Dynamics of Ocean Structures
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Lecture - 45
Response Spectrum

So, in the last lecture we discussed about (Refer Time: 18:00) how to check the (Refer Time: 20:00) in a given process, and we also understood that, what is the advantage of making the processes stochastic process.

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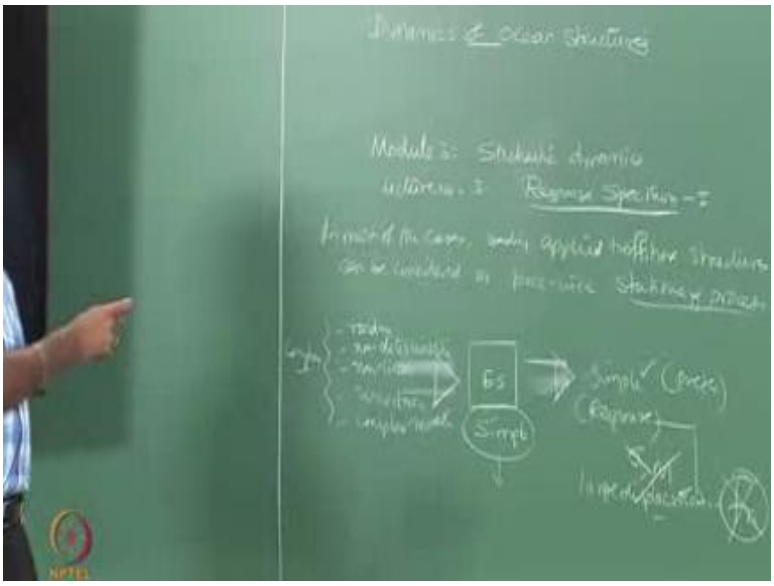

So, in most of the cases, the loading applied to offshore structures, can be considered as piece wise stationary process, now the first question comes here is, what is the stationary process, and what is the advantage here is the process become stationary, and therefore, how one can connect the response to the load. So, what we are actually interested is like this. I have a system, an engineering system, may be a platform etcetera. It is subjected is an input loading, which is random, non deterministic. May be the loading in nature is non-linear, because (Refer Time: 01:52) variation etcetera, highly uncertain cannot be predicted, and complex models in terms of equations, suggestions etcetera. When you apply such a complexity, or set of complexities to the system, which may be simple, even if it is simple the outcome what you expect, need to remain simple; that is what we prefer.

Usually, the outcome what I want, is the response. It is actually not the stresses and moments, we do not look at these, because if you looked at the stresses and the moments you can always design the system, to take care of the stresses. There are many different methodologies available to take the level of stresses from elastic limit to yield level to breaking failure etcetera. So, we had looking at this stress levels at all, we look at the responses. These responses will induce large displacements, large displacements will cause undesirable stresses and moments, but large displacements affect functionality; therefore, we have problem. In the large displacement do not affect the functionality of the platform; we actually do not bother about that at all.

There are systems which have been designed for very large displacements and they exists, but in our case we do not allow this, because it causes lot of uncomfortably for me production in process equipments etcetera. The marine are the pipe line risers which are disconnected. So, there is a very serious challenge on functionalities. So, therefore, we go for a form design system, where the functionality is not compromised, only the form is altered. So, we wanted to have with the response in a simple order, for a given input which is highly complex in nature. Now look at in the analysis process, if I have a stochastic model or random model let us say, even though the system remains simple, the results what a derive from the analysis, will also the as complex as the inputs.

When you talk about the non-linear input, it is expected, psychologically at least that the output will also be non-linear. To add to the complication my system is never simple, my system is very complicated than the loading itself, because the system itself will induce non-linear loading, where just know we saw in TLP case the system is response dependent the whole system design is response given. So, the system is not simple, it is very complicated. Now if under the given scheme of this window, is there a probability or a possibility of making this connectivity linear. The connectivity is between to parameters; one is the response, one is the load. Load is uncertain non-linear, response is uncertain non-linear. Can I make it linear; can we have a window or bridge which makes it linear, and time invariant?

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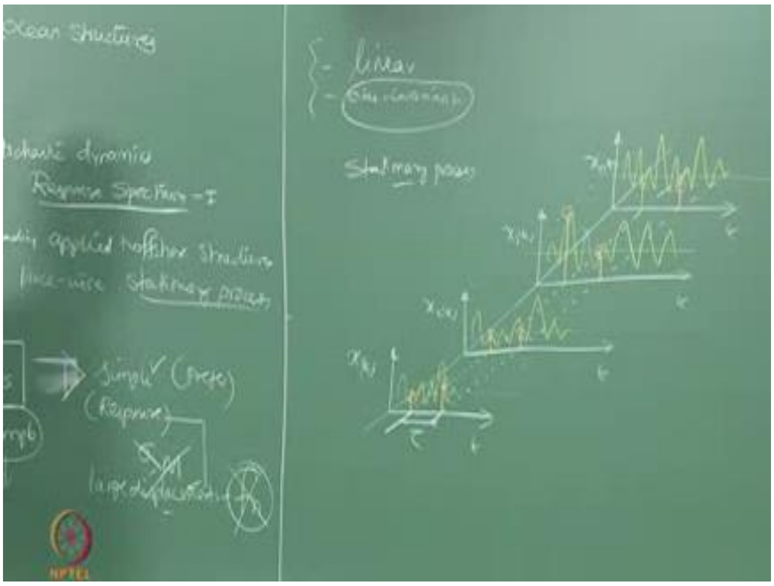

Now, one can ask me a question can I make this connectivity linear, and time invariant the movement it a time invariant, then you will say sir is it a compromise on dynamic analysis, because dynamic analysis actually a question on time variant properties. Still dynamic analysis will exist, because I am only compromising on the time invariance property of the complexity, but not foregoing or compromising on the dynamic characteristics system. Therefore, the dynamic analysis will still be existing, but I want to make a compromise on these two parameters. The best answer to this compromise is stochastic analysis, you cannot do otherwise also. To make it compromise on this area, I must assume that the process which is encountering the system, is stationary. It is not static, it is stationary. Then immediately the question comes in mind is, what is the stationary process.

Let us say I have a window, which is having series of data's, let us say I have a data here, let us have another data here, let us have another data here, another data I call this as x 1 of t x 2 of t x 3 on t x n of t, where x is the variable not the response, just a variable, and of course, in all these. In symbol the x the x axis in this case is our time scale, I have a plot like this. Now I am cutting two windows here; one at any time, let us say t one, and another time t 2, which is different by tau, and I am trying to measure the response of these two windows. Let us say this window is responses this value, and this window responses this value, let us say this value, let us say this value, let us say this value, and let us say this value. These are the points may be (Refer Time: 8:33); one is average along n symbol, the other is average along the time window; one is average along n symbol, other is average along the time window. I want to take a statistical property of first order for this.

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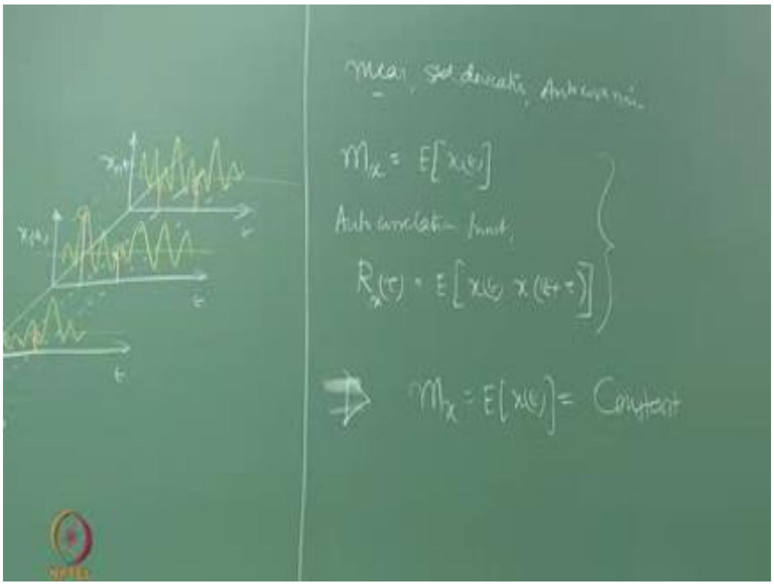

So, the first order statistical property may be mean and standard deviation, and auto covariance. Let us talk about them if I say m of x which is the mean of these variables, is generally the expected value of x t. If I take the auto correlation coefficient r of r x of tau, which is expected value of x of t x of t plus tau. If these two values are equated to; that is mean value of x, which is expected value of x of t, remains as constant; that is showing time in variance. How this is appearing? This is possible only when mean along the n symbol, and mean at every window along the n symbol, gives me the same mean. So, it becomes time invariant, provided tau is very close. So, it is become time invariant. So, the mean is only along the data, not along the time history. So, if this becomes constant, then my process can be termed as a stationary process.

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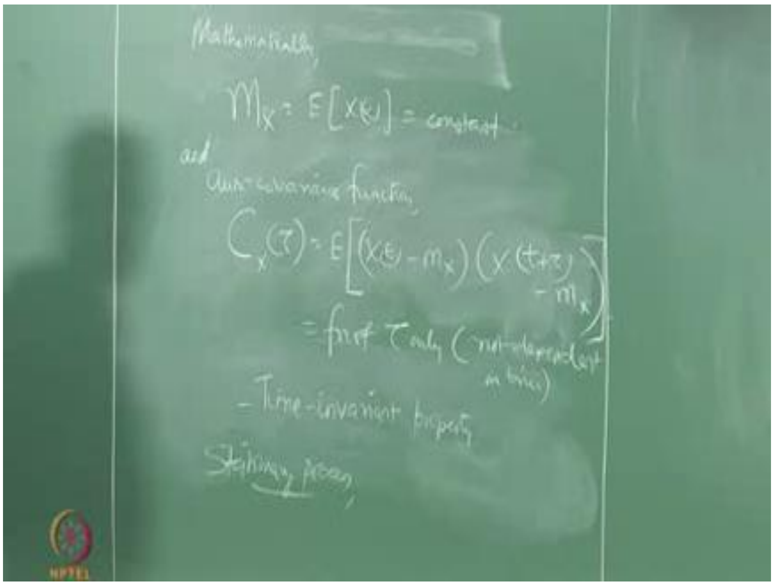

So, mathematically m of x is expected value of x of t, is constant, and auto covariance function c of x, which is along tau is nothing, but the expected value of x t minus m x x t plus tau minus m x is again a function tau only, not dependent on time. So, they demonstrate time invariant property, and the data is averaged only along the n symbol, only along the data. Then the process can be turned as a stationary process, mathematically. Now, one can ask a question, what is the advantage of turning this process as a stationary process, provided mathematically this is valid? Let us say this is proved we are able to show, that they are time invariant, what is the advantage. The advantage now is the following.

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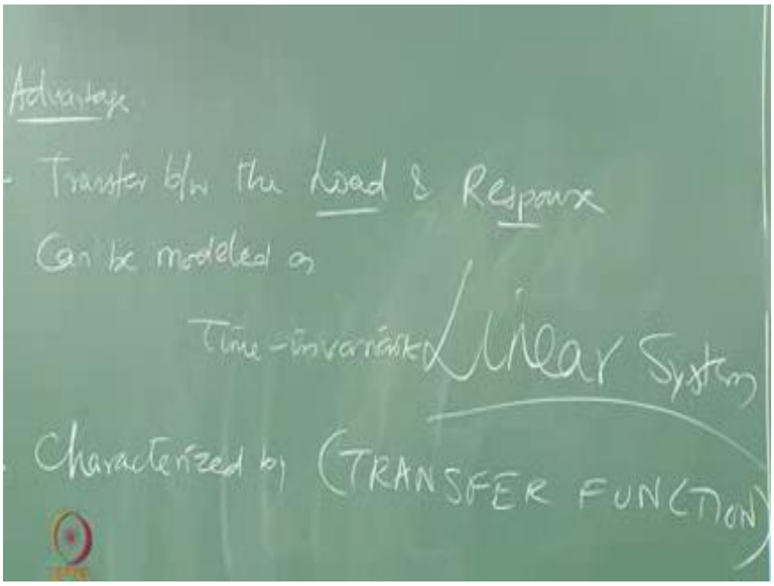

One of the main advantages what to gain is the following. The advantage is once you have a stationary process the transfer between the load and response, the transfer or the connectivity between the load and the response can be modelled, as time invariant linear system. So, you have two properties achieved automatically; one is the time invariant or the connectivity can become linear. Now, one can ask me a question, if I have connectivity of this order, how will you characterise this. This can be characterised by what we call a transfer function. So, one can characterise this connectivity, by what is called as the transfer function. Now the question comes in mind is what is the transfer function I have to look like; how it is characterising or connecting response and load in the linear system; that is the question coming in the mind.

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Now, let us say A is the variance of the response. We already know what is the variance. We have defined already in the last lecture, what is a variance. Let us say the spectrum of variance; that is the variance spectrum of the response, this is nothing called response spectrum. Various spectrum of the load which is called the load spectrum, the relationship between the response spectrum and the load spectrum, is connected by Trans function. Demonstrate this mathematically by an example, because now there is an apprehension in every body's mind that, the movement you make this connectivity linear and time invariant where is a question of non-linear characteristics coming into play, number one.

Number two, where is the dynamics coming to play here, because I am making it time invariant. Let us look at the quality of this transfer function, and then you will know that I will ultimately connect transfer function to be as same as the dynamic amplification factor. So, you will easily understand the dynamic analysis is not killed or compromises in stochastic process, except that this becomes the easy connectivity between l s and r s, where l s stands for the load spectrum, and r s stands for the response spectrum. This is the very important domain of solution for people, in frequency domain analysis.

Generally you will not get the load history; you are always got a load spectrum, given to people. You do not want the time history, you generate a response spectrum. Now, one can ask the question what is the easiness of developing response spectrum, compare to that of a spectrum what we generate from the time history, because we have a time history response, we can always convert that into frequency domain easily while doing a transformation. What is the advantage? The advantage is response spectrum is a qualitative indication of response of a single degree freedom system. It is the qualitative representation, of the response of equivalent single degree freedom system. Single degree freedom system is always a focus on dynamics. We know how to solve them very easily it is a looking at a complication of the system itself, look at the response spectrum of single degree. So, now, let us try to see how the character of this t f looks like, by taking mathematical example.

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So, let f of t denotes stochastic stationary low acting, low process acting on the engineering system. Now, it is assumed that f of t acts as linear time invariant. How I can say this. Because already said it is a stationary process, which will have an impulse response function, indicated by h F x t d, be very careful in writing in nomenclature. It is written as h F x t. It is not h f x of t, this is wrong, this is not h of x of t, these are all wrong, it is written like this. There is a very interesting meaning for this; I will let you know later may be after few minutes. So, the impulse response function is given by this, but this is now the response of the system, why it is response of the system, because it is actually the outcome of the system, when such a load is acting on the system.

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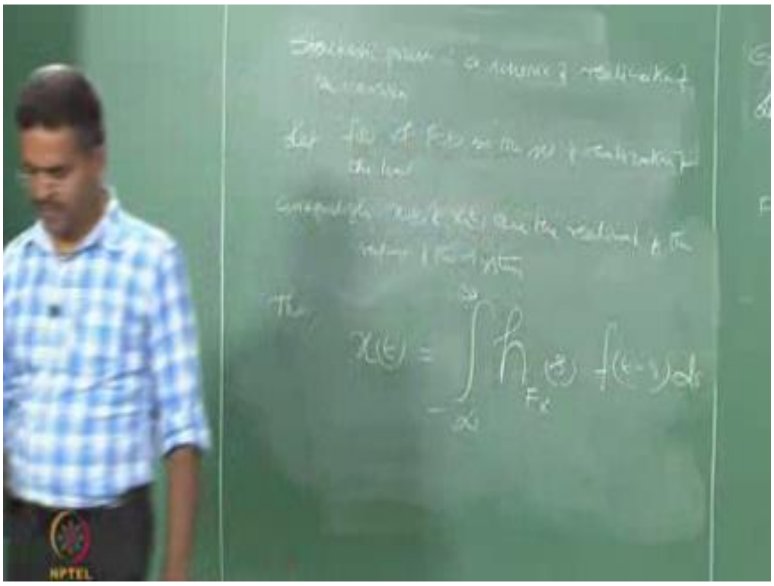

Now, we already know stochastic process, is a scheme of realisation of the variables. So, let f of t of f of t d the set of realisations of the load, let f of t of f of t, being the set of realisations of the load. There are many f of ts available out of which these are the realisable we already discussed what is realisation value, f of t. Correspondingly x of t of x of t are the realisations of the response, of the system, then the following equation holds good, because we already wrote this equation. The realised value x of t can be written as minus infinity to plus infinity, the impulse response function which is f x s of f of t minus s d s, I call this equations number one.

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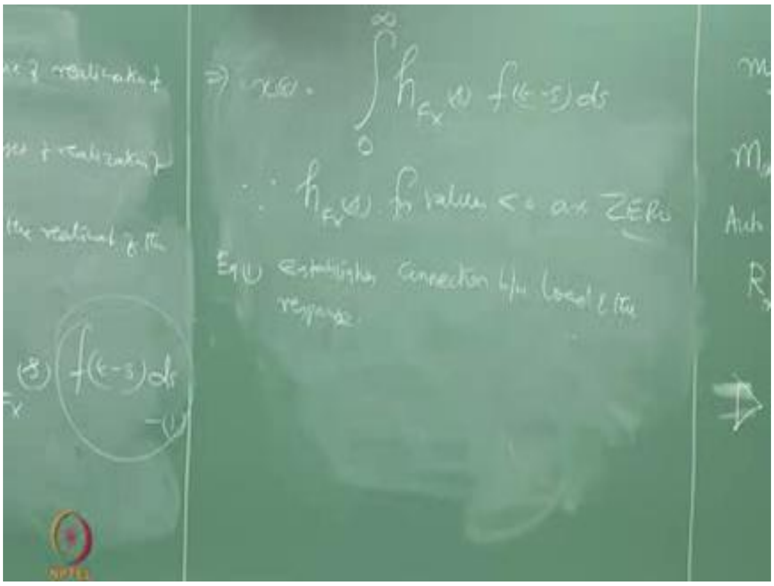

Which can be re written as x of t 0 to infinity h f x s f of t minus s d s, because h f of x of s for values less than zero are (Refer Time: 21:32) negative values have no impact. Now, equation one establishes, connection between the load and the response. One can ask me a question where is the load appearing here. As I have already said f of f around physical realisations of set of f of t; similarly, x of physical realisations of set of S of t. Now, one can ask the question say this t here, but this is t minus s here; since these both processes are stationary, these term or a statement is valid. Now we have connectivity between these two. I already said this time invariant and linear, so therefore, the integration need not be applied on the system at all. Let us rewrite this equation again now.

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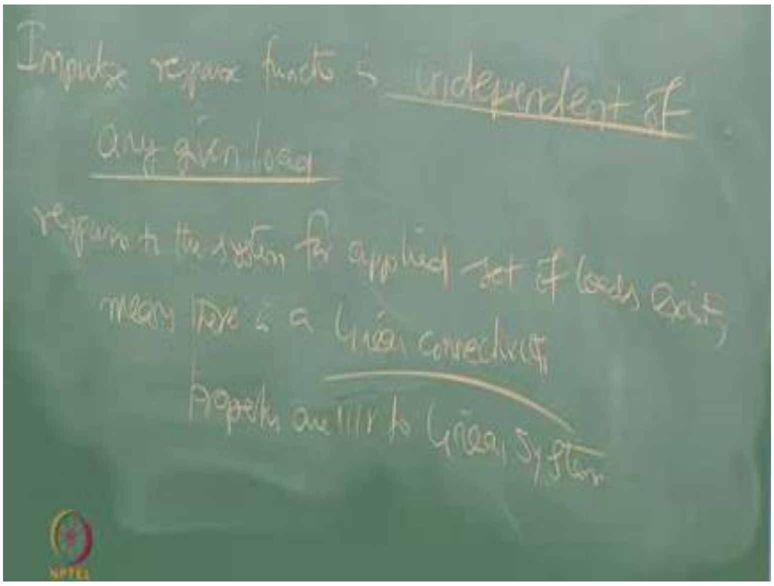

Hence x of t which I am now talking about the whole set, not the realised value, whole set is still valid between the range of zero to infinity, because negative realisations have no consequential meaning in the analysis of h of f x s f t minus s d s equation two. Now, one can ask me a question, what is the difference between equation one a, or one and equation two. Equation one a or one, is realised value of response, connected to realized values of the process whereas, equation two is connecting the full set of response, to that of full set of the forces. It means that equation two implies that, for response and load pair, has corresponding realisation. It means within a band of zero to infinity positive whatever may be the load applied, it will have a response.

There is nothing like an invalid response for a given load, because it means that small x of t or capital x of t small x of t are almost equal in terms of one a and two, stating that for every load applied there is a corresponding pair of response in this band, which is connected by the transient function, or by a impulse response function. Now the simple response function has got some special characteristics. Let us write down that, the impulse response function is independent of any given load; that is the first point. The second is, the response to the system, for applied set of loads exists, meaning there is a linear connectivity, or the properties are similar to, linear system.

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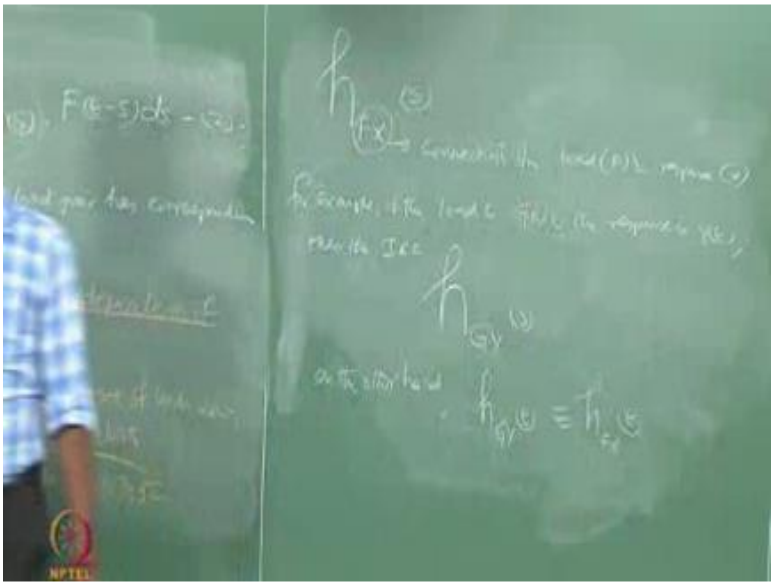

Now, interestingly let us talk about this suffix, let us say h f x s denotes, the suffix here denotes, connectivity between load and response. It is nothing than non-linearity of this; for examples, if the load is g f t, and the response is y f t, then the impulse response function could be indicated as h g y s. On the other hand h g y t is as same as h f x t. Now let us us talk about the mean of this stationary process, mean value of this process.

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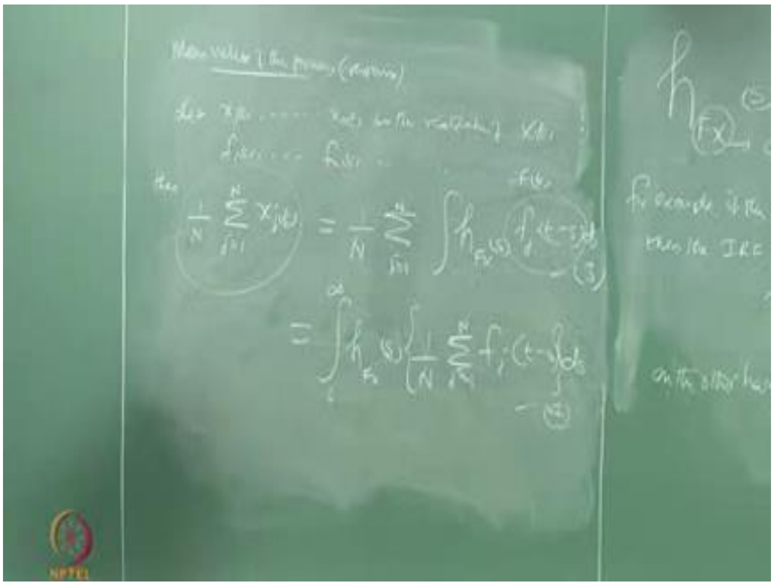

Let us talk about mean value of the process means the response, because we have got two processes now; one is the low, one is the response. Let x 1 of t x n of t be the realisations of x n of t f 1 of t f n of t be the realisation f n of t, simply x of t and f of t n, because n is already here. Then 1 by n of summation of j varying from 1 to n of x j of t which are the realised values of the responses by taking mean, for the n number of such realisations in this total set of x of t can be expressed like this by n of. We already know the response is connected to the load using a response function, or the impulse response transfer function. So, 1 by n of integral of zero to infinity (Refer Time: 29:24) of the summation is here j equals 1 to n of integral of h of f x s f j of, because the count has to run at j, f j of t minus s d s, I call this equation number three, which can be now said as, integral zero to infinity, which f x s.

I take the summation inside, because now there are two processes here; one is the response process, one is the load process. I can only take a mean of the process; I cannot take the mean of a function which is a transfer function. So, I must take this function outside the summation; take the summation into the transfer into the function of the process. So, I should say 1 by n of summation of j equals to 1 to n of f j of t minus s d s. So, this will classically give me a statistical meaning, like this. So, I call this equation number four, h of x s, so this will lead to. Now this already had expected value of lead to the following expect the value of X of t, why X of t, because amongst the total value of x of t, I already have only this much realisations of this value.

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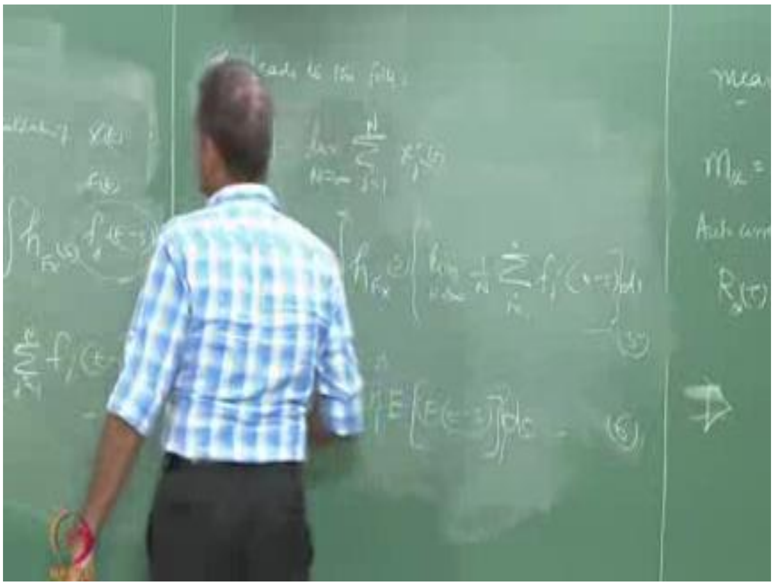

So, whether we write this as the mean of the whole value, or the mean of the realized value in statistical processes or stationary process is going to be having the same meaning. So, e expected value x of t, which is the left hand side of this equation, which can be now, limit tends to infinity, because as a integral here there is a summation (Refer Time: 31:53) here the integral which is j equals 1 to n of x j of t; that is what the understanding of mean is, which can be now equal to h f x limit n tends to infinity 1 by n of summation of j equals 1 to n of f of j t minus s d s, equation number five, limit tends to infinity 1 by n of e, 0 to infinity. So this can be further simplified as h f x s, that is an expected value like you have in the left side here. So, expected value of f of t minus s d s. I am using F here, because f summation is nothing, but the realized value of the process of f of t, which is as same of x of t. I call this equation number, may be five a, it is easy from is refer back later

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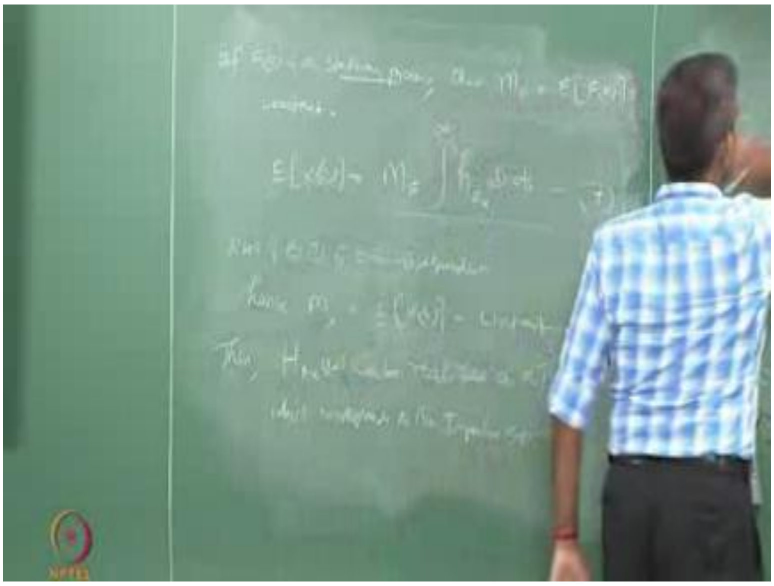

So, now if f of t is a stationary process, then mean value of f which is expected value of f of t is constant. Therefore, expected value of x of t which was in the left hand side of this equation, is now equal to m of, because that is constant integral of zero to infinity h f x s d s equation number seven. Interestingly, you can see that the right hand side of this equation is time independent; the r h s equation seven, is time independent. Hence m x is the expected value of x of t should also be a constant, because x is also a stationary process. Remember we are imposing stationarity condition, only on the load, not on the response. We are deriving that the response will also be stationary process, because the connectivity is constant. We are imposing stationarity only on the load not on the response. We are saying that both are stochastic process. Let us try to understand the difference clearly, when this statement is accepted, then h of f x of omega can be realised as a transfer function, which corresponds to the impulse response function.

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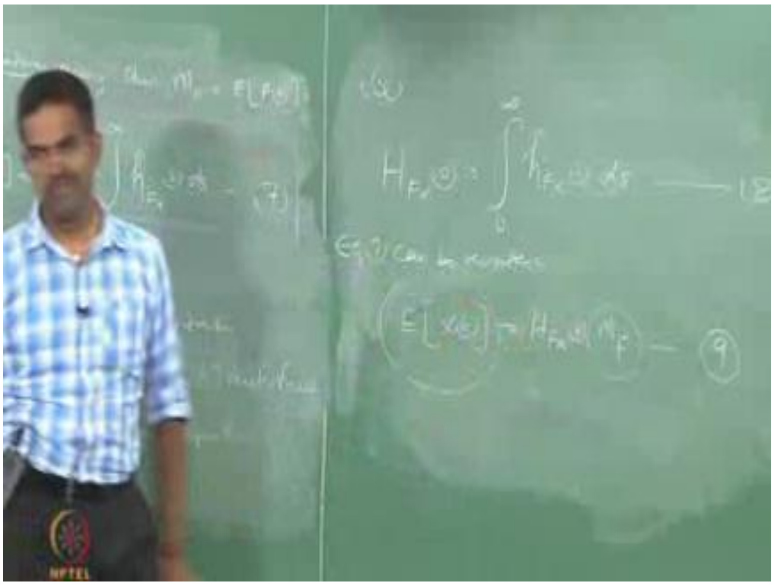

That is h f x of 0 is integral of zero to infinity h f x s d s equation eight. Now let us talk about equation seven, equation seven can be re written as expected value of x of t is m f h f x 0, or let us say h f x 0 m, equation nine. Now the mean of the response and the mean of the load, which are first order statistical properties of the stationary process, are connected by a bridge which is transfer function, which is h f x 0, which is nothing, but the value of the impulse response function, for the positive domain zero to infinity for the realized value of x, for the realized value of f, which is time independent and not load dependent, they are not neither load dependent nor time dependent.

Now, we will extend this derivation next class to connect, how to get h f x realisation in interns of dynamic amplification factor. So, if you know d a f with some modification to d a f, I can find transfer function h f x easily. This d a f we already know for single degree, because we already have derived this expression we already know that. So, if I am able to connect or touch up on, the connectivity between the transfer function, to that of d a f with that idea I can connect the first order statistical properties of the response and the load easily with this expression of stochastic process. So, the analysis becomes more realistic, which accounts for all uncertainties and variabilities to the given system; however, the response, is derived in a more simpler fashion, compared to that of earlier case of detailed time history analysis; that is the idea. So, we are expecting to get a response spectrum, if I get the load spectrum as an input data.

So, if we have any input data from the load spectrum, which is known to be maybe pairs and (Refer Time: 39:12) spectrum may be (Refer Time: 39:14) spectrum which are a load spectrum (Refer Time: 39:17) etcetera Amith Azmi, I will be able to get the response spectrum, if I am able to identify the corresponding transfer function which bridges response and the load. We may not have to worry about the transfer function, because transfer function has improved to be load independent. It can bridge anything, any load to any response; that is the beauty of this whole exercise. So, we will talk about this more continuously in the next class, and we will understand how h f x can be bridged or connected, or relative to the classical dynamic amplification factor, which we already know in the dynamic analysis. So, any questions, any doubt here. Generally the terminology you got be very careful, the nomenclature to be very careful, and you must understand the justification of certain assumptions made in the derivation, because certain cases we said it is going to be a constant time dependent, because we already said that this is stationary process.

So, one must know what is the stationary process what is (Refer Time: 40:25) etcetera that is I gave you entire definition of slowly how they have been done. Once the realisation is understood, you will be able to appreciate the frequency domain or stochastic dynamic analysis very easily, when you know the two time domain analysis exactly; that is connectivity will be established easily; that is the idea, any doubt.

Thank you.