Dynamics of Ocean Structures
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Lecture – 43
Stochastic process

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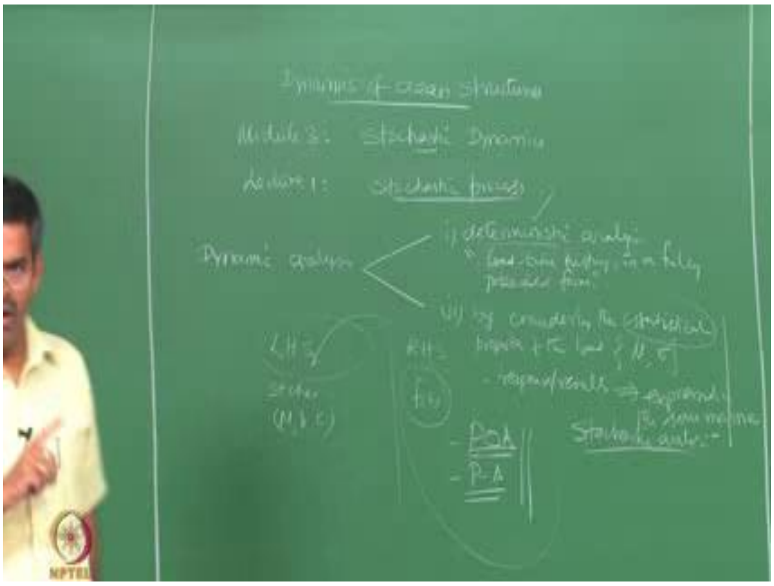

So, we will talk about the last module in Dynamics of Ocean Structures, where we are going to talk about the stochastic dynamics part. We will discuss very briefly about the stochastic dynamics as applied to offshore structures before that, let us try to introduce what is actually meant by a stochastic process why stochastic process is different from a random process and how comfortably a stochastic process can be chosen for doing dynamic analysis of structures and then once we understand this once, we relate the necessity of doing a stochastic analysis or stochastic process application for loads applied on offshore structures and one can easily justify why stochastic dynamics can be successfully applied to analysis of offshore structures.

So, we will actually not demonstrate any example or any let us say solution we are talking about stochastic dynamics as applied to offshore structures, but nevertheless we will talk about some fundamental relationships and some equations and some terminologies as applied to dynamics which is essentially derived from stochastic process. So, the first most question comes in this mind is what is actually meant by a stochastic process. We all know the dynamic analysis can be actually done by two ways, you can carry out dynamic analysis essentially by two ways; one is what we call as a deterministic analysis which we already know the prerequisite to carry out deterministic analysis is simple. You must actually know the load time history in a fully prescribed form.

So, if you know the load time history in a fully prescribed form why we are talking about load because, when you talk about analysis generally there are two ways; one if you look at the left hand side and right hand side of equation of motion, left hand side talks about essentially structural characteristics. For example, mass stiffness c they are all structural properties mass is actually a structural property because mass is second moment of area of the given members. So, one can find out for every given member one can find sigma x square by sigma a and try to find out the assembly of this, I get the mass matrix stiffness matrix is actually a functional relationship of the characteristic of the structure itself, it depends upon the form at the boundary condition as well.

So, it is again there is a property related structures of course, damping is derived from m and k. Therefore; obviously, damping will also be a function and characteristic of structural properties maybe not only necessarily material because, k is involved with e l a and I therefore, material characteristics will also play an important role, but nevertheless you see k m essentially are derived from the form the structural form. Therefore, it is important that they form essentially focused on the structural geometry where as if you look at the right hand side of equation of motion essentially this talks about loading characteristic.

Of course in certain classical cases like TLP or spar or even triceratop we have a dependency of f of t on x, where x is also a factor which is governing the derivation of k m and c there is an interlinking or what we call coupling between the r h s and l h s that is only on special kind of structural system otherwise usually the practice is r h s or loading is decoupling from the system that is generally a case in almost all kind of structural phenomena. But in special cases which are formed driven not function driven then you will have this dependency or coupling effect of mass k and c dependant on the response itself and therefore, they form some complexities which is very special in certain cases.

So, in on the other hand when we talk about the load time history we are not generally confused with the material characteristic with the mass characteristics or with the damping situation in a given system because, generally they do not vary with time; however, loading will definitely will get time that is why we do dynamic analysis if a loading is also not varying then that becomes analysis becomes either static or worse by worse there was static analysis we do not do dynamic analysis at all.

Therefore, in the whole context the right hand side of the equation of motion is strongly dependant or influenced by the time variation therefore, we talk about that I want the complete prescribed loading time history to perform a deterministic analysis that is the reason the second method by which you can perform dynamic analysis is considering by considering the statistical properties, properties of the load statistical properties means generally first order. For example, let us say mu that is mean and standard deviation these are the first order characteristics of any given system. So, if you know the statistical properties of the given load then; obviously, the response and the results of the analysis will also be expressed usually in the same manner.

So, if you look at the whole scheme of explaining the method of analysis in dynamic scenario it is governed only by the load, it is generally not governed by the system it is true in dynamic as well as static analysis and it is also true for higher order analysis like non-linear. For example, push over analysis for example, p delta effect all these if you see the classification of analysis in general in structural generic algorithms are derived only based on the loading criteria it is never based on the form or the geometry of the structure at all, why they wanted to make it because once you know the loading criteria history define you must be able to apply to any form of the structural system available on earth.

So, they want to make it more generic that is the reason why they do not depend on the form, but of course, form derives a loading from the right hand side of the equation of motion. So, in both the cases when, you talk about deterministic or stochastic we are only bothered about how the load history is varying if you are not able to prescribe the load variation in the time domain as detailed as this case, but you only have the mean and standard deviation of variation on time domain for a given loading then one can do what we call stochastic analysis.

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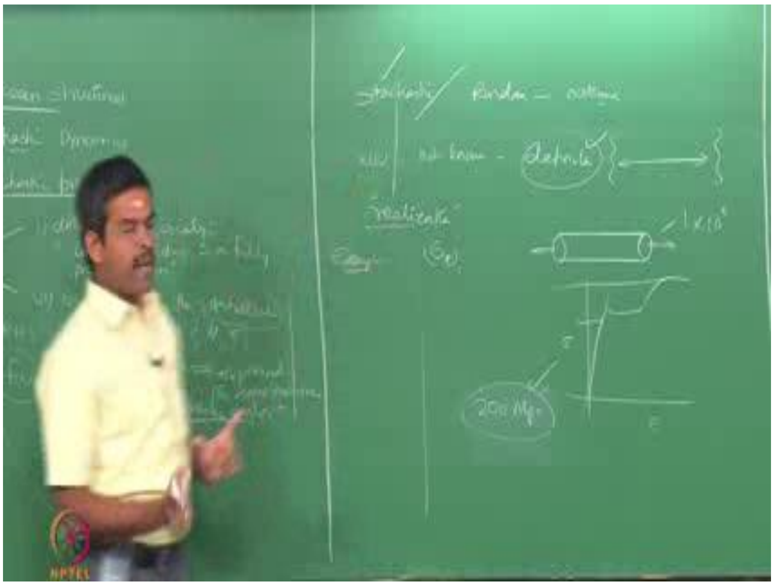

So, there is a very serious question here how stochastic analysis is different from random process or why stochastic process is different from a random process, it is very I mean both of them are intricately define more or less in a same manner there is a small catch to call the process as stochastic or to call the process random.

Now, many people generally have a basic confusion the moment we say random analysis or random vibrations or random process, they always think that the variable in the given process is not known that is what they say that to the variable in the given process is unknown it can assume any value. Therefore, it is random that is actually not the definition random or the term random has does not come from the basic request or requirement that the variable can assume any value the variable actually has a definite value it actually has a value you do not know the value. Therefore, the process is random there is a very classical difference here the value of the variable is definite it does not change, but it is definite between a large bracket it can assume any value it has already assumed a value, the value is fixed since you do not know that value as in the case of vice versa here you call that process random. So, you apply a tool by randomly you fix this variable or try to find out the value of this variable in the given scheme that is why it is called random process.

Now, in random process we generally call the results as outcome you do a random process you call the results as outcome stochastic analysis is also a stochastic process also a random process except that the outcome, what you refer in random is called realization in stochastic what does it mean stochastic process has got more physical definition compared to random process. Random process has got mathematical definition where as stochastic processes have thought physical definition because these values are physical realization of the given process. So, that is the difference between a random and a stochastic scheme. So, otherwise both of them will have the same mathematical algorithm to follow they will also dependant on mean and standard deviation and covariance etcetera.

Random process also depends on the same algorithms. So, there is a difference here it is mathematically intrinsic where as this is realization of physical intercity. So, this is more or less a realization of a given problem. So, now, one can ask me a question, when do we apply a random process when do we apply a stochastic process we can give a classical example, I am talking about of course, please understand in mind we are talking about both this process or only this process applied to the loading history we are not talking about this process applied to the left hand side.

One can ask me a question sir what is that beauty? If I apply this stochastic scheme to the left hand side very interesting you take a simple bar subjected to axial tension do a tension test in a u t m plot the stress strain curve for a given material you take about one million sample you will; obviously, see depending upon the grade rate of loading temperature ambience humidity pressure etcetera. You will not exactly get the superposition of all the curves identical there will be some variation. Let us pick up only one point let us say the heal point let us say the ultimate load any one point of interest for us, you try to form a table the table is nothing, but the values of healed point sigma y p sample one and sample 2, sample one; 10 power 6 you take a mean.

So, therefore, you can still apply a mean and standard deviation of the same scheme on left hand side also because you are assuming is going to be which is the slope of this curve is going to be always 2, 10 power 5 Newton per mm square mega Pascal 200 and 10 mega Pascal. Let us say 200 mega Pascal mp. So, you are assuming this value to be fixed, but all the time this value may not be fixed. Therefore, you can also correct this which is also one of the important input for stiffness matrix derivation which is also an important input for mass because density will also vary similarly take a piece keep on varying it find out the density you will keep on varying you can now.

So, all these concepts of first order variation can be also applied to material characteristics which are actually the left hand side of the equation of motion, but still we will not call that as realization. So, a stochastic process is only applied to the load variation that is very, very clear is very important because when, I move to fatigue analysis I will talk about the characteristic variation we solve this stochastic later. But in general stochastic process is referred only to the variation of load in terms of mean and standard deviation. Where as you can also have material properties also varying with mean and standard deviation.

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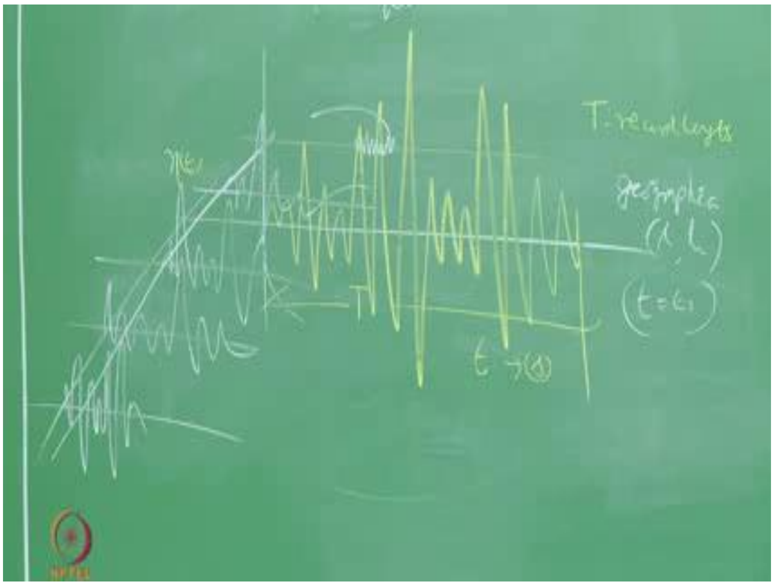

Now, having said this let us come to an example of stochastic process a classical example in terms of load variation of mean and standard deviation could be a sea surface elevation which we call at eta of t a typical sea surface elevation can look like this, let us say this is time t in seconds this is eta of t which gives me the amplitude of variation and of course, this is going to be capital T which we call record length. So, you pick up a record length and look at the variation now, one can immediately see you can associate this as a stochastic process because easily you can find mean and standard deviation you can always find out what would be the variation of amplitude from the mean this process is actually done in wind load analysis where we convert the process into 2, one is called the gust component other is called the mean component.

So, what we do in a given history of wind load we draw a mean line we took up we take only the variation respect to the mean line and call that component as a gust component and the mean component, what we do that is why in wind analysis people generally do quasi static analysis they do not do dynamic analysis except that the structure is very flexible and one of the character dimension structure is very less compared to height of the structure. If it is exceeding 5 then, people do dynamic analysis otherwise not required.

So, in a given time history like wind loading it is also converted in the using the scheme. Therefore, for a sea surface elevation it is very easy for us to know that I can easily estimate mean and standard deviation, now interestingly one can ask me a question. Sir this is a surface elevation is drawn or is observed at a specific geographic location maybe the latitude, the longitude is known to me. But; however, even in the specific location it is observed at a specific time t equals t 1. If the t is equal to t 2 it will vary it means you have infinite number of samples appearing how will you take the mean because samples will change as per t samples will change as per l 1, l 2 samples will change throughout the year samples will change throughout n number of years.

So, it is a very complex process because you will not be able to take a mean. So, easily in such a manner can we design the system with maximum loading the design will be foolish because, you are interpreting a load which does not occur at all can we design a system for the minimum loading the structure has a probability of failure which is not requested in a design. So, how do we design therefore, there is a problem the input load is very complex in nature which is undefinable, but we are defining it by using random theories available in the literature to avoid this people move on to the stochastic dynamic stochastic process, where people said this process is considered to be an ergodic process.

So, they are redefined the scheme then.

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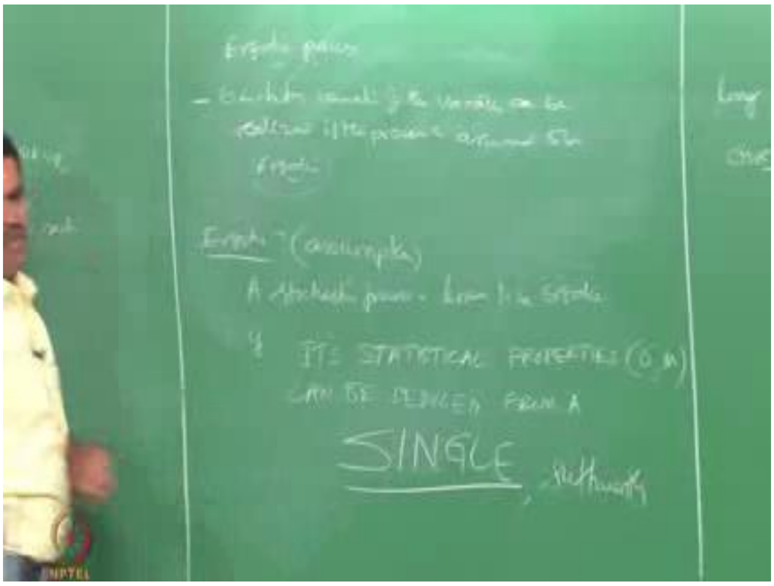

What is ergodic. So, first let us understand if I convert or assume or idealize this scheme into an ergodic process the major advantage is the time history variation, please understand this variation of the sea surface elevation in terms of amplitude is only on time scale, I am not looking at the other scale what could be the other scale other scale could be location can be period rainfall may be other other parameters also, I am not looking all of them in the parallel axis I am looking only one axis where only variation with time. So, I am looking at the time history variation of the variable can be realized if process is assumed to be ergodic.

So, the main advantage the main advantage is realization of this - what is the realization from the given variation of time history of amplitude or wave heights or sea surface elevation. I want to know one specific input data which I will use from the analysis that is called physical realization. So, to make this I am now getting into a stochastic process in randomness this convergence to realization will not happen in stochastic process is going to happen I will tell you how. So, the process need to remain ergodic now the question come what is ergodic process what is ergodic a stochastic process is known to be ergodic, if its statistical properties initially I am bothered only about mean and standard deviation, but then we will talk about coefficient of variation etcetera later if its statistical properties can be deduced can be deduced from a single sufficiently long sample of the process.

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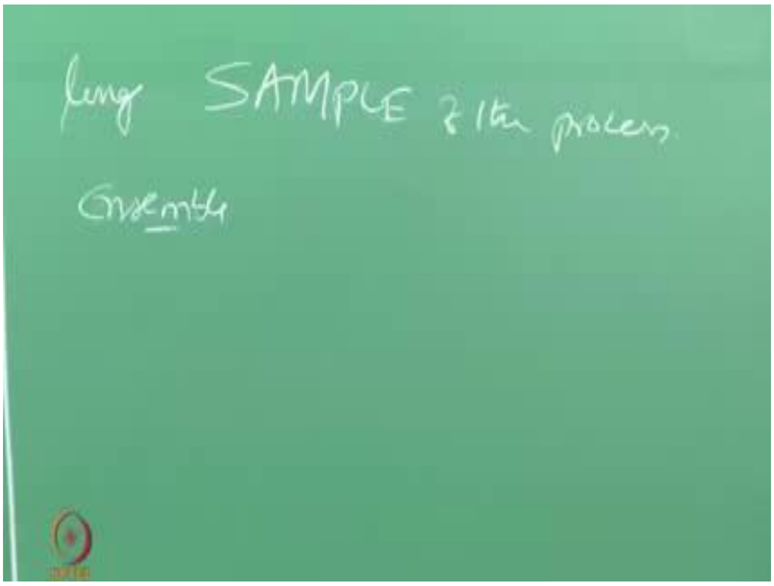

Now, one can ask me a question how do we physically understand this statement. Now we already know that stochastic process is dependent on statistical parameters the foremost fundamental statistical parameter which I am interested is for a given variation is mean and standard deviation. We all know how to compute them for a given mean is nothing, but an average statistical average and standard deviation is the variation of any variable with respect to the mean and take a square of that, it will variance and standard deviation very simple arithmetic s we can easily find out this nowadays calculators also have this value really available in the given data if you give the data.

So, the first level of statistical property is the mean and standard deviation I have got infinite number of samples if I am able to classify one single sample from the infinite series whose mean and standard deviation is same as that of the whole process hypothetically. I can pick up that sample for my analysis because that sample will become a representative value of the whole scheme it is not the value of the segment you understand the point. For example, I have n number of schemes I take any specific point t where I try to find out mean and standard deviation of all these and in this sample one of them has the mean and standard deviation as same as that of the mean and standard deviation of the entire scheme. So, I need not have to do analysis for the entire scheme do the analysis only for that sample if the sample exists then that process is called ergodic process.

You will not exactly get that same trace, but closer tolerance now the advantage is the biggest advantage is you can pick up that sufficiently long sample from the process which has the same statistical property as that of the scheme, where do you get this you deduce this you find out this from the given sample.

So, how do you do for everything you find out sigma and for everything you find for all at given t you find and pick up which is comparable, pick up that sample then the whole scheme is ergodic then the stationary process if it is ergodic then, my problem is solve because my f of t is now defined though it is random in nature though it has got stochastic variation along different parameters, but still I am able to pick up one sufficiently long sample from the ensemble from the ensemble which gives the mean and standard deviation or the statistical characteristics as same as that of all the processes then it is called ergodic. So, what is the advantage of being ergodic now what is the advantage if the process remains ergodic.

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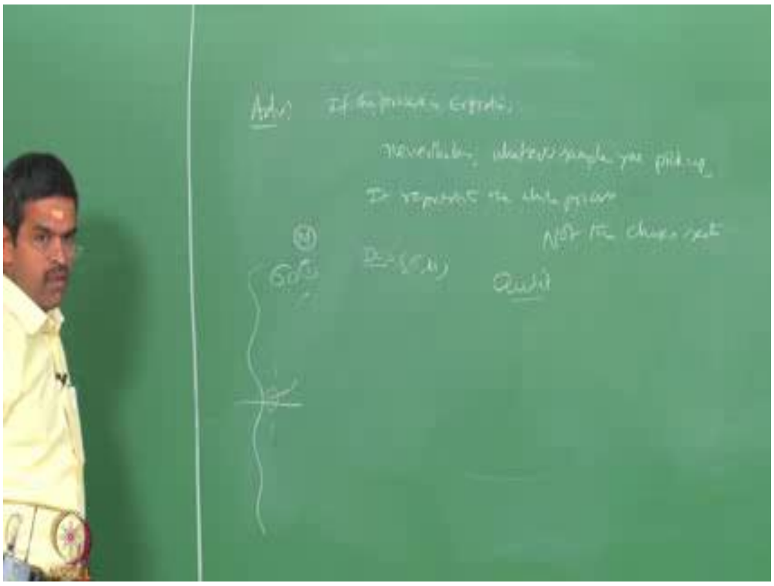

If the process is ergodic nevertheless whatever sample you pick up it represents the whole process not the chosen section that is important for example, let us say we have a class of 60, we conduct exam on dynamics of structures we are trying to work out the mean and standard deviation of the marks obtained by the 60 students. If among the 60 and of course, this 60 will have variation, if I conduct the exam on every Mondays, every Tuesdays I mean another parameter is there I find out the variation of every student in terms of same mean and standard deviation in one student matches with my mean and standard deviation of the entire class.

So, if I want to actually show the performance of the entire class to an external auditor say I want to express that, how my class is good in answering I will not allow the auditor to ask question to anybody in the class I will ask him to ask questions, only to that point who is representing the whole process and not the individual section then that process is ergodic. So, if I am able to generally get an ergodic process assumption it means actually is an assumption, but by luck if you see in reality sea surface elevation assumes to remain ergodic.

So, it is advantage that we need not have to bother about the variation we pick up any one sample it does not mean that which sample you picking up because by ergodic definition any sample present in that scheme of ergodic should represent the same mean and standard deviation as that of the whole process. So, does not matter what sample you pick up, but it is very clear the sample does not give a self understanding within the section, but it is representing value of the whole process. So, that covers by enlarge a larger domain of variation which is uncertain in the given described loading which was not available and described in a classical dynamic analysis because, all the time we talked about f of t we never looked into the complication of f of t. So, intrinsic in the earlier analysis, but in reality it is true that it is. So, intrinsic that is an advantage of being ergodic process.

Now, the question is how you will define ergodicity mathematically, how you will define.

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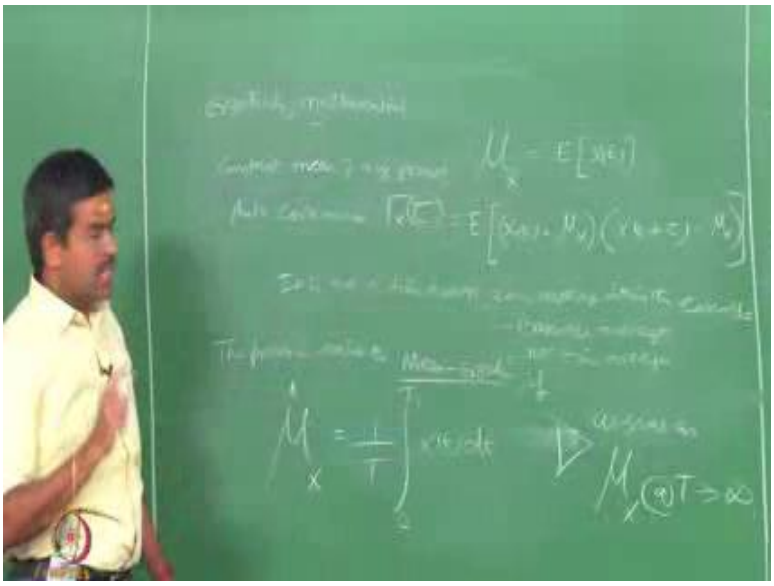

Ergodicity mathematically ergodicity can be defined by 2 ways; one by mean other by co variance let us say the constant mean of any process is given by mu x I will write the symbols very clearly my handwriting may not be that classical, but I will write try to write the symbols very clearly that is important because everybody has some demerit like my handwriting when I write faster is very difficult.

In fact, sometimes I am not able to read my own handwriting, but that is the fact, but; however, I live to make up by reading the words before and after. So, what I am actually meaning mu x actually, we know it us an expected value of the variable that is the mean value by definition autocovariance which is expressed as gamma of x of tau which is the expected value of x of minus mu x multiplied by x of t plus tau minus mu x these are 2 classical equations for finding out mean and autocovariance in statistics please understand the autocovariance actually is dependent on the lag tau not on t it means, it is not a time average it is actually an average within the ensemble because here tau is important it is not t. So, they are called ensemble averages and not time average.

The mean and standard deviation of an ergodic process is actually an average of all the ensembles at any specific time, it is not time average you understand the point, where as in random scheme it is time average. So, we are converting that slightly in a variation that I want to realize. So, now, you will really understand stochastic process become a realization of a physical value though there are 100 samples, million samples available I am able to I should be able to pick up one sample which gives me the physical meaning of all the processes which I can use for the analysis. So, it is called stochastic process where as realization is on the time average where as this is not on the time average it is on the ensemble average.

Now the process is said to be mean ergodic, if mu hat of x which is of that of the process is one over time of 0 to the record length t of x of t d t which is as same as mu of x at t tends to infinity. Now this is where the bridging is please understand that the mean and standard deviations sorry mean and co-variance or ensemble averages not time average. But if I have a scheme where the time average has the same value as that of the ensemble average then it is called mean ergodic process that is what we want we want to pick up a process where the time average and ensemble average should reflect more or less the same value similarly if you want to call, if the process is said to be autocovariance ergodic.

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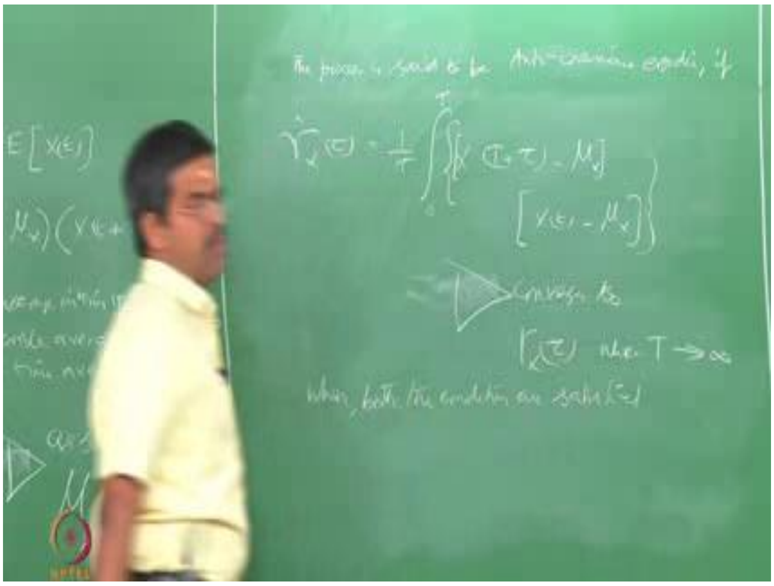

If gamma x of tau hat is a time average of the record length t x of t plus tau minus mu x multiplied by x of t minus mu x which converges to gamma of x tau when t tends to infinity.

Now, your process is considered to be ergodic one when both the conditions are satisfied when, it is both mean ergodic and autocovariance ergodic. The process is said to be an ergodic process you can also do ergodicity check for a discrete random process. Now this is a continuous process because I have got the variation of record for the entire domain I can have discrete values also ergodicity can also be checked for discrete random variables for a discrete process.

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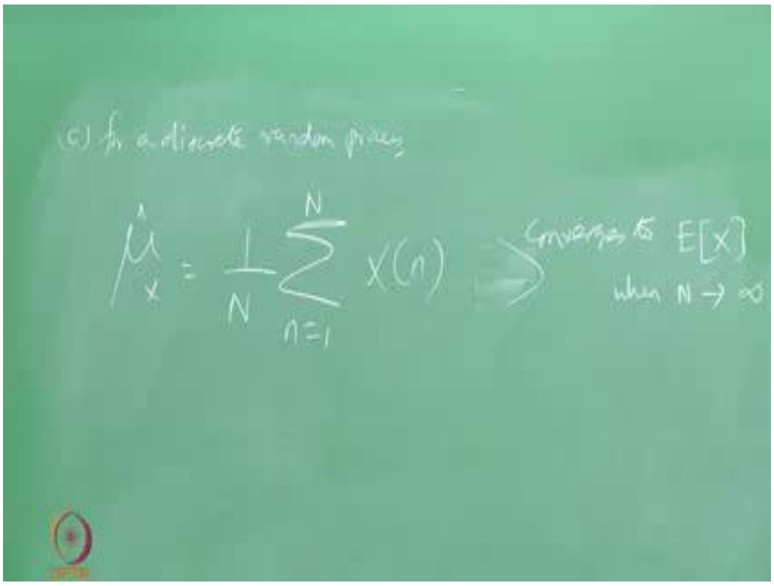

The mean is said to be ergodic, when it is between the sample of variable small n to capital n as n is a record length that is time number of records x of n, which is as same as or converges to mean of x when n tends to infinity. So, one can check ergodicity for a continuous scheme for a discrete scheme as well.

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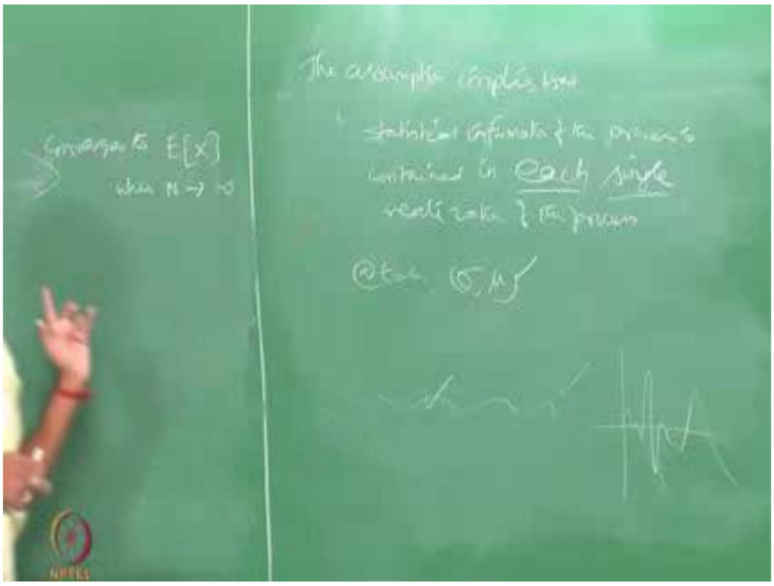

So, this assumption implies a major parameter the above assumption above means the ergodicity implies that what does it imply to me? In my analysis statistical information of the process is contained statistical information of the process is contained in each single realization of the process, I will ask you a very interesting question, let us say I have a sea surface elevation random data available to me at t equals; let us say t one you have an average available to you in the record, you have such averages available to you for about 100 samples. Amongst this you have picked up any one signal for an analysis this is an input data now, this input data is typical to what you have pick up this and if the designer or the auditor or the examiner or the faculty or the colleague wants to know that how? Are you sure that you have not missed out any other signal, which is more vulnerable than the signal then your design is failed.

If you are able to prove that the statistical characteristic of this realization is a part of the entire scheme then that question becomes null and void. So, if you do a stochastic analysis by proving that the process is ergodic in nature both mean as well as co-variance ergodic, then all these uncertainties in terms of accounting for large variation in the ensemble will be null and void. So, that is an advantage when you do a stochastic dynamics or dynamic analysis using stochastic process for a given scheme because the right hand side of the loading is very highly uncertain and the process is highly random in nature to avoid this confusion. People should know that how I can check my ergodicity for a given scheme.

We stop here, we will discuss it in the next lecture continuous may be another 6, 7 classes we will have. We will stop any questions here?