Dynamics of Ocean Structures
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Lecture - 47
Return period and Fatigue damage

So, we already discussed about the Stochastic Process, Stationary Conditions (Refer Time: 00:26) and advantage of assuming the process to be a (Refer Time: 00:30) and we gave estimated the first order statistical properties for a stochastic process, and we in the last class derive the connectivity between the load and the response what we call as the transfer function.

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We already said that the variance of the response is actually a function of the white noise which is modified input of the modification of the input spectrum which then, depends essentially on the square and we already know that the response are narrow binded process for weaken damped system for weakly damped system response spectrum or response is narrow banded. It implies that it depends essentially on the transfer function which depends on the ratio of omega, omega bar and zeta. Because transfer function is closely mapped to dynamic amplification factor which is the ratio of the frequencies and the damping ratio in the given system.

So, essentially this conveys me the exactly the same meaning as that of a conventional dynamic analysis, where I do not bother about the statistical characteristics, I only bother about the input value in terms of time history and try to find out the response either using the model response analysis etcetera. I also have the similar meaning there that the system response will be dependent essentially on the amplification factor of the system, which is factor or factor of the ratio of the frequencies and damping present in the system. I have also got exactly the same meaning in terms of physical understanding even through a stochastic process where we convey the same meaning and we call this as white noise approximation. Where the input spectrum is modified because the variation of the input spectrum compared to that of transfer function is not very sensitive.

So, we capture the sensitivity of the transfer function and keep it out of the integral and this integral is from 0 to infinity I mean minus infinity to plus infinity and so on. This is what we are discussed, now we will talk about classical definition and understanding of return period we have already highlighted this partially in the first module and we recapture it again with understanding of stochastic process. Then we will see how stochastic process can be useful for estimating the fatigue damage in the given system using linear approximation of cumulative damages.

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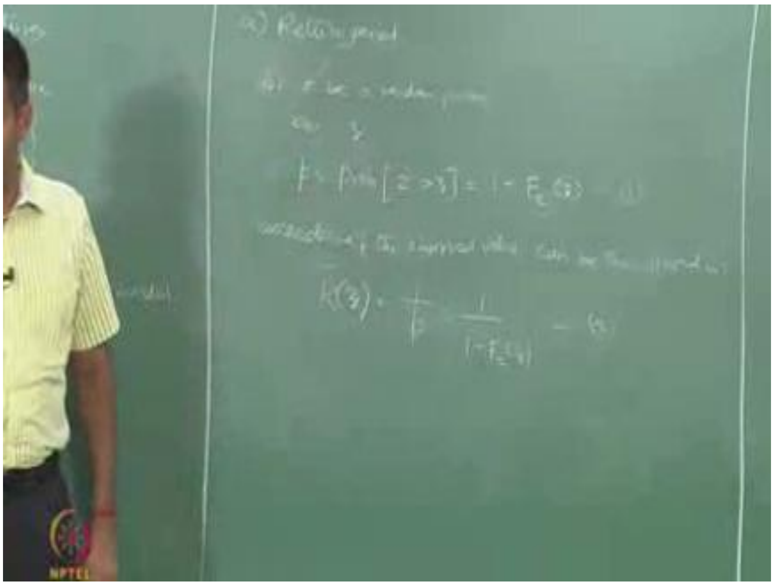

Let us talk about return period, now because period return, the moment I say return period; people have confusion about the reference period and the return period a lot of confusions are there return period can be expressed in 3 manner, one on the time domain manner one in terms of the risk associated with the return period. The third could be a general definition of any value in a given set let z be a random process the random process like we all understand it can be a wave height, it can be wave period, it can be a stress value, it can be a bending moment, it can be a deflection any value.

Let it be a random process and I want to compare the value of z of p with some of the threshold value small z and I want to see what is the probability of exceedance of z of t in a given set with that of small z, I want to see what is the probability of the value what I am observing in a given system exceeding any pre-assigned prefixed pre determined values small z then I will call that as period of exceedance that is what we call it as return period, period of exceedance. Let that probability p be probability of z exceeding, z which is given by one minus f capital z of z that is a nomenclature, z is the set of observation. z is the threshold value with which you are comparing equation number 1.

So, the probability of the exceedance of the value observed by you which is actually a random process by comparing it with some standard reference value because, when you say you have to compare it with come value that comparison value a fixed value and you are observing a set of values which is randomly distributed and probability of that exceedance takes place by this relationship. Now probability or let say exceedance of the observed value can be then stated as return period which is r bar z. r bar z which is one over the probability of this value, which is 1 by 1 minus f z of z equation 2. So, if you know this probability of exceedance then you can easily find out time period which is inverse of this, so inverse of this probability.

Obviously it is very important factor here which people generally associate this for return period you have observed set z, set of values which is a randomly distributed now there is lot of uncertainty in my recording z you must know one important factor associated with return period is at what intervals, you have got record z because the observation of z should be statistically independent it should be statistically independent. So, to make any observation statistically independent you have to have a sufficient delta t between the observations.

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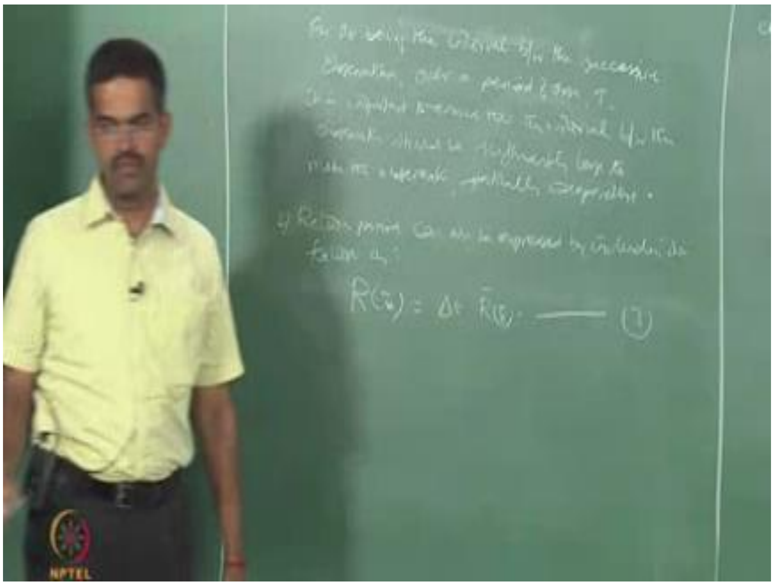

So, if delta t is for delta t be the interval between the successive observations between the successive observations, over a period of observation t it is important to ensure that the interval between the observations should be sufficiently large to make the observations statistically independent.

So, in that case the return period can be expressed this is a second form of return period, this is the first form of return period is this the return period can also be expressed by including delta t feature as given below. So, in this case we say that r of z here it is r bar here it is r, which is indicated as delta t of r bar I call equation number 3. Because this is also called as return period, but this includes a feature of time interval between the observations now we can also understand this as an example.

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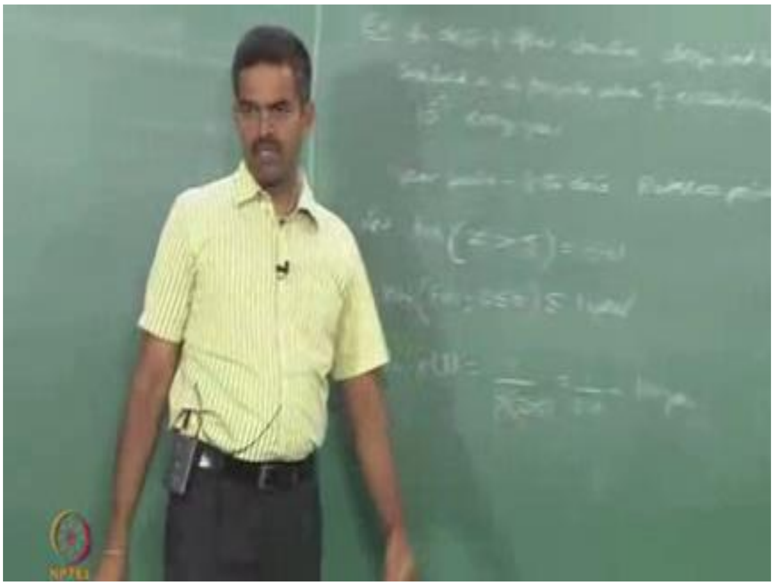

Let say in design of offshore structures, in design of offshore structures this is the design load which is also a variable is considered as a probable value of exceedance of 10 power minus 2 every year.

What does it mean 99 percent the design load, what you apply to the design will not exceeded 0.01 percent it will exceed that is in every year. So, there are 2 period associated here; one if we invert this I get a probability I get return period because that is how we got the return period in the first definition the other period. Which is associated with this is now this is called reference period and probability of exceedance of this which will give you the return period. In my case it is going to be let probability of the value which is a variable exceeding zeta is not the damping, zeta is some threshold value in the design load which is going to be in my case 90 percent it will not exceed.

Now, I have to define z, z is actually a design load there is a threshold value where; z is going to be where z is going to be the maximum of because I have to consider the maximum combination of the given loads which is considered as the maximum combination from the given set of node f of t for any instantaneous time for any instantaneous time within a given reference period of 1 year. So, that is the condition therefore, r bar of z, hence the return period r bar of z sorry will be one over probability of z exceeding zeta which is 0.01 going to be 100 years. So, that is how we calculate the return period that is going to be 100 years that is the simple example to illustrate, how we can compute the time period and what is the difference between the reference period and return period.

Now, we can also compute return period with terms of risk associated because every load has a got the risk the moment I say the value will not exceed by 99 percent. Now the counter question is if we exceed in a 1 percent, let say the 1 percent exceedance is possible what would be the level of risk, if it exceeds because we are worried about the safety ultimately and safety and risk are contemporary exceedance.

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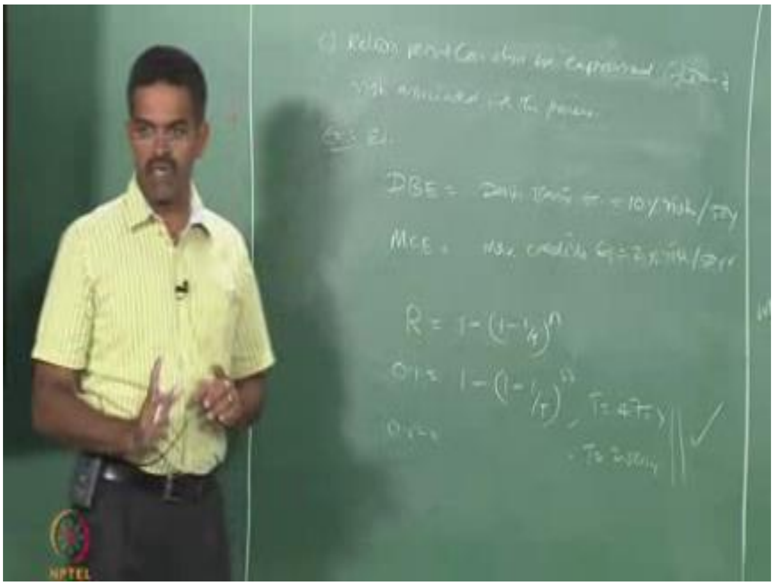

So, return period can also be expressed in terms of risk associated with the process.

For example, let us take earthquake as the earthquake is loading process which is stay stochastic process. So, there is something called designed basis earthquake maximum credible earthquake. There are 2 earthquakes which are used in the analysis design basis earthquake design basis earthquake maximum credible is for the redefined as configured also maximum considered earthquake, what is actually credible earthquake? Now they are given with respect to the risk associated the design basis earthquake has got 10 percent risk occurring at 50 years. Whereas, the maximum credible earthquake has got 2 percent risk or it is associated with 2 percent risk.

What does it means? So, there are say there is an input signal which we are going to consider for the design if I want to call the input signal as a design basis earthquake signal then, it has got a probability of exceedance by 10 percent within 50 years, Whereas, if we have signal which I want to call or qualify as maximum credible earthquake then, the exceedance of that signal it may gives a life or it may give a system is only 2 percent. Now I can compare these 2 and get back my return period on these 2 like the equation here. For example, the risk associated is connected to the return period by a given expression which is 1 minus 1 minus 1 by t of n.

So, let us say risk is 10 percent here that is 0.1 that is what I am looking for my d b e design based earthquake. So, 1 minus 1 minus 1 over t t is the return period I do not know; whereas, the reference here is fifty if you calculate t you get t as 475 years whereas, when I substitute this risk as 0.02 on the other hand for the m c e then I will get t as 2500 years. So, this will now give compatible physical picture of understanding it is that earthquake which will be exceeded in it is process not in the system in it is process by only 2 percent, which will reoccur once in 25000 years, but the 2500 year can be this year also where you have designed your platform. So, you cannot say since I have designed a platform today in 2015 my return period is 25000 years therefore, my service life is only a 40 years or 30 years they did not bother it is not taking about the design life of the structure it is the return of the process we really do not know when last 2 percent exceedance occurred.

So, but if it would have occurred if we know that we are alive during that time we will be sure that 25000 years after only it would get returned, but since you do not know that because the process is random as I said random process is values fixed you do not know the value. Therefore, it is random. So, if it is random process you do not know the value. So, that occurrence period can be this year, where you commissioned a platform therefore, risk are always there. So, risk when associated with return period will give you the fairly a practically meaning of understanding the connectivity between the return period and risk where n is the reference here; that means, return period cannot be quantified in a given design if you do not have a reference period. So, do not try to give a reference period return period without reference period there is no meaning this how return period is generally qualified in stochastic process.

The second one could be how this process can be used for fatigue damages. Now why we are bothered about fatigue damages fatigue is a process where the amplitude is not exceeded instantaneously. So, there is no damaged caused by excessive amplitude the difficulty with fatigue is the amplitude is within the threshold value, but the number of cycles the material subjected to has exceeded the threshold number.

So, you the amplitude is very small, but you are doing that amplifier n number of times you are whispering on some body's year 1 million times a day. So, he gets irritated. So, that irritation is the fatigue damage whispering is a lower decibel level maybe one decibel or 2 decibel you are not you are not disturbing anybody, you are going and simply whispering on his ears about 3, 4 million times in a day. So, I get irritated. So, that is the damaged caused to the ear because the amplitude is. So, low, but the number of cycles what the ear can sustain the whispering is very very high phenomenally. Therefore, you cannot sustain or the ear gets damaged or person gets impaired.

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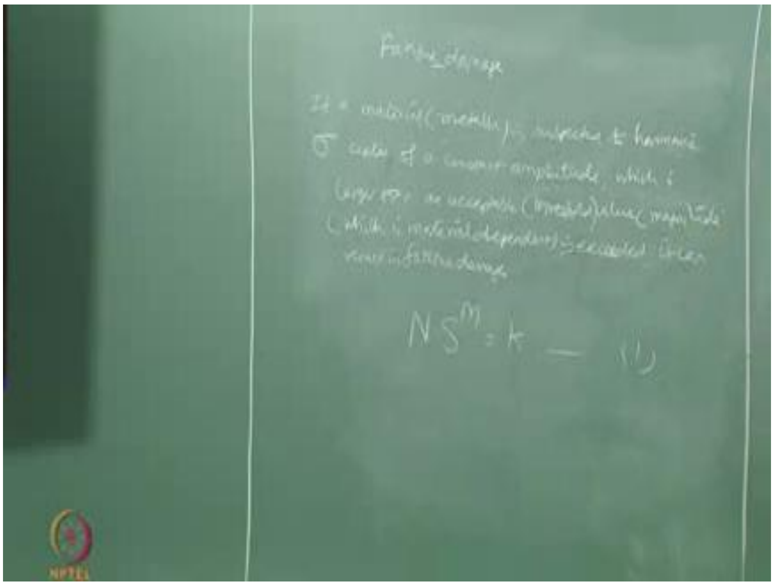

So, let us talk about fatigue damage fatigue damage can also be estimated using a stochastic process like this let see how the fatigue damage is if a material essentially metallic composites all these are not included in this we should have the sustainability for reversal of loads all material do not have this your friend should able to listen to you 1 million times if he closes his ear no fatigue. So, material should allow reversal of forces. So, essentially metallic material unfortunately comes with this inbuilt characteristic. So, metallic materials is subjected to harmonic stress cycles why is it harmonic? Harmonic has got cruff, I mean trough and crest harmonic stress cycles if a material is subjected to the harmonic stress cycles of constant amplitude of constant amplitude which is larger than certain number of acceptable value.

What otherwise you call threshold value we can always say this as magnitude value does not make any meaning because, I am talking about the stress I should talk about the amplitude now one can ask me question, how will you decide what is the acceptable magnitude of the stress value this is material dependent how can we compare this. For example, a whispering on ears of a and ears of b a can sustain one million cycle b will not sustain even one more the reaction of a and b to the same whispering decibel of the same number of cycles of the same amplitude is different because material characteristic of a and b are different.

So, material dependent can cause fatigue damage is exceeded, it can result in that is the statement. Now if a material value subjected to harmonic exceeded it can result in fatigue damage interestingly there is very fundamental characteristic about the fatigue damage because people talk about let say fracture mechanics there they will talk about fatigue and fracture in detail, we are not touching about that because we are doing fatigue discussion only for about 15 minutes.

So, this is very difficult to qualify a understanding on fatigue damage and fatigue responses in just fifteen minute, but since I am interested to touch upon this topic this is one of important areas in stochastic process touch up on this important character of fatigue damage is damage is always cumulative, you it is irreversible and cumulative that is the problem every fatigue damage has to be computed as a cumulative because the fatigue damage being a low amplitude value it does not demand irreversibility because, the damage does not occur actually it is always cumulative that is a very serious problem here and most of the successful models where people use fatigue damage estimates are linear.

i will talk about these 2 points now. So, one is cumulative one. So, now, let us see how to compute this is given by a simple relationship which is n s power m is k i call equation number one where m and k are constants depending upon the material type of loading.

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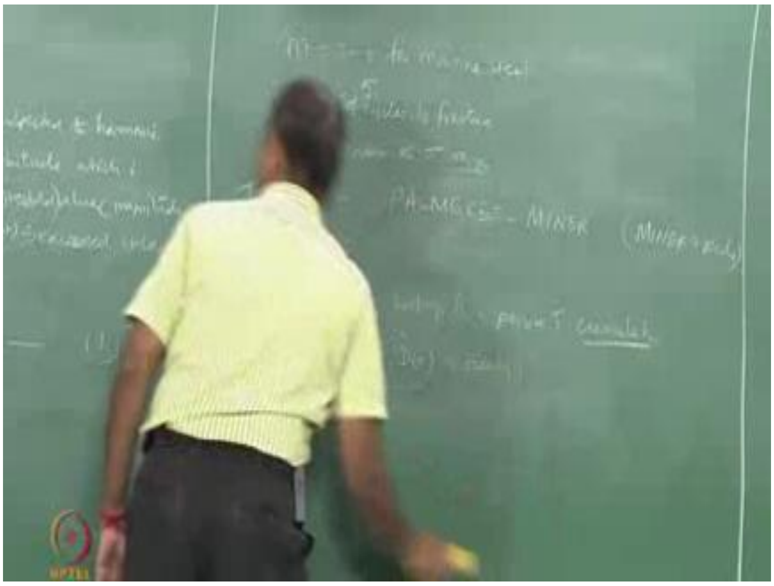

Simple number, I am worried only about the marine steel that is 3 to 5 do not for all the material same number varies n denotes the number of cycles to fracture . So, let us be very specific the numbers of stress cycles we are taking about stress magnitude, here of course, as I said you have to fix up a threshold value. So, s denotes the stress range these are all stress symbol do not write as 6 stress value otherwise you will read this as 6 range and etcetera 6 cycles we will complete the confuse I think you are clear with this notation stress.

I am writing avoid some writing the word I am writing the stress value. So, while we are talking about range in stochastic dynamics you cannot always qualify an exceedance by a single number why because of whole process is random you have to give a range the moment I say the range always I say range width or bandwidth. So, from that plus or minus, so s minus a and plus a the range is 2 a that is, how we fix up the range that is always it is the other way if you say s plus 2 a no fatigue. Fatigue is only a reversal. So, I have a threshold value plus and minus of that value is actually fatigue we have only one value and say it is exceeding only not coming down no fatigue. So, bandwidth is always plus and minus of half of the range. So, you get the range double of this value say a k minus this.

This hypothesis was suggested by was suggested by pamgress minor called as minors law called as minors law in fatigue estimates this assumes the damage to remain linear, for a given loading history. So, that is the important input you need you may ask me why this input is required if you do not have the loading history you cannot calculate the threshold range of stress value number one number 2 from the loading history you will not know how many times it is exceeded and it is not exceeded. So, you have to have the history the moment you talk about the history I must always have a period for a given loading history for a period t because I cannot consider infinite history for a period t cumulative fatigue damage which is d tilde t is given by small n j by capital n j where small n j is a number of stress cycles in the time history associated within the stress.

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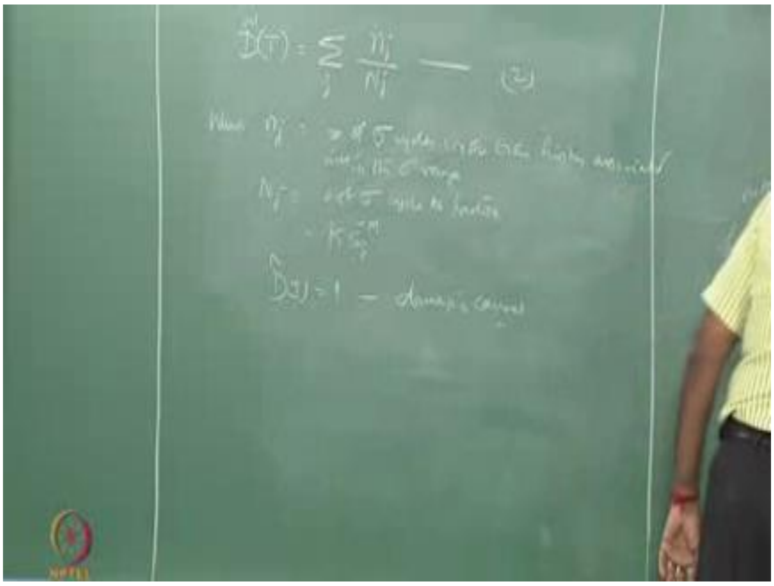

We are not bothered about the value of stress values beyond the stress range only in the stress range the number of cycles are associated we are interested this is not fix this again stress. Stress cycles associated capital n j is the number of stress cycles leading to fracture which is given by a simple expression which is k s j to the power of minus m we got it from equation 1 I just reversed it. The cumulative damage or the damage is occurred when, d tilde is, one when it occurs damage is caused I mean for d tilde equals 1.

Now, one can ask a very interesting question why fatigue damages are cumulative in nature.

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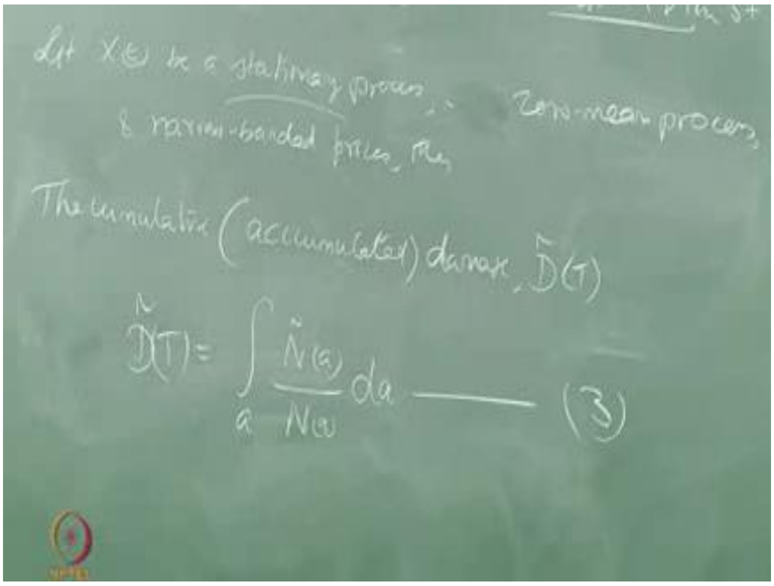

Why fatigue damage it is cumulative why I should consider cumulative effect when because when n j may be one of the element of the capital js is more or less equal to the exceedance of stress value within the given range it heeds away the life time of the structure physical example one, cigar smoked has eatened away for example, 30 seconds of your life. So, 2 cigars already 130 have gone the second cigar again 30 seconds. So, it is cumulative damage though smoking 1 cigar alone will not cause fatal stress exceedance is there. But it is within the threshold value. So, the reverse effect is not visualized by the human being.

So, when you keep on smoking continuous cigarettes and you can say today you have smoked only 5. The doctor would say you have smoked 30 minutes cigarettes earlier, therefore length is spoiled. So, cumulative damage the threshold value is exceeded cumulatively because, it eats away the life time of the structure or the human being parallelly exactly that is why the damage in fatigue is cumulative. So, let us take a stationary process x of t let x of t be a stationary process because, when we wrote this sentence for the first time in stochastic dynamics class we were worried that what is stationary process whether it is the same stationary as we are getting, not here worried about that. Now we really know what do we actually mean by a stationary process because we have lot of information about that.

So, we know that stationary process and 0 mean process I can add more complication to this because we all are expert in what is a 0 mean process, 0 mean and narrow banded all will happen because we already know all this things will happen in a given system process. Then the cumulative actually literature uses the different word they say accumulative that is what the literature says for me I understand easily this word cumulative accumulated damage d tilde t for this case we will be given by over a stress range a over a stress range a, it is a continuous variation n tilde a by threshold value n a d a equation number 3 the denominator n a where n a can be given by the equation.

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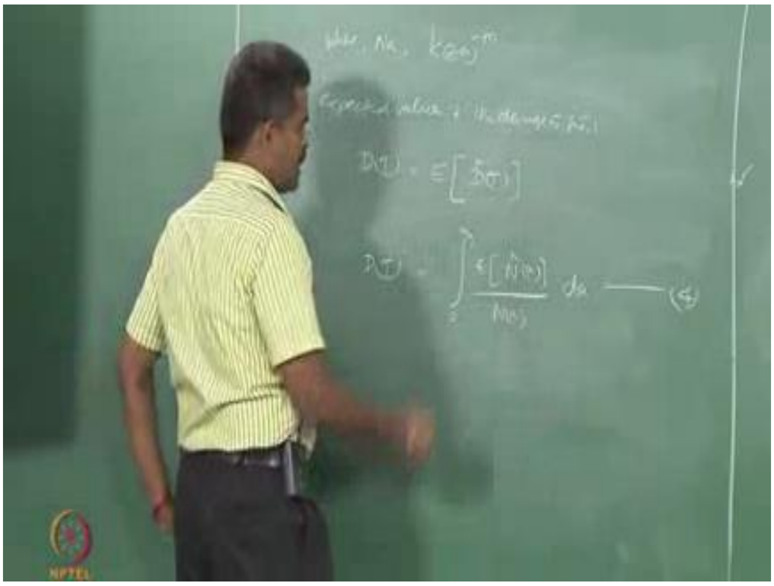

We already know which is k the stress range to the power minus one, why I am putting 2 a here because the stress value a and plus and minus a I get 2 a whereas in this equation I must put the full range 2 a minus m and m is the material property of t now the expected value there is where I moving to stochastic's the expected value because, always I am looking for a mean value. So, the moment I say expected value of the damage is given by b f t which is expected value of d tilde t which is expected value of n and d a equation 4.

Of course, this goes 0 to infinity because mean is only the possible response of the system.

Now, here we are worried about the expected damage of this which is n hat which is actually the observed value. So, this can be computed for a given von mises stress failure. There are different kinds of stress theories available in the literature von mises stress theory is one which is popularly used for fracture damage estimates.

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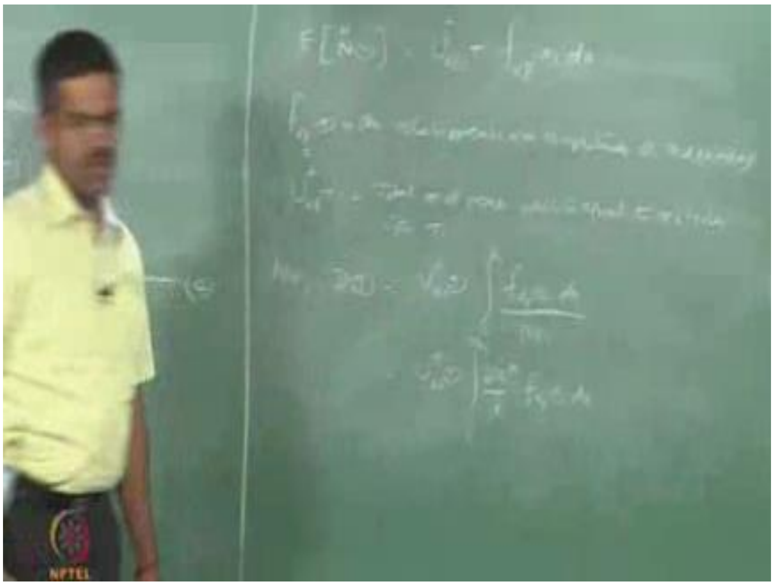

So, we are talking about the expected value of n hat or n tilde of which is easily given by the multiplier of the response amplitude function and the power spectral density function, which we already had in the first lecture of stochastic. So, I am re replacing it with a different symbol. So, v x 0 t there are not multipliers there are actually one term, v x 0 of t only the positive value v x 0 t of that of f of x p a d a. Where f of x p a d a or f of x p a is actually the relatives of the peaks with the amplitude is the relative peaks with the amplitude that p stands for that peak value in x in the process relative number of peaks relative number of peaks with amplitude a and of course, there is the range.

So, with amplitude a and a plus d a increment it whereas, v x naught t is actually the total number of peaks which is equal to number of cycles, which is going to be same you can either count the peak exceedance or the number of times it has exceeded. It is one and the same actually in a given value for count either they have threshold exceeded or the number of times exceeded practically gives the same number which actually as same as the equal number of cycles exceeded within t. Because we have look at the window of capital T now d of t, now d d of t can be given as because I have e expression expected value of n of t here I can substitute this back.

So, this is going to be a constant total number. So, v x 0 t integral because I have an integrated function integral with function there which is f x p a 0 to infinity f x p a d a by n of a and already we have a expression for n a then, substitute that back here therefore, that will become v x 0 t 2 a m by k f x p a d a now, for a narrow banded process already we said that x of t is a stationary narrow banded process for an narrow banded process it is generally Gaussian distributed.

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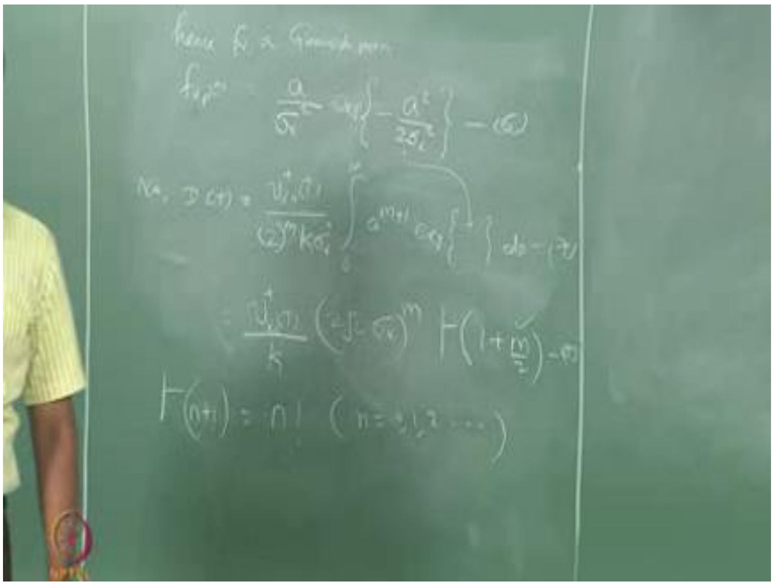

Hence, for a Gaussian process the power spectral density function f x of p a can be given by a standard expression available in the literature which can be given by a by sigma x square the variance exponential which is e power minus a square by 2 sigma x square e power minus I call this is equation number I mean, I do not know if you are left 5 somewhere this is going to be 6. So, now, substitute back this p s d on equation d of t get d of t back. So, d of t now d of t will be given by which is accumulative damage estimate I will write down the expression in short form which is v x 0 plus t 2 minus, I am writing in a specific format which is comfortable for me to explain later k sigma x square integration 0 to infinity a m plus 1. Because there is one a here there is one 2 m here am plus one exponential is of this function.

Now, this integral this is a standard integral which is again it is a da which can be evaluated as a gamma function. So, I write that value here v x naught t plus by let say k 2 root 2 sigma x 2 root 2 sigma x to the power m gamma function one plus m by 2 m is the material property. Now for our understanding we have very good engineers and mathematicians gamma function of n any variable n or let say be very specific n plus one because we have an gamma function here like this is nothing, but factorial n for n equals is it going to be 0 no 0 1, 2 that is the standard equation function available in the mathematics, gamma n plus one. So, if I know the value of m I can estimate this. Now I will call this equation number let say this is 7 and this is 8, 1 is actually not interested in estimating the damage accumulative damage ultimately. If the structure is getting accumulative damaged if a person is smoking a cigarette continuously what is the life time of that person. So, life time estimate.

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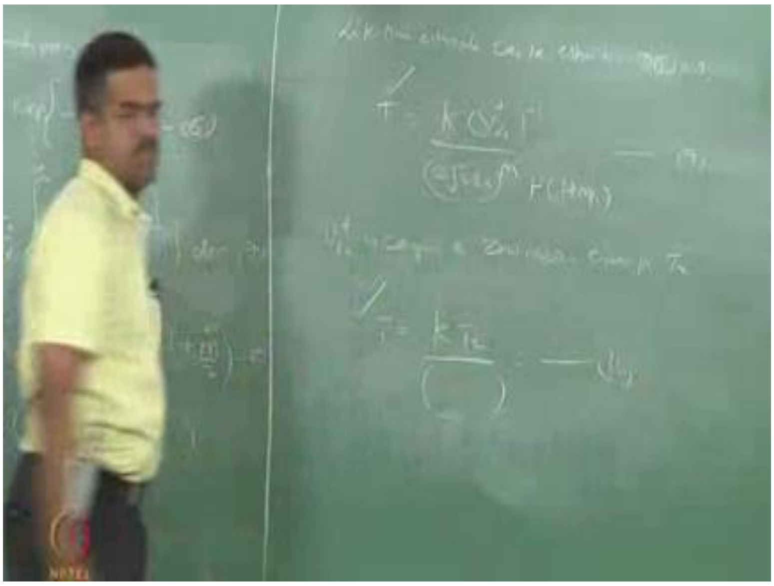

The life time estimate because we know that to get the life time estimate damage has occurred. So, I must say d of t is one damage has occurred. So, substituting d of t as one in the equation 8 can be estimated by setting d of t as one in given in equation 9. If I do that I will get t there is no bracket here t as k v x 0 inverse 2 sigma 2 2 root 2 sigma s m by the gamma function. So, I get this one life time call equation number 9. So, one can get the life time estimate if you know this.

Now, interestingly it is difficult for us to compute v x 0 for a given system. So, v x 0 can be in stochastic process comparable to 0 mean crossing period b x naught is comparable to 0; me crossing period which is known as t z in the literature. Therefore, t which is nothing, but k t z equation I get the life time estimate. So, I can use stochastic process to estimate fatigue damages also using the minors rule or using this kind of relationship, where I know all of them because I know the variation of the process the standard deviation, I know the material characteristic this is the material property again type of loading k is known to me t is let us of course, the count available from the stationary process can easily find the life time estimate for this. So, it is very easier and closed form solution which can give you the damage estimates using stochastic process.

So, these lectures of 4 to 5 lectures in stochastic dynamics have made you to give an introduction to how dynamics can be done. Even though people claimed that the given input loading is not defined, it is not well prescribed and people say the variation in wave heights are. So, high it cannot be considered as random stationary process, there are arguments they can be considered stationary process. If you taking if you took long term statistics they can be called as stationary process, there are arguments plus minus. So, stochastic dynamics is in the verge of let say a research application in offshore industry. So, we have computed all the 3 modules.

Now in the module 1, we talked about the introduction dynamics we moved on further from first degree to that multi degree. We solved problems with 4 and 5 degree here then, in the second module we applied the dynamics understanding to emphasize fluid structure interaction, we picked up various generic modules then we picked up specific examples of offshore platforms.

We derived the complete equation of motion terminologies, we discussed in length about the damping properties different varieties of damping and we did lot of numerical examples to understand structural dynamics in terms of fundamental applications and appreciation. Then we moved on to higher understanding of dynamics using stochastic theories or stochastic process like this which is also one of the methods of appreciating. We have discussed time domain solution, we have discussed iterative frequency solution, we have discussed about the transfer function approach which can be used for frequencies domain approaches etcetera.

So, we have got and we have discussed the Riemann zeta algorithm, we have solved examples and we have given research papers for reference to read and we have made a 1 to 1 correspondence. What has been published to the paper? What experiments have been done? What numerical studies have been established? How they are connected in new generative software structures? That IIT Madras, where we did lot of experiments on new forms of offshore platforms which has been also told we have got extensively a good understanding through this course on structural dynamics applied to ocean structures with various salient examples. Where we stand vertically special on certain areas of ocean structures, where we conducted experiments and published literature we have authenticity we can say that yes we are able to at least relate the fundamentals of dynamics to application problem of offshore structures using this kind of analysis and system. So, we also showed in between the classes and lectures that how design can be also sufficed using structural dynamics as an analysis tool.

So, why dynamics is important how designers look dynamic analysis as a primary tool in the design stage itself we have sufficed that also. So, it has got a very good let say horizontality and of course, in certain areas very great verticality's on certain subjects of interest, I think I am very confident that whatever lectures we given in turning back again and looking at this and we have got about 2 books, authored at IIT Madras, on this area there are many courses available many text books available on structural dynamics many authors have contributed all of them are equally important we have referred more than 85 paper in the NPTEL website. All papers, all books, all notes, all will converge to a single understanding on enhancing confidence in dynamics. If that is the achievement what we are able to do in the end of the course. I think we have done our job.

Thank you very much.