that is the restoring force offered by the body, in addition to that, if you apply MX double dot in the direction, opposite to motion of the body, the body is moving this way, that is why restoring force opposite. If the body is moving from left to right, I must apply a fictitious, imaginary force, which is proportional to it is acceleration, in the direction opposite to motion. So, I must say, this is MX double dot. Now the body will be addressed correct.

Now, in combine them, MX double dot, plus KX is 0. Which means that, MX double dot plus KX is 0. X double dot plus K by M of X is 0, or X double dot plus omega square M, is 0. Is as same as the one what we already have with us. So, D Alemberts principle is only a concept. It cannot be applied to complicated systems, because it is very difficult to find out, what would be the mass contribution on every mode of vibration, to bring the body to rest. I can give an example. Imagine there is a stick model which has got only one degree of displacement, which is the tip.

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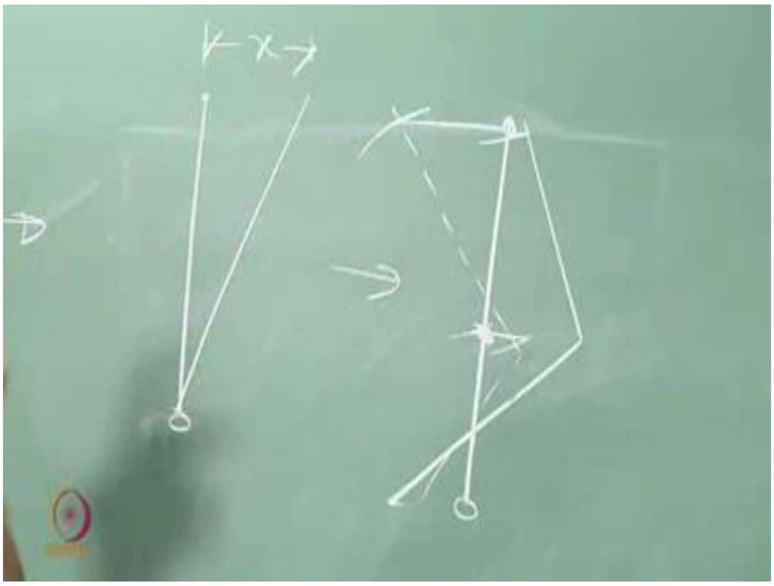

Let us say under any force, this hinged or fix whatever may be, because the shape may be varying, I will make it as hinge, if it is fixed the shape will vary. So, it will lie, oscillate like inverted pendulum. There is only one degree of displacement, which allow to this mass, when the mass is pushed to the right, it will oscillate and so, on and so forth.

So, therefore, it is easy for you to apply a negative force, which is proportional to it is inertia, in the direction opposite to the motion of the body. On the other hand, imagine a system like this, where there are 2 mass lumps, may be this is hinge. On application of a force like this, the mass behaves like this. So, for example, this mass does not move, but this mass moves. On the other hand, this mass can move and so on. So, there are 2 displacements which are different, that is why it is called degrees of freedom. Degrees of freedom are independent displacement required, to explain the position of on vibrating motion, under a given force cycle or force system.

Now, it is very difficult for you to apply a single inertia force, which is proportional to, which acceleration, and which mass, ok? Or, then in that case, we must find out, what is the mass contribution, in total, and then apply. Therefore, D Alembert's principle, being a principle convenient, is applicable to single degree freedom systems. For higher degrees, this method is not followed. Under a given 5 methods of writing equation of motion,

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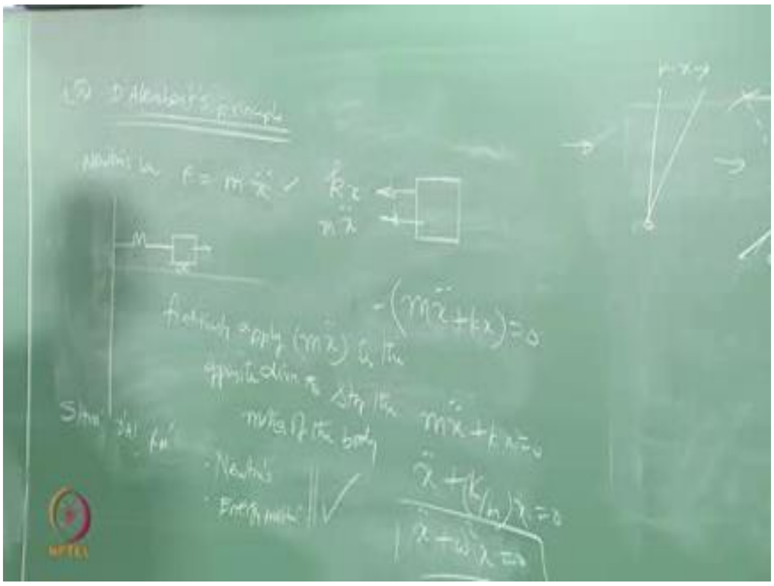

we have seen that, Simple Harmonic motion seems to be single, and easy, for a single degree. Similarly D Alembert's principle seems to be easy and simple, for a single degree. And Rayleigh's method seems to be only a concept, which is again a repetition of an energy method. So, we are left only with 2 methods where we can write equation of motion easily, one is a Newton's method, other one is the energy method. So, these 2 methods are generally practiced, in dynamics to write down the equation of motion.

Now, one can ask a question, we are interest in finding the response of the system. Why are we writing equation of motion? Where are we getting the response? Because our interest is, to mathematically model the system, identify the degrees of displacements and degrees of freedom, and then write the equation of motion. That is what you are saying, but we are interested not in writing equation of motion, but to find the response of the system, which is function of time. Interestingly if you agree, that this is my equation of motion, for the given system which is an ordinary differential equation in time domain.

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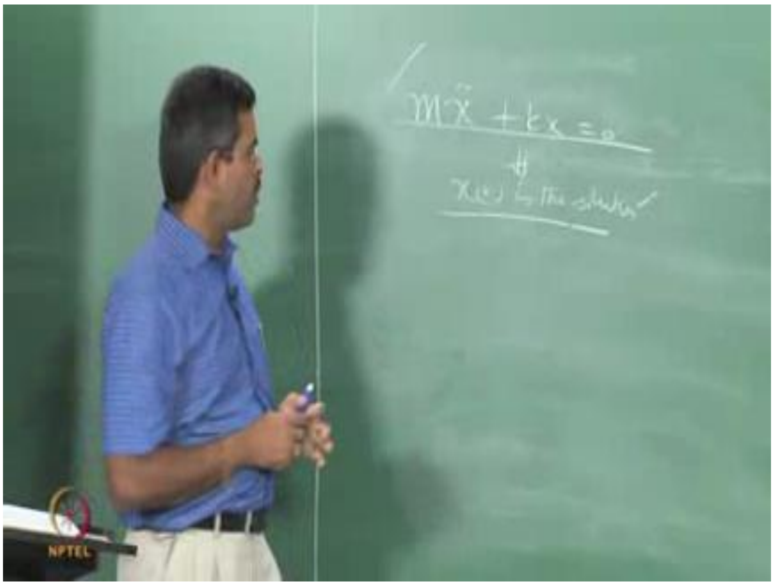

Solution of this will be, obviously, X of T is it not? So, which I want, on the other hand, the steps involved dynamic analyses are the following. There are 2 steps involved, 1, for a given system, for an identify degrees of freedom, write down the equation of motion by some ways or other, there are many ways to do this.

One, understands, and writes equation of motion yourself, with any one of this 2 method. The second is let others understand, you copy form them and write the equation of motion. So, first step is to write the equation of motion, by hook or crook. Second is to solve the equation of motion, to gain the response. This is where we are mathematically interface play. So, this is physics, this is mathematics. Now if it is very simple, ordinary differential equation, we have a handful method of solution, where I can write the complementary function, the particular integral, then apply the initial and boundary conditions, solve them and get X of T in a full domain. If this on, each one of them becomes a matrix may be 6 by 6, 3 by 3, 2 by 2, they require a matrix, when they will become a matrix? When they become a vector, when they will become a vector? When there is more than one degree of freedom.

On the other hand, when the systems become multi degree of freedom, then solving this, or finding solution of this, through standard solution, it is not possible. So, what we have to understand first is, let us write equations of motion for different systems, solve them understand them, for single degree first. As you move to higher degrees of freedom, we have to neglect or ignore the method by which I am writing accepting here, that is the standard ODE solution procedure, then I have got the follows on numerical procedures, which is available in the literature, because this is anyway mathematical. One of such method is the standard ODE solution. There are many other methods to find solution for a differential equation. We all know that. If we do not know that, we should know that. So, there is, in this is not the only way you can write the equation of motion solution, that is one of the ways, there are many other ways which we will see in the class.

So, to start with, we will pick up single degree of freedom system, and then we will try to write down the equation of motion, solve it using standard procedure, and understand, how qualitatively, and quantitatively, the results look like.

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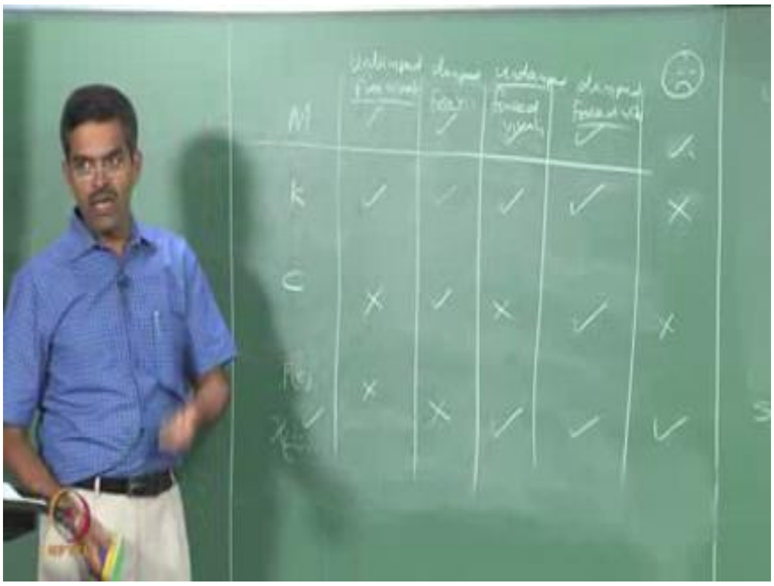

Now, let us get back to, what are the essential characteristics of single degree of freedom system (Refer Time: 28:19) Same question asked back again, which started this class actually. The essential characteristics are, mass, stiffness, damping and force. these are the essential characteristics. Now let us see, mass has to be present in any given system, otherwise there is no dynamic analysis, because we already said dynamic analysis is possible only, when it is got a representative inertia force present, than mass proportional. So, mass need to present in a given system.

If the mass is vibrating, or if the mass is displaced, obviously, the mass has to be brought back, to it is original position. That is what we call as Re-centering Capability. That is one of these structural design requirements, of any given system. You left your house, to come to IIT for graduation. Your father is expecting back to re-center. You will go back to your house. If you keep on moving ahead then problem is there, right? So, Re-centering is always required at every part of life. You come from the hostel, you expected to go back to the hostel, and expected back to come again, to the class room. So, there is always a cycle.

So, same way the structural system has to have a Re-centering Capability, because I do not want my system to keep on moving as I keep on exploring. Which is very ideal, I have a very, very interesting system, where, it keeps on moving, I keep on producing oil. I travel around the world like XYZ scientist and keep on accumulating barrels of oil and come and sell it India, sometimes Indonesia, sometimes China, which is very fantastic idea. Does not work like that. My system has to stay, I have to do exploration, my platform has to re-center, of course with or without a permanent defamation.

I cannot allow my platform to move, as I while, as it keeps on going, through it may be floating, very clear. So, re-centering is essential, for that I need restoring force. The restoring force is not there re-centering thought, cannot be invoked in the design. Remember, these conditions of equation of motion in structural engineering, applied to dynamics, are all from physics, from the action of the structure. They are nothing to do, with the dynamics as a subject. I am now posting requirements, which are all require to be present in a given system. Then I will come back, whether to dynamics analysis or not.

Now, damping is one part where, I can mathematically represent the frictional force. I can even ignore it. Let us say, my body is, completely kept on a smooth surface. It does not develop heat, there is no friction, there is no loss of energy, and there is no dissipation of energy at all. So, I can say, the damping component which is dissipating part of the energy is not represented significantly, compared to inertia force, compared to the restoring force, relatively. Now F of T may not be there. You may be wondering sir, F of T is not there, how the system will move? If I have no F of T and no C, I call this as, undamped, free vibration.

The word free vibration stands for, the system vibrating without any force. Now again you have a serious question, if the system has no force, how do it vibrate? You set an initial vibration, and leave the system free. You cause an initial vibration, let us say, call the body to the right, and then leave the body free. Oscillate the pendulum, let the pendulum tilt, once you move the pendulum to the right, and leave it, the pendulum will keep on tilting on it is own. So, you have to set the motion on, but you do not require a constant force to keep the motion on. Therefore, I can say, this is not present, continuously, initially this is certainly present, if that situation comes it is called free vibration.

If damping is not present, it is called undamped system. Now the second case, I have anyway mass, I have anyway restoring force, I have damping also I may not have the force. I call this as damped, free vibration. Free because, F of T is 0, damped because C is present, in this case C is not present which is undamped. Let us see the third case, M is present, K is present, C is not present, but F of T is present. It is called undamped forced vibration. Undamped because C is not present, forced because force is present. Then do not be confused, say here also force was present. **F**orce is continuously present; there is a time history of force. It is present all the time, like a wave, like a wind etcetera. The time history of forces which is permanently, or continuously, presents in the system, but in this case force was only allowed to initiate the motion. Then force was removed, right?

The fourth could be, mass present stiffness present, damping present, F of T present. This is damped forced vibration. Now I have a fifth system, where mass is not present, stiffness present, damping not present F of T present. This system is unacceptable. There is no mass. I have another system where mass is present, restoring force not present, F of T present - unacceptable, because I have a Re-centering Capability.

Remember I am assigning these characteristics of dynamics, in a physical sense, to a given model, not in mathematical sense. So, never understand dynamics in terms of these clusters, understand dynamics in terms of a physical meaning. If I understand the physical meaning, they will be associated to them automatically, in your mind. So, do not try to understand the system, by these clusters of words, though they are important. But understand them with a physical meaning, of the characteristics of single degree freedom system, which is essentially required to classify a system as a dynamic system. Which already said in the last class, ok?

So, let us take up, for start with, can I start from any vertical columns there, of course, this we are not anyway doing, because this is an unacceptable system. Can we start anywhere? The degree of complication of understanding dynamics starts in the same hierarchy, easiest, easier, slightly tough, and toughest. So, it goes the hierarchy in the same manner. So, for making it very easy, we must start with this system. So, we look for undamped free vibration model. What we are interested in? To find X of T, why we are interested in dynamic response analysis? We already know, using these 4 characteristics, if I form an equation of motion, which is ordinary differential equation, and solve it mathematically, I will automatically get this solution. Just know us that. So, will pick up an example, write equation of motion, apply mathematical concepts to that, solve it as an ordinary differential equation, get the X of T, and try to understand qualitatively, what do you mean by response analysis of, undamped, free vibration model is it clear?

So, remember these tags are all associated, in a physical sense to, the characteristics of a system, which is classified as dynamics system. They are not associated to, for example, you should never say, free vibration means force is 0, no. Free vibration means, one of the characteristics present in the system is not invoked. They are all physical parameters. All of them are equally important. There is no one hierarchy over the other, but unfortunately, this and this, are the parents, and remaining all are children, where sometimes nowadays children take over their parents. Because of their intelligency, because of their arrogancy, because of the way they have been brought up, because the food what they eat, because of the education what they receive. They take away the parents, but ultimately they remain children only.

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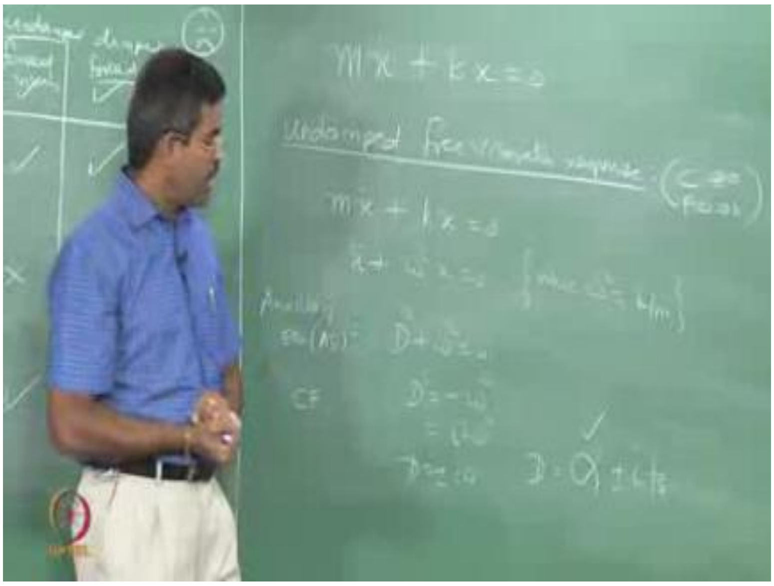

So, we will take up undamped free vibration, We already know that the M is present the equation of motion should be MX double dot plus KX plus 0 is equal to 0. So, I am looking for undamped, free vibration, response. That is what I want this is what I want. So, I can even write in bracket mathematically, C set to 0. F of T set to 0. Can I write this? Because this implies mathematically that this statement is correct. So, I pick this MX double dot plus KX is 0. We already know how we wrote this equation, there are 5 methods, we already explained. So, X double dot plus omega square X is 0 where, omega square is K by M.

I can write something called auxiliary equation which is abbreviated as AE, in mathematics - ordinary differential equation. We written as D square, plus omega square, is 0. Where D stands for DBDT of the variable, D square is D square by DT square of the variable. Second derivative of the variable, variable here is the X, which is the displacement term, which is nothing, but degree of freedom X1, X2, X3 etcetera. So, the complementary function should be, let us say, the root is minus omega square, which is I square omega square. So, D will be plus or minus, I omega. So, this can be compared to, alpha, plus or minus I beta, where alpha is called the real part of the function, and this is called imaginary part of the function. When I have both real and imaginary part, I can write something called the complementary function for a given ODE as solution, which is given by this equation.

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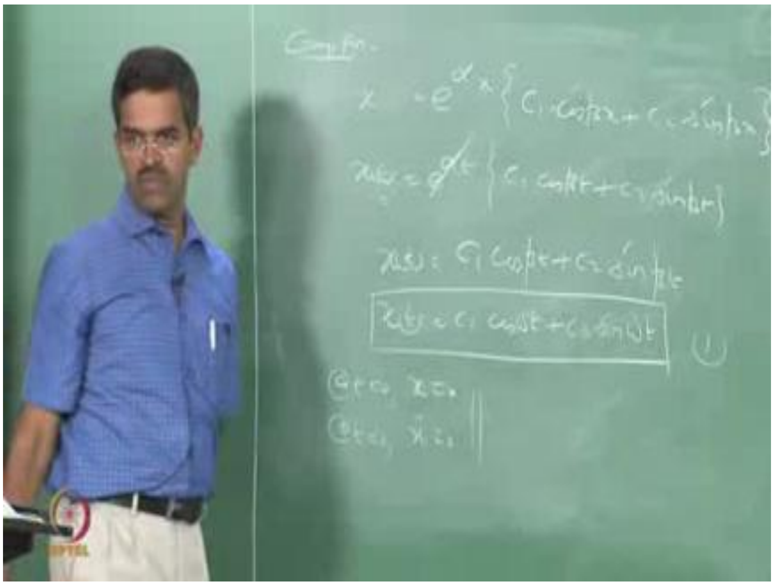

So, I write the complementary function, for this, which will be, E power alpha X, C1 cos beta X, plus C2 sin beta N.

So, I should say X of T is E power alpha T, because that is the variable, C1 cos beta T, plus C2 sin beta T because I am writing this in the time domain. This is not TE; this is X, simply the solution. And we know alpha 0 in our case, there is no real component. So, X of T simply becomes C1 cos beta T, plus C 2, sin beta T. Beta in this case nothing, but omega. So, X of T is C1 cos omega T plus C 2 sine omega T. Let us call this as equation 1.

Now, there are two constants here C1 and C2, which depends on the initial conditions of the problem. What are initial conditions of this problem? When at T is equal to 0, there was no displacement, at T is equal to 0, the velocity is also 0. Let us apply this.

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So, the conditions are, X of T is C1, cos omega T, sin omega T, X dot of minus C1 omega, sin omega T, plus omega C2, cos omega T. Let us say at T is equal to 0, X0. This term goes away, C1. At T is equal to 0, X dot of 0. This is omega C2, correct? So, I have now both the constants. Let me substitute in equation 1 and get the full solution. Say X of T. Now I should say, substituting, the above in equation 1. I get or let us say we get.

That is the very interesting term we are available in the engineering generally, we always say we assume, let us assume, you will always see the teacher or the book author or the student does not say, let me assume, have we ever seen a book like this? See it is a very interesting statement; it is very sarcastic to say also. We always say we because, we unite all you in the same boat. If you consider there is solution is answer is wrong, and you say this is idiotic solution, you are also an idiot. Therefore, there is nothing like I, we, let us assume. So, we are all joining hands, in making a mistake if at all. So, so are with me all the time, you may not be with me physically. So, X of T is nothing but, C1, that is X naught, cos omega T, plus X dot omega by omega sin omega T.

Equation number 2. So, I have the full solution now. Now still one may not be happy to know, sir where is the solution here, I am not able to visualize it physically. Yes it is possible because, I have not substituted T is 0, X is 0. I said X is nothing but, X naught, that is, you can always start the problem with an initial displacement. You can always start the problem with initial displacement, which you know. So, these are known values. These are known physical parameters, applied to the system; this is physical parameter of the system, because omega is square root of K by M, because omega square is K by M. Now omega is called there are 2 meaning of omega. Omega is called circular frequency, expressed in, radiance per second, where M is in KG, and K is in Newton per meter. It is also expressed as F, F is called cyclic frequency, and omega and F are related, omega is 2 Pi F. F is 1 over T. Therefore, omega is 2 PI by T, since they can be related.

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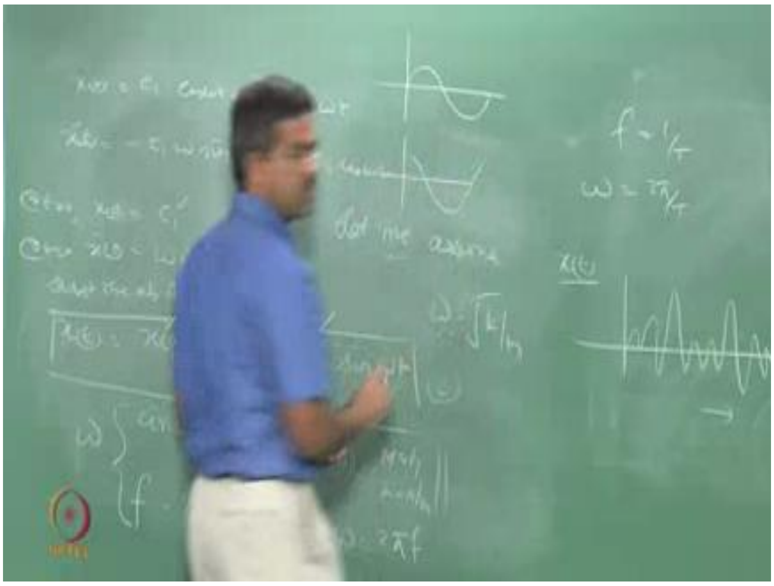

Now, we have learnt two interesting parameters and results of this. What are they? one is the response of the system, which is the time history of the response. Because it is time history it is varying with time; Obviously, it should look like, it should look like, a history, which varies with time. But this will be unfortunately super position of, a regular cosine, and a sin wave. Sin wave and cosine wave has specific characteristic, we all know. That this is the typical sin wave; this is the typical cosine wave, super impose them, and run for n number of seconds, you get a combination of, the final result which depends on, what is the weightage of the cosine function, what is the weightage of the sin function, in the final result you understand the point?

Depending upon the multiple of X naught, and X dot naught be omega, the weightage of the sin and cosine function of final solution, will amalgamate to get a result like this. But this will definitely have, a variation, along it is time domain. So, we call this as, time history response because it is varying with time. So, by writing equation of motion is Simple Harmonic or, in any other method, we got equation of motion, which is fortunately for us an ordinary differential equation of second order, because the variable is of highest orders 2, in this case D square Y by, DT square it is a second order. I have used complementary function and, particular integral in this case PA is 0; because the right hand functions of this equation of motion is 0.

So, there is no particular integral for this solution, I have only one answer, which is complementary function which is X of T , where there are 2 constants, I derived or, I determined both the constants, from the initial condition, which I have assumed here. Very easily, for a given mathematical model, I can write equation of motion, solve the equation of motion; get the solution which is, dynamic, response, analysis of, the single degree freedom system, under the given condition of M present, K present, C naught present, and FT naught present. Similarly, we look into the other remaining 4 condition one by one, next classes, and we will see qualitatively will plot them and see, how they look like. So, it is very interesting and very important for all of us to know, that these solutions should not be memorized. Further you will have more terms to come in, right? Do not memorize them at all, try to keep on writing them repeatedly and get it here. Do not memorize anything and do not try to reproduce them when it is asked, right?

So, try to memorize anything. Whenever the question is asked, understand undamped free vibration. What are the conditions present? Write the equation of motion, solve it from the first principle, and try to understand the solution, and then plot. And try to see what is the physical contribution of these terms there, why it will look like this. So, if you relate this from the first class onwards, your dynamics class will be very interesting, and you can approach any new innovative structural system, where dynamic analysis need to be done by you as a research scholar.

If you simply follow this mathematically by some standard text books, where people, all these equations all methods are explained in almost, all texts books available on earth which teach dynamics. But the way it is explained are entirely different. So, do not try to get confused, one should have mathematics background, no doubt, but still this is the very simple mathematics. If you are not able to differentiate this, I think, let us assume that we are gone. But if you are able to do this and write this, and extend it slightly and remember, we should be, I am always associating to you, remember the solution from the first principles of m present, k present, c naught present, therefore, no C term, F of T naught present therefore, there is no PI. Always relate it physically, and then it will be easy for you to understand.

Otherwise do not get carried away by the solution, and do not remember, you will never ever able to remember these terms. You will always make a mistake. You will put X dot here, and X naught here. It is always obvious. Or your friend will do that and therefore, you will also do it. So, it is very easy to remember, very simple to follow. Therefore, dynamics is a very, very easy and easiest of all the subjects actually is very, very easy. Ok you have to follow it sincerely from the first class onwards. And my book is also there, other books are also there, which ever language is comfortable to you. Just follow it accordingly. Slowly read the papers, not the text books, read the research papers also, then parallel text books, then the class your knowledge level will slightly go higher and higher. If you do not do parallel reading, whatever I am doing here will look like a biggest mathematics ever even the life actually, These are very, very elementary.

Then you will only leave with appreciation, no learning. By the time you start learning, I will finish the class. So, that is where, generally, people make a mistake. Do not learn, understand and try to reiterate and do this repeatedly, therefore, next class you will have one million questions to ask. You ask the questions, and we will clarify, that will be easy.

Thank you.