Dynamics of Ocean Structures

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Lecture – 41
Numerical Integration

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So, today we will discuss about the solution procedure for solving equation of motion. We already know the standard equation of motion for a multi degree freedom system model is given by this. So, if you are talking about multi degree, then I should say these are my matrices and my vector. Each one of them we know how to evaluate for a given system. For example, the mass matrix, the stiffness matrix. And, based upon the Rayleigh model, we can also work out the C matrix, which is the damping matrix. And then, of course I can work out the F of t for different kinds of forces. For example, in the last lecture we discussed about forces arising from impact and non impact waves, in the previous lecture we talked about forces arising from seismic effect on tethers, we also talked about conventional forces which is coming from springing ringing response (Refer Time: 01:10) loading from different spectrum. So, we know how to work out them.

Now, the question is in the entire equation of motion, you will notice and appreciate that. x is actually the one which is requested or required to be evaluated, which is the displacement time history because it is going to be dynamic analysis; this is going to be a time history. So, I must get a time history in terms of time. I must get the variation of how the displacement varies as the time progresses. once I have this, I can always do your transformation of this to a frequency domain. And, try to find out the frequency content of this. In the last presentations, we saw the results on both the time domain and frequency domain. One can easily convert them the either way to find out the content of this because frequency response will give you the energy content and specific frequency; whereas, time domain response will give you the amplitude in terms of the maximum value. So, one can request term; can have both for better understanding.

So, in such equations of motion it becomes very interesting that I must solve for the displacement vector as a time history. So, there are many schemes available in the literature.

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The most common scheme and popular is the direct integration scheme, which is one of the most popular methods used for solving equation of motion in a time domain. Now, the word direct actually stands for or 'direct' refers to; the equation of motion is directly solved using numeric algorithm or numeric integration procedure, integration scheme without transforming the equation of motion into another form, before the numeric integration is attempted. That is why it is called direct. So, we do not do any transformation of the coordinates for this. Directly we get the answer.

So, numeric integration is actually a step by step integration procedure. The moment I say step by step integration procedure, there are two catchwords here; what should be my step size; because the step size should be in such a manner, it should represent a continuous solution more or less. That is number one. Number two, the step size chosen by you should have some relationship with that of the overall t of the structure. That is capital T. There should be some relationship between the step size and T. Thirdly, the step size chosen and the integration scheme should give you unconditional stable convergence. Otherwise, there is no point in having any numeric scheme. So, we must ensure that.

Now, there are some assumptions made in this case. What we try to do generally is we try to solve the equation of motion as any time instant.

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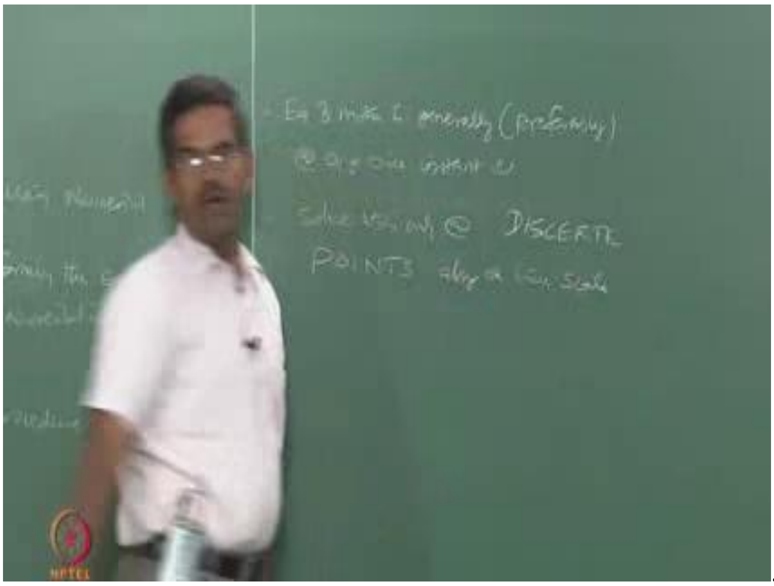

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Equation of motion is generally solved, is generally or preferably we would love to solve this equation of motion at any time instant. That is what actually I want. I want the solution of u of T or x of T at any time instant. But, when you adopt or apply a numerical integration scheme, you will solve this equation of motion only at discrete points, along the time scale. The moment I make this statement, you will have a question in the mind that when the solution is valid, what is going to be the solution? I get x of T. By applying some scheme, I find x of T which is a vector in this case because there are six degrees of freedom I get the vector, may be at T equal to point one second. I get a vector. Similarly, the next vector can be at T equal to point one two second. The step size is 0 point 0 two seconds. Let us say we fix the step size.

We can fix it as close as 0.001 also, to make it more or less continuous. That is all in our hands. But, however the solution what you obtained using the scheme will be discrete. It is anyway cannot be continuous. Now, the problem comes here is when you have got the solution applied or the validity of the solution is at every discrete point, what does it mean? When I substitute x of T at point one, it should feedback and it should give me the equation of motion valid at that particular point. What does it mean is that x, x dot and x double dot multiplied, pre multiplied with this should give me F of t at 0.01 second. That is the meaning actually.

So, though F of t is continuous, F of t is not discrete. F of t is continuous because F of t, you know, it is actually a continuous action applied from the waves or any action coming on the structure. But, you have to pick up a discrete value and try to compare and say the validity of this solution. So, there is a very serious problem here. The problem is that when you know solution at x of t point one, x of t is point one two, you are always assuming the behavior between this band of your choice. You may say it is linear, you may say it is non-linear, it is cubical, it is hyperboloid, you are assuming that. The solution does not give any nature qualitatively about this variation within the time step. That is very important.

So, you cannot interpolate between the time step. If you have a value at point one, yes it is available; if you have a value at point one two, it is available. Somebody asks you what is the value of point one one two, you cannot interpolate. Then, in that case you have got to rerun the whole scheme again and taking step size as multiple of point one one two and get the value. But, you cannot interpolate because the distance between or the interpolation nature between the time steps is arbitrary. That is one of the important catch here. It is seen sometimes as a demerit. Sometimes people do not mind because people say I am making a step size so close. How does it matter? If you make it so large, then it really matters. But, if you make it so close it does not matter.

But, what is that step size, which will make an unconditional stable solution; we must know that. And, of course this step size should be having some fraction of the overall T of the system, the time period of the system. This T can be either the excitation wave period or it can be either the structural period also. So, you have to choose that. So, let us see what are those conditions available in the literature. Newmark has given a very interesting algorithm, which is named after him called Newmark's integration scheme which we will see now in this class. And, I will show you the coding also for this.

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So, Newmark N M; that is the reference, this is named after him as Newmark's method. It is actually Nathan Newmark's method. That is the method what is proposed by him in early 80 s, no early 60 s.

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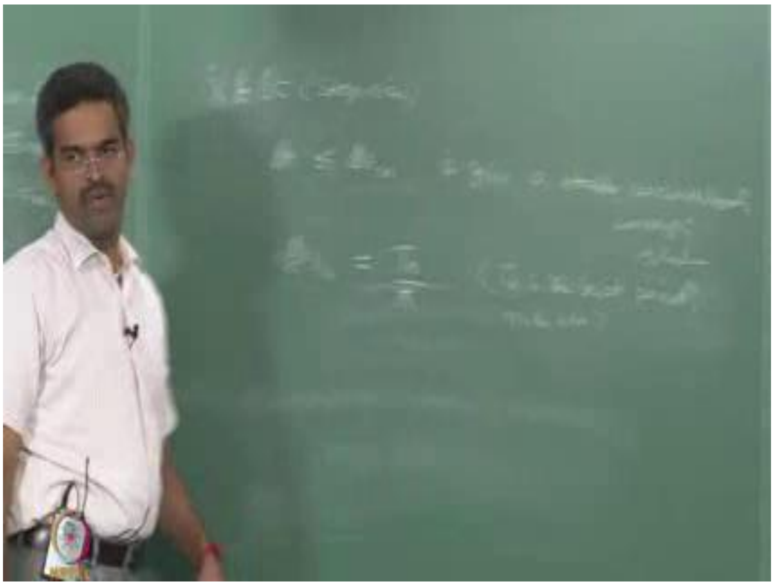

This gentleman said or given an algorithm saying if your delta t, the most vital part of this scheme is the step size delta t, if the step size delta t is lying in the range of delta t critical or less than equal to this, then it gives you a stable unconditionally converging solution. Now, delta t c r is actually T n by pi, where n stands for the natural period. Now, if there is six degrees of freedom or multi degrees of freedom, T n is the lowest period; the least of all. And, of course n is the order or degrees of freedom, whatever it is.

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So, he has given a scheme. The scheme is like this. Let me write down the scheme here. Step number one: form k, M and c matrix, and of course F of t also, which is known to us for a given system. Initialize x dot, sorry, x 0 and x dot of 0 and find x double dot of 0. It is very easy to know how it can be done. Let us say if equation of motion is M x double dot plus k x is 0 or F of t. If, let, I am not allowing damping (Refer Time: 12:14) in the system, let us say for x 0, this becomes 0. So, I can easily find x double dot 0 as simply mass matrix inverse supposed to multiply by F of t. I can get x; this vector easily. There is no problem. So, I can always compute this.

Once I get this, I must choose or select the step size delta t. We already have a governing given by the scheme manager saying that delta t should be within delta t critical. So, I know the time period because I do natural vibration, free vibration response. I will get the periods of all the degrees of freedom. Pick up the lowest one and check that. And, I can always check on. And, I must also select and choose alpha. There is another variable or a parameter in the scheme and delta. Delta is usually taken as greater than point five and alpha is computed greater to be point two five of point five plus delta the whole square. Now, let us call equation number one.

If delta is taken as point five and alpha will become point two five, as you see here. It is going to be one, one square one and point two five, then the scheme is called Newmark's beta method. The scheme is renamed as Newmark's beta method. Then, there are some constants required to operate the scheme. Let us see what are those constants.

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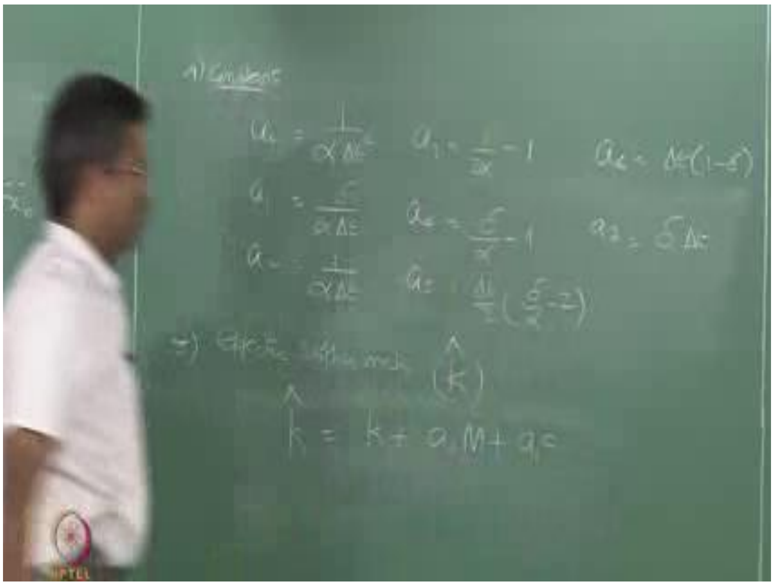

They call them as integration constants - a 0, a 1, a 2, a 3, a 4, a 5, a 6, a 7; 8 constants. Eight constants are there. One by alpha delta t square, delta by alpha delta t, one by alpha delta t, and these are some integration constants recommended by the manager to operate this. Step number five: so, till now for a given equation of motion, you know this step number one. It is computed. Then, you can initialize these values assume them. Obviously, you will assume them as 0 because that is how we generally do for free vibration analysis also. And, one can compute x double dot naught, then one can fix up the scheme, find out these constants. Then, form effective stiffness matrix, which I call K hat. K hat is given by a 0 M plus a one c.

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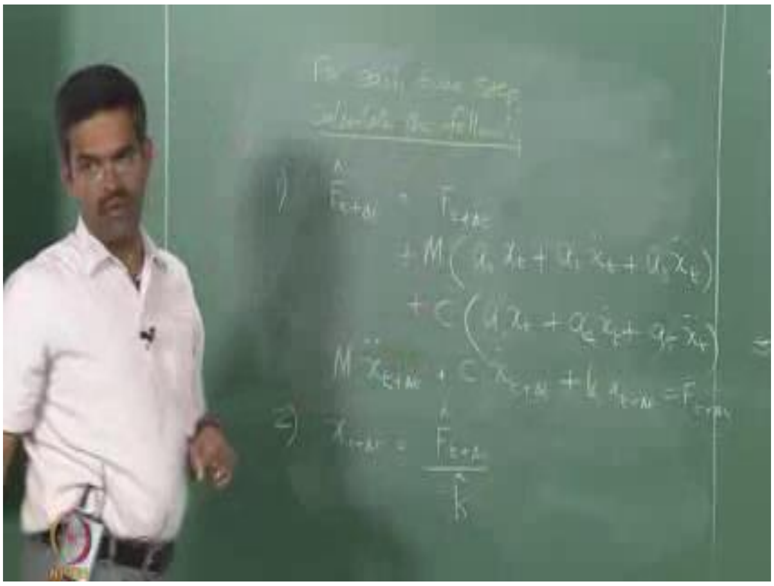

For each time step, calculate the following. Let us say one. F t plus delta t, which will given by F t plus delta t plus M times of a 0 x of t plus a 2 x dot of t and a 3 x double dot of t plus C of a 4 a 5 x t x dot x double dot. This is okay and I will put the plus here.

Now, to compute a new revised force vector you have to have these constants available to you, which is known to me. You should also have the values of response, displacement, velocity and acceleration in the previous time step because this is t plus delta t, this is t. So, this I will know. The one which I will not know is this; because I want the force at the new time step of t plus delta t. So to get this, it is very easy; M x double dot t plus delta t plus C x dot t plus delta t plus k x t plus delta t is actually equal to F t plus delta t hypothetically. But, I cannot get this value because I actually do not know x t plus delta t x dot of this and x double dot because I am actually calculating them. To start with, I will always say this value will be equal to the force at the previous time. So, that is the first one. So, x t plus delta t, which I wanted now in the new time step will be given by F hat of t plus delta t by K hat, which I know from equation five or from step number five.

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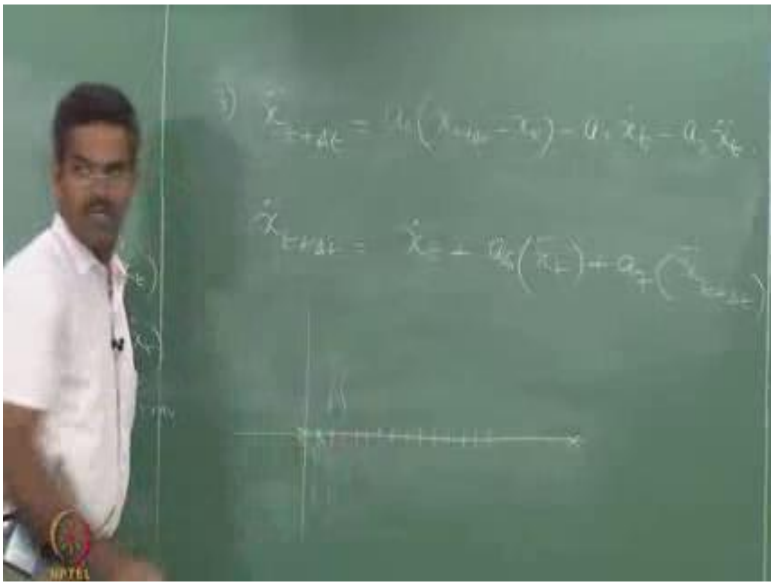

Then, the third one will be x double dot and x dot. There is a quick resemblance here. One can easily understand. We will look at the acceleration term, which you want to find at t plus delta t. The coefficients recommended by the manager are same for acceleration which is mass associated, a 0, a 2, a 3. That is the catch here. So, you do not make a mistake there. And, to compute the velocity at the new time step, you require acceleration in the new time step. So, you first compute acceleration, then get the velocity.

Now, to compute the acceleration in the new time step, you must know the displacement in the new time step. So, you calculate the displacement first, then go to acceleration, then go to velocity. That is the scheme. So, by this way in a given time starting from 0 to any value you want, maybe 10 seconds, 15 seconds, 100 seconds, whatever you want. Divide them into equal number of delta t s. At every delta t, get the vector which is displacement velocity and acceleration, which will give you more or less a continuous solution. But, however the behavior between this displacement and this displacement at this interval is arbitrary. That is very important. So, you can make it as close as possible to look like continuous. This is the scheme suggested by Newmark.

Now, let us implement this and understand how we can solve a simple problem. Then, I will show you how this problem becomes more complicated, when you attempt to solve this problem for compliant structures.

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So, let us take the problem simple with example or exercise, whatever you call. Let us say I have a mass matrix. To make it very easy just for solution purpose, two 0 0 one. I have a stiffness matrix; six minus two minus two four. Do not bother about the units and do not bother about the significance of these numbers. Just for understanding, we are trying to adopt this scheme. F is 0 and ten. So, you have the steps with you. Let us try to do this.

So, the solution for this I want to solve M x double dot plus k x is F of t. That is what is asked in the problem. So, I have to find x of t for different integral. So, step number one; as you see from there, I have the mass matrix k and F. So, let x 0 be 0, x dot 0 be 0. So, x double dot can be mass inverse of F of 0, which is available here. Can you just find out this, Mass inverse of this? So, mass inverse is going to be one by two of 1 0 0 2 of 0 and 10. Is it going to be 0, 10?

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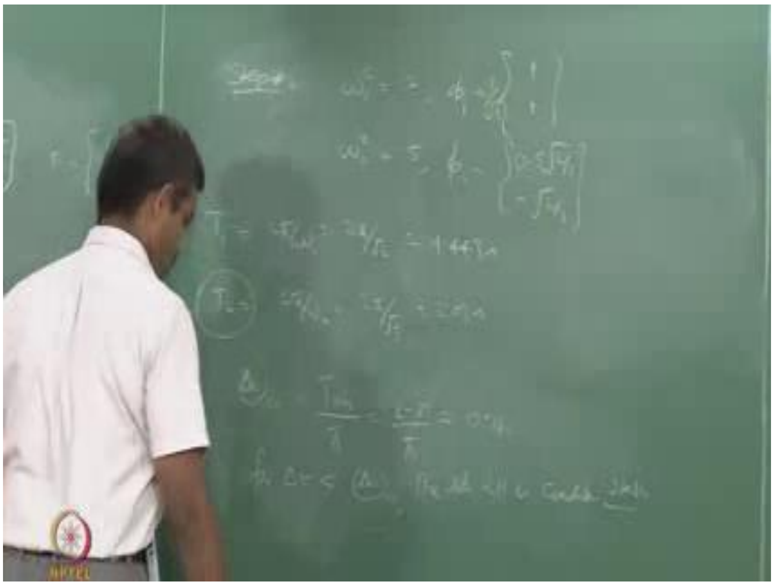

Step number two: I want to find fixed delta t. To find delta t, I must know the time periods of the given system. So, if you try to adopt any standard procedure what we discussed in earlier module, I am getting omega one square as two and phi one; the first mode shape - One by root 3 of 4 and 4. Omega 2 square is 5, and, the second mode shape 0.5 of 2 by 3 and minus 1 by 2. So, let us work out T 1 and T 2 in this case. Two pi by omega versus two pi by root two in this case. Similarly, two pi by omega one omega two two pi by root five. Can you give me these values? It is four point four four three. And, this was two point eight one seconds, is it ok.

So, the lowest of this is T two. So, my delta t critical as advised by the manager is T n by, T minimum, by pi. Is it not pi, which is two point eight one by pi. Which comes to how much? point nine something, point eight nine approximately. So, my delta t should be closer than this, lower than this. If it is there it will give me an unconditionally stable converged value. So, for delta t lower than delta t critical, the solution will be conditionally stable.

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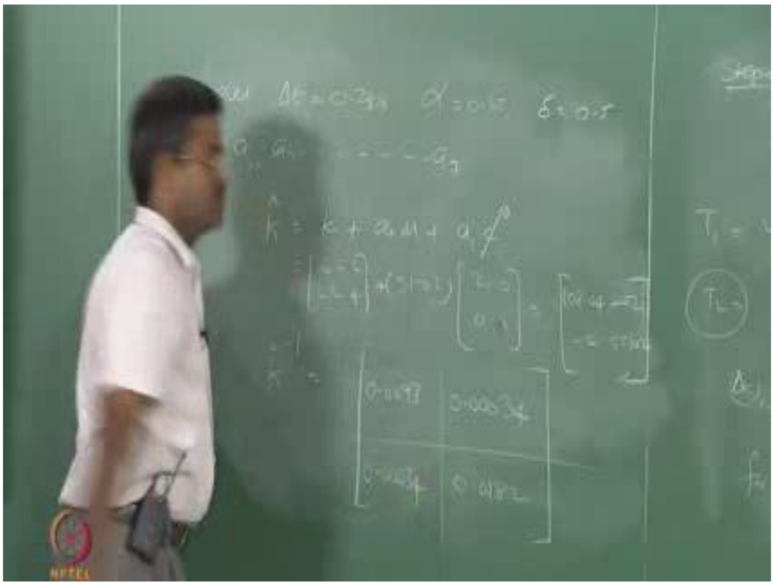

So, let delta t be point two-eight seconds, one-tenth of this actually. One-tenth of this and, alpha be 0.25 and beta, sorry, delta is 0.5. So, I am operating a standard Newmark's beta scheme. So, I can also compute a 0, a 1, a 2, etcetera till a 8 or a 7, sorry. I can compute them. Quickly please compute them. Step number three: I want to find K hat, which is k plus a 0 M plus a 1 c. c in this particular problem is 0. So k, already I know, which is six minus two minus two four plus, what is a 0?

Student: 51.02.

51.02 of 2001, which will be again the matrix of 2 by 2, please compute and give me the matrix; minus two minus two 55.02.

Now, I want to find K hat inverse; as required for finding out my displacement. So, can you quickly get me the inverse of this? Two by two matrix will be 0 point 0 0 nine three, 0 point triple 0 three four, 0 point 0 one eight two, and it is symmetric. Please check whether these answers are okay. Most of your calculator should have a capability of inverting a two my two matrixes at least. So, you should be able to do. Now, I have come to a part where all fixed answers are placed.

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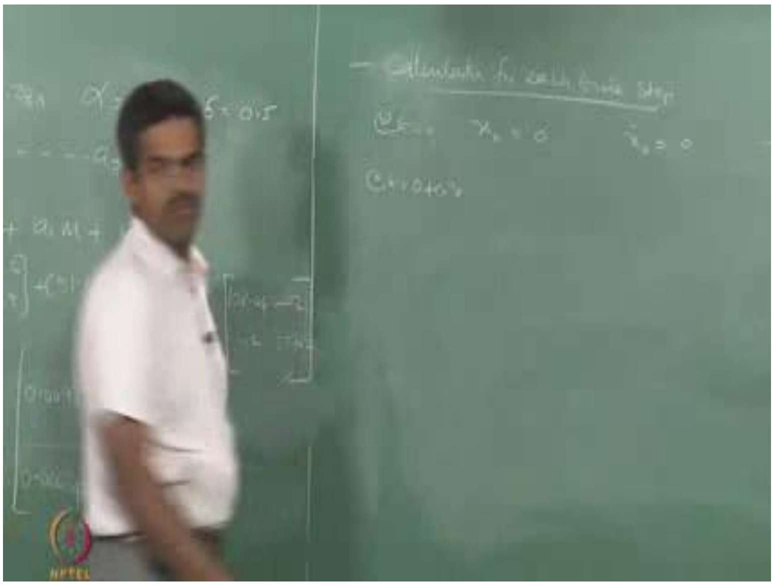

Now, I want to go for a each time step, calculation for each time step. So, at t is equal to 0, I know x 0, x naught 0, x double dot 0. I know this.

Now, I want to find at t equal to 0 plus 0.28. That is, 0.08. I want to know this. So, already you have the equations. It is given to you in step number three or four, given by the manager. So, you can easily find out x t plus delta t, x double dot t plus delta t and x t x dot t plus delta t and keep on. Let us see the scheme here easily, which can be done in MATLAB. So, I am going to this. Please, see the screen now. Go to the MATLAB.

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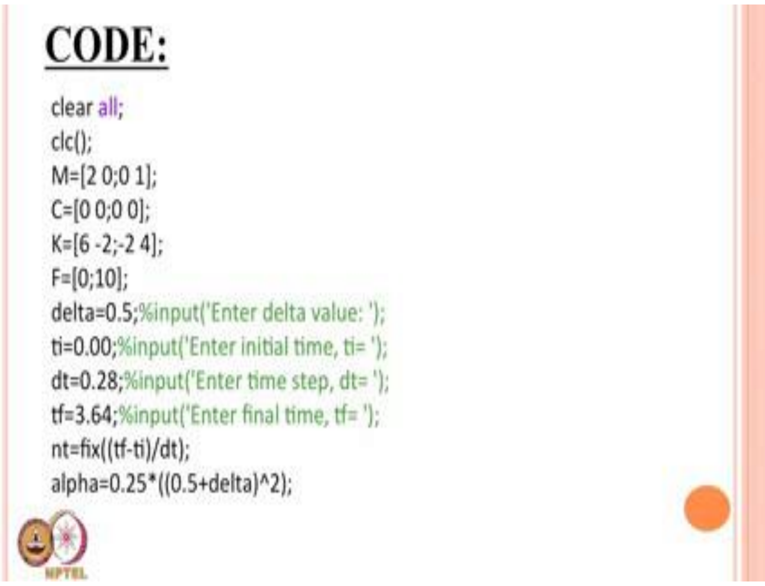

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So, the code is available there.

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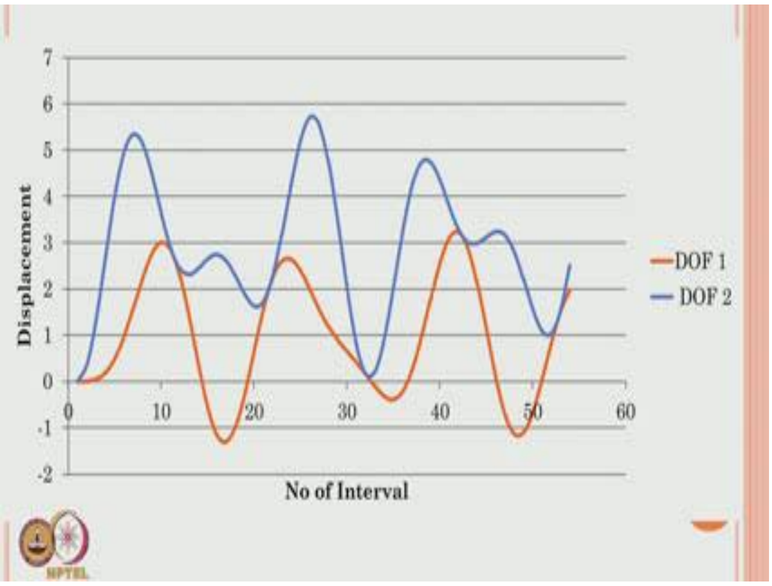

For about let us say 55 seconds or 50, 60 seconds, close from 0 onwards. The degree of freedom one and two is marked on different legend colors; displacement, velocity and acceleration. Now, looks like a continuous.

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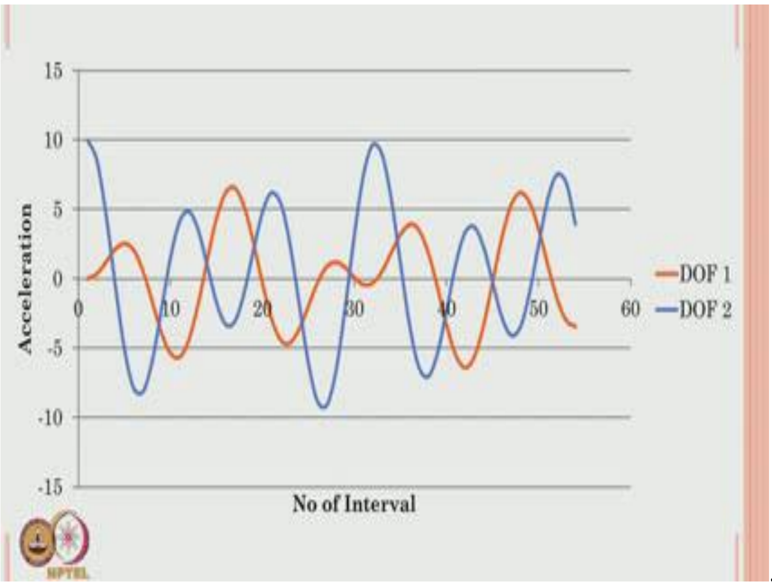

This is how you get the time variation of the displacement, velocity and acceleration using a numeric integration scheme as suggested by Newmark in 1960 s.

Now at every point here, at every time step here, you get only one value of x, x dot and x double dot. But, when you apply this scheme to a system, where the compliant structures are involved, let us say. Now, let us see what is the complication we get when we apply this scheme; by applying this scheme to a compliant structure. Take for example, TLP. Why TLP because we have shown the results of TLP in the previous lectures. So, let us see how, what is the difficulty here.

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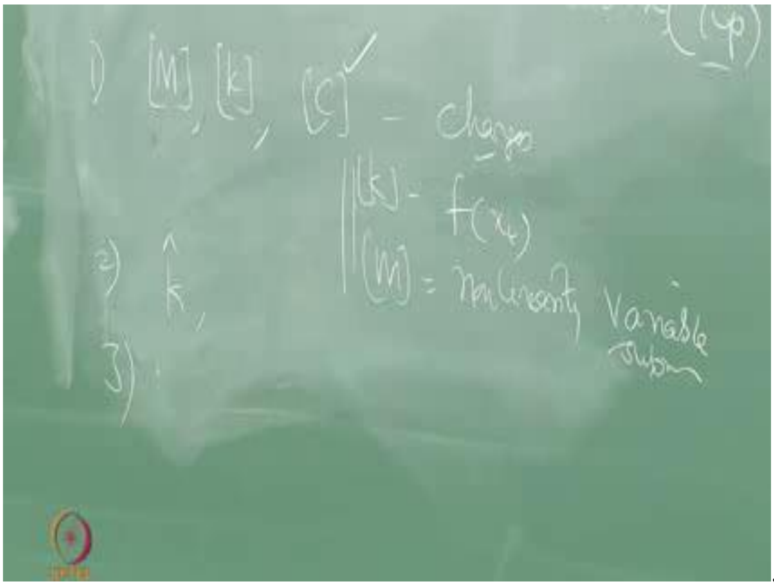

The first difficulty comes here is very simple. In TLP, the mass matrix and k matrix, and therefore the c matrix because it is a 0 M and a 1 k. Keeps on changing at every time step. And most importantly, k actually is response dependent. We have already seen the equations for TLP. We know it is response dependent. And to some extent, some of the coordinates of M is also buoyancy dependent. What we call? Non-linearity arising from due to variable submergence, therefore these two is not a fixed value. As you had in this problem, they are changing.

To start with you will have M, k and c for the initial value of x only, which is at static equilibrium condition. So, initial values you will have, then you keep on updating. It means when your M, k and c is changing, the whole scheme of this will get iterated, will not be unique. Two: now, look at the equation for K hat. K hat is a function of k which is function of x. K hat in this algorithm is constant. It is out of the loop of the calculations in every time step whereas, now a case K hat will get into the loop because K hat becomes the function of displacement, where displacement is actually an unknown. So, this is another issue; K hat.

Now, more serious issue is this. by any chance, if your integration scheme is not proper in your analytical programming, inverse of K hat will fail will not be able to get the answer actually. It shows divided by 0, it will exit out of the program. It will not give you the answer because there is a difficulty where you have done mistake in your algorithm and K hat inverse does not exist.

The scheme will stop. You will not be able to get the answer. And, the fourth problem; very serious is I want the convergence. Why I need a convergence? In this case, if you see at every time step starting from 0 or 0.28 or 0.56etcetera, I have only one value, whereas, in my scheme of TLP at point two eight, I initially have a k. I initially have a k, which is based on x at 0. Whereas, my displacement at exact point two eight should have a k at point two eight. So, my k will get updated. The moment the k gets updated, delta t x at delta t will change. So, I get a new value. So, this new value should have a k at that x of t. Where, I have a k only at this x of t. So, keep on iterating. So, I must iterate till there is a convergence of at every time step, one x of t and another x of t is exactly converging. So, this problem will be there in case of compliance systems.

And, additional problem; in all the scheme if you see, the whole issue K hat or x of t, there is a divider at F hat t plus delta t; whereas, F is also dependent on x because I am having in Morrison equation, x dot minus u double dot. I have a; I have relative displacement. So, if you do not have the acceleration at t plus delta t, i cannot find actually the Morrison force. This will also come into the loop again here.

Now, the question here is if I do not know this in my algorithm, I cannot find u dot because it is F t plus hat by K hat. Both are now getting simulated or getting iterated.

So to start with, at every point I will have nothing except the values of the previous point. But, all the values at this point should be available at that point, which I do not have. So, I believe that the previous value points are as same as the value points here and find out the answer. See whether this answer is what I am assumed in the beginning and getting the same back again. If you do not get, keep on iterating. Iterating in sense, not only the displacement, you have got to start iterating from k, M, c, F of t. The whole equation of motion is getting completely iterated. Is it clear?

And, now the most important last confusion is this convergence should happen at velocity displacement and acceleration. Now, there will always scheme there, any one of them will converge, the other one will not converge. Therefore, it is very difficult and complex that people do not apply this for compliance structures. And, you will find very less analytical papers available on solving this by this procedure. So, when your acceleration velocity and displacement do not converge in these schemes, we focus on displacement. One can ask me a question why displacement. Compliance systems are displacement bound. They are not acceleration bound. The systems are actually meant for large displacements. That is why they are called compliant. So, we compare with the displacements, then checkout.

So, all these earlier solutions what you saw in your presentation for TLP solutions were based on analytical coding done on Newmark's beta scheme, which showed a very promising convergence. And, we had seen the results very easily, where the schemes has given in most of the reference papers listed by me in NPTEL website.

So, when you apply this scheme for a system which is compliant in nature, where the LHS and RHS of equation of motion are interconnected and response dependent. This scheme is not as simple as you see in the coding here or in the understanding in your mind. It is very complicated.

So, however when you apply this for the coding, you will get an answer of this order, where more or less the solution looks as if it is continuous. However, the behaviors between the time steps are arbitrary. I can even plot them, I can even join them as a straight line, as a cubical line, any line; plain, curve, anything as per my choice. That is the catch here. So, I made it very clear that how the TLP solutions are obtained for equation of motion, what are the different kinds of forces which has been attempted.

So, in the next lecture we will focus on new generation offshore structures for ultra-deep waters, where the design and development also goes with dynamic analysis parallely. So, we will take up a new kind of a generation platform, which is meant for ultra-deep waters in sea. What would be actually the complication deriving the stiffness matrix for that system and how the stiffness matrix or how the response behaves. Behavior is seen when it is subjected to random and regular waves experimentally, analytically and numerically. Of course, in that solution we are not adopting Newmark's beta method.

We are going for a inbuilt algorithm schemes available in the software like ANSYS Aqwa. So, I will show you that. I will show you even the screenshots how to model them in Aqwa, what are the difficulties arise when you model them in Aqwa. I will show them. I will also show the model. I will show you an experimental video. I will show the solution experimentally, analytically and numerically. Compare them. Then, I will show you how this was patented and how the project was; I mean the problem is accepted internationally by many agencies. So, that would be the focus for the next two lectures to understand. Once we understand that we have done enough on FSI. We will stop there; move on to the last module, where we will talk about stochastic dynamics.