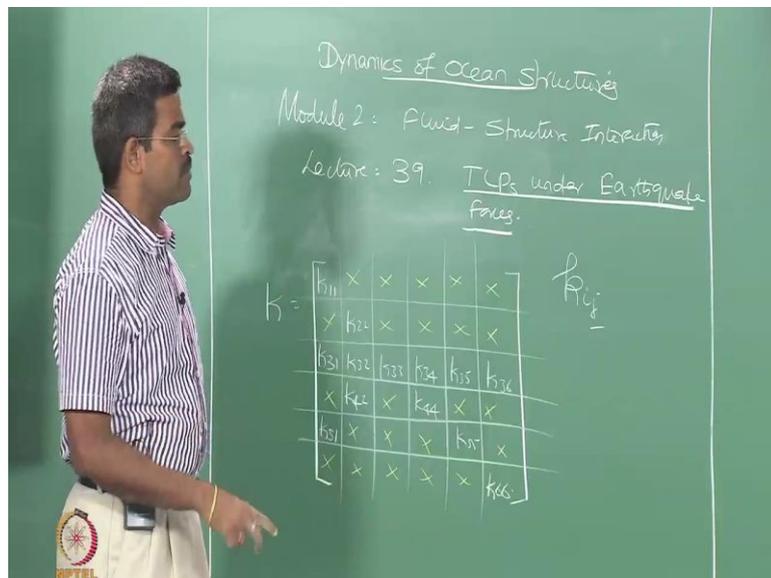


Dynamics of Ocean Structures
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Lecture - 39
Fluid Structure Interaction

In the last lecture we are discussing about the Derivation of Stiffness Matrix for TLP.

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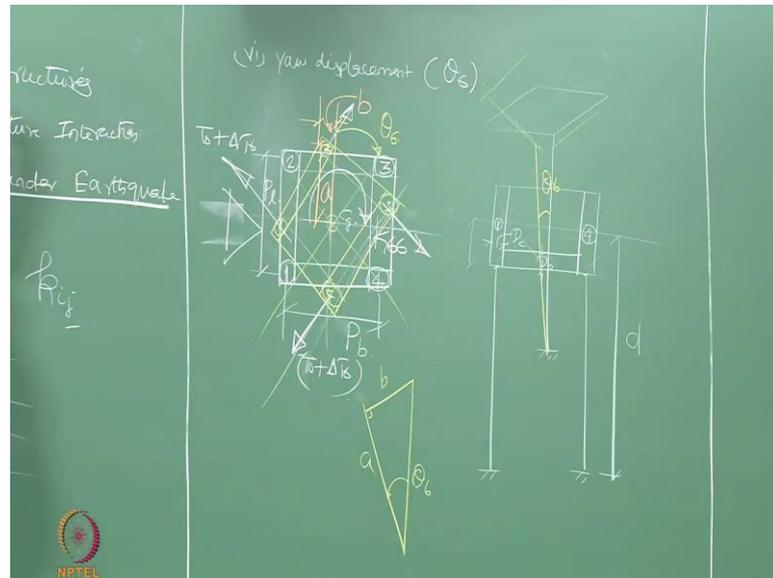


If you recollect the stiffness matrix was derived column wise we are one more degree to derive we will do that. Now let us look at the revisit the basic. So, we have k_{11} which is from the surge k_{31} and k_{51} and we have k_{22} k_{42} and k_{32} and we have k_{33} which we have derived yesterday and k_{34} and k_{44} , we have k_{55} and k_{35} , now we will have k_{36} and k_{66} . Remaining all elements which have been not marked here they are 0.

There are some special characteristics about this matrix we will talk about this slightly later once we complete this column of derivation which is for a yaw motion. As we described to derive the elements of stiffness matrix I must give unit displacement of the j th degree and I was get the forces in i th degree keeping all other degrees a freedom

restrained. In this case I must give you in rotation about the yaw plane and I must get the forces which are activated or influenced by the yaw motion.

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So, let us try to draw the plan of a four column pontoon supported TLP. So, I should say a 6 yaw displacement θ_6 . With respect to the cg we give the rotation, so we rotate about the y plane as the horizontal plane to give yaw motion. And we know to give this we need a force which you will be actually k_{66} and this will be contracted by two planes if it this one by a force of this order which can be $T_0 + \Delta T_6$, T_6 that is how it is rotated. And we know the standard nomenclature we call this as this may wave approach direction, we call this as P_b . This has my planed length P_b p l center to center and we call this distance as a , and we call this distance as b .

If we look at the elevation of this platform that is initially the platform is a horizontal in static equilibrium. Let us view the platform from this direction, so this column will be 1 2 is behind 1, this column will be 4 and 3 is behind 4. This may be the still water level; this may be the designed draft for this given which we call as \bar{h} . And they are connected to the tether with the bottom and the seabed where we call this as my water depth which is small d and these are the diameter of the columns, so these are d_c and of

course this is a pontoon member which can be d p, but in this case both of them are equal this is usual practice for welding comfortability.

Now, imagine that this platform is having a three dimensional plane like this, I am looking at these points on the points where the tether is connected I am drawing this plane somewhere here, there is the plane what I am looking here. I am drawing it separately. So, let say connected only one tether, when this platform yaws when this is a yaw motion to the platform so the column get shifted, but they will happen on the same horizontal plane. And the angle between them it should be as same as theta 6.

So, now if you look at the geometric calculation of this; let say I have a tether, I have a tether vary and I have a normal distance. If I call this as my theta 6, I will call this as b and this as which will be more or less correspondent to a and b qualitatively. Quantitatively not because this is wave in elevation this is plan, but qualitatively they come with the same meaning. Let say now the new length because you now when the platform yaws actually when the platform moves in the horizontal plane rotation in horizontal plane the tethers will get twisted. Now tether will definitely get a new length now.

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Let new length of the tether be l_1

$$\Delta T_6 = (l_1 - l) \frac{AE}{l}$$

$$K_{36} \theta_6 = 4 T_0 \frac{(l_1 - l)}{l} + 4 \Delta T_6 \left(\frac{l}{l_1} \right)$$

d.

$$K_{36} = 4 (T_0 + \Delta T_6) \left(\frac{a^2 + b^2}{l_1} \right)$$

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Let us call that new length; let the new length of the tether be l_1 . l_1 can be computed vectorially if you know the value a and b . One can use a vectorial algebra to compute l_1 , which will of course have a marginal value of l plus l plus Δl that is going to be l_1 because cable is inextensible, but this l_1 will be different from l certainly. Therefore, ΔT_6 is going to be that l_1 of the axial stiffness of; let us put it like this the change in length minus the original length multiplied by axial stiffness that is going to be new tether force caused because of the yaw rotation.

The moment I do this in yaw plane though it remains horizontal, but there will be some set down effect happening because of this motion so k_{36} will be invoked. You may ask me question how he will be invoke. Now any change in length will always tried either pull down the platform or release the platform which is going to be happen in the u direction, which we call technically a set down. Though you may imagine you may not accept because yaw is going to be happen in horizontal plane anyways the platform is not heaving at all, how I will get k_{36} . Any change in length will always be directly connected to heap motion. There is no other rescue except there he only up to compensates this, so k_{36} will be invoked.

So, k_{36} of θ_6 for dimensional stability can be now derived directly from the new tethers which are the original length. We will take a proportion because I am looking only for the increment plus ΔT naught of a e by l , there is no a e by l because ΔT naught is already there so a e by l the ΔT_6 will have a e by l . I should say ΔT_6 . So, $4 T_0$ of the proportion of that plus ΔT_6 l by l_1 of l by l_1 minus proportional, only the proportion we will get added to mass stiffness k_{36} can be now found out from this relationship because T_0 is already known and ΔT_6 can be known if you know l_1 . ΔT_6 can be known if you know l_1 , because remaining all known to you. And l_1 can be vectorially computed if you know a and b .

K_{66} the second component will be the restoration by the movement value on the horizontal plane which will be T_0 plus ΔT_6 4 times of that rather the near cables will have different angle of twist the father cables will have different angle of twist. So, one can say this is ΔT_6 and this will be ΔT_6 bar or dash, we have already said in the previous elevation let us equate both of them, equalize them therefore I take this as

four value. I could have also written this as $2 T_0 + \Delta T_6 + 2 T_0 + \Delta T_6$ as we did in the last time then we equated them it becomes 4 multiplied by let us call this as new length of l_1 .

So, now I have k_{36} and k_{66} remaining all we already discussed in the last lecture. So, I have the full stiffness matrix now. Now we can discuss what would be the salient points of this stiffness matrix.

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MATHEMATICAL DEVELOPMENT

$$[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{F(t)\}$$

- $[K]$ is square, non-symmetric matrix of size 6×6
- Heave has strong coupling with all degrees-of-freedom
- Presence of off-diagonal terms reflect coupling between various degrees-of-freedom
- coefficients depend on change in tether tension which affects the buoyancy of the system
- Hence $[K]$ is response dependent
- Coefficients of $[K]$ are nonlinear (for example, k_{31} , k_{32} etc)
- Hence $[K]$ is not constant but changes with instant of time

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Now, one can see here k is a square matrix, we can see this is square matrix where we got 6 by 6 it is a square matrix. K has non-symmetric city see k_{31} is present, but k_{13} is not there. So, k is non-symmetric or asymmetric or size 6 by 6 . You will see that you give unit displacement any degree 1 2 that is surge sway heave roll yaw pitch or pitch and yaw this invokes automatically and understanding of the force in the heave degree. It means heave is strongly coupled with all degrees of freedom; it has got a very strong coupling.

Now if you look at the diagonal band and look at the off-diagonal terms whichever is present apart from the band; the off-diagonal terms reflect the strong coupling between various degrees of freedom. Now, one can easily see how they are coupled. For example,

if we look at this term pitch and surge is coupled, look at this term roll and sway is coupled and so on so forth. You can easily find out the off-diagonal terms will tell you what is the influence of coupling between various degrees of freedom. Heave, however is coupled with all degrees of freedom. Other than that there is a combination between certain degrees of freedom get coupling that can be seen from off-diagonal terms.

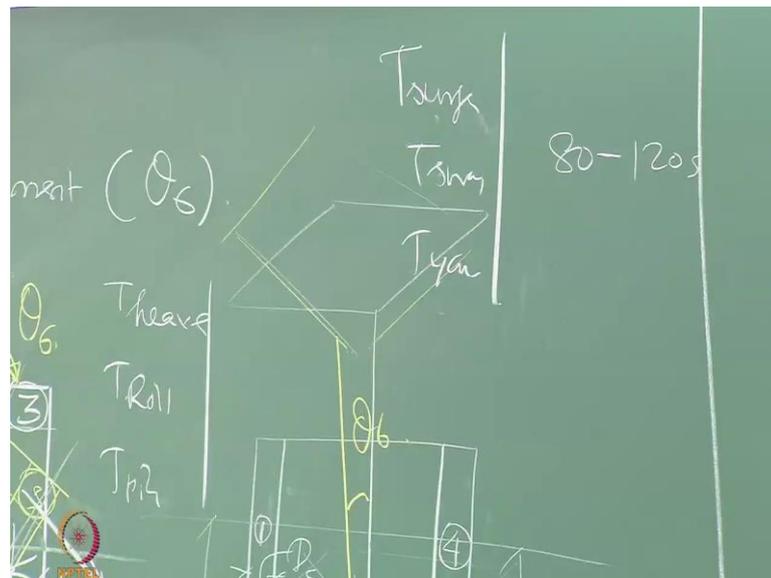
And you have seen for example, let us take here the change in length of tether tension affects of course the buoyancy of the system, because it is invoked in the heave response, heave response will cause set down, and set down will alter buoyancy. So, you will have force challenge the buoyancy of the system. And if you look at all the terms available in the stiffness matrix let say even this a and b etcetera, l_1 , etcetera or even in earlier cases you will see that most of them are response dependent. So, response is not known for a given system, therefore k instantaneously is not known. K matrix is not known on the given problem that is very interesting. If you do not know k though you may know m you cannot find the frequency of the system, so cannot design the system actually. So, k is not known because k is most of the terms in fact all of the terms are response dependent. If you do not know the response you do not know the k .

And most importantly if we look at for example, k_{31} I wanted to open and see k_{31} or let say there are cosine and sine terms available in this which are non-linear, so it has got non-linear terms. Since, it is a response dependent the stiffness matrix is keeps on changing as a wave approaches the system and pass the system. So, k is not constant, but it changes with instant of time. It means the platform will have a different band of frequency sweep from a to z as the wave passes, because as the wave passes set down and heave response will be invoked. As heave response invokes change in tether tension will happen. Change in tether tension will ask will demand change in different elements of stiffness matrix. The stiffness matrix elements changes ω change though m may not change but m will also change I will show you m will also change, but if identify that m is not changing k is changing ω certainly will change.

Therefore it is very very interesting as per the design conceptualization TLP is will never get into a resonance band at all; TLP will never be in resonance band at all, but then how TLP is failed. That is very interesting problem what we will talk about in this lecture. So,

TLP's will not fail because of resonance, because it is a sweeping omega. And TLP's frequency time periods if we look at; if we look at the periods which we remember we already said that.

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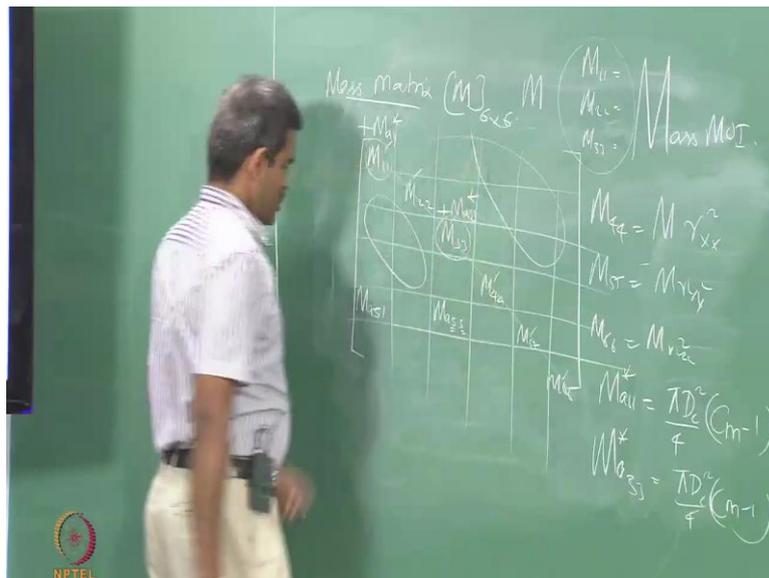
Let say T surge, T sway and T yaw, because most these are all on horizontal planes. These two are displaced in this rotation they have been in a horizontal plane surge, sway and yaw. They all in horizontal plane it is very complained about this, complained means, highly flexible. Flexible means large period. So, it is about 80 to 120 seconds very very high. If we talk about the on the other hand the contrary the design let say time periods of heave, time period of roll and time period of pitch the T, takes these values they will happen in the vertical plane because roll is a rotation of x axis and y axis and heave is in the vertical plane.

So, one is very complained about the horizontal plane the other is very stiff about the vertical plane, the moment I say the stiff about the vertical plane the period is going to be very low. This will be within 2 to 5 seconds, so there is a drastic band width between the flexible frequencies and stiff frequencies in a given system that is why it is called as on hybrid platform. If you look at the real wave which is coming in the system in any sea state starts from 6 seconds to that of 60 seconds they will never be in the band of TLP at

all. So, TLP will have no excitation frequency matching without the natural frequency even though the frequency of the TLP does not change with k .

However, in this case since k is response dependent we have just now saw k invokes change in ω therefore, TLP will never be in the resonance band with that of the excitation frequency. Then how TLP is actually failed? That is interesting question, but this is very important that they will not. So, you have got one set of degrees of freedom far away from the band width of the excitation period. One set of frequencies are the period which are very low from the excitation period, therefore they will no possibilities of having a resonance tuning with that of the excitation frequencies or excitation forces.

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Now, let us talk about the mass matrix. As you surely know mass was also going to be 6 by 6 and we know that we have going to lump the mass of the whole system at the point where degrees of marked or measured. In this case of machine degrees of freedom or at the cg point on the deck I can measure anywhere I want. The moment I do that then I got a lump all the mass calculated at the cg of the deck. Though I have shown here in plan deck as a point, it is a 3 level deck let us say you have to hypothetically imagine that there is a point in the deck plane where the cg are the mass moment of inertia of the

whole system is getting concentrated at that particular point I am measuring surge, sway, heave, roll, pitch and yaw which I have been marking all my derivations.

So, mass matrix as usual will be 6 by 6 square and we know it is going to be diagonally dominant because, the reason is I am lumping the mass at the degrees of the freedom we are marking therefore off-diagonal terms are generally 0. Let us say m_{11} or let us say m_{11} m_{22} m_{44} m_{55} and m_{66} this is what you generally encounter in any structural system where the degrees of freedom are lumped at the mass where or the mass has lumped at where the degrees of freedom are measured in the given system and dynamics. However, in TLP you are got again one more complication from the mass matrix also. There are few off-diagonal terms represented in mass matrix also I will come to that how they are coming.

Now first let us see how we compute m_{11} for example, all let say m_{22} or m_{33} . These are nothing but mass moment of inertia or second moment of area of all the objects present in the system with respect to x y or z axis aligned so one can easily compute it. σ_x^2 where σ_x where is x is distance for surge that is m_{11} , similarly σ_y^2 where σ_y where is y is distance for sway that is m_{22} and m_{33} , I can easily find out this. If I really wanted to find out m_{44} which is mass moment of inertia about the roll degree-of-freedom I must say it is m, fortunately you will see for symmetric balance system in a static condition or equilibrium you will see that all this three mostly will be equal to same value simply m. There will be no variation between this.

Now, one can ask me a question why there should not be variation. There will be marginal variation in heave because of the top side details. However, the variation will not be much is varying is about 5 to 10 percent. So, for practical cases one can say for example, a given story in a building a certain story generally you see the mass is lumped every floor the analysis like this, if you want to measure the mass here half of this floor half of this floor which is equal to one floor height; so this mass is m_1 . Similarly m_2 m_3 m_4 will be all same. However, we see this is going to get only half, so this is going to be half of the existing mass this (Refer Time: 21:39) there in buildings also. Wherever you lump a mass where you have machine the degrees of freedom this complication idealization was there earlier also.

In this case also there is a variation heave mass will be slightly different which is more than these two, but the variation has been seen only by a margin of 5 to 10 percent. So, let us hypothetically understand that all this three will be equal to m a single value which can be computed geometrically. Now I want to look for m_{44} which is here, this will be nothing but this mass multiplied by r_x square where r_x is the radius of variation which is i_x by a root of that. i_x is already known because for finding out m_{11} I wanted a x_s I already know that area also I know, so I can easily find m_{44} .

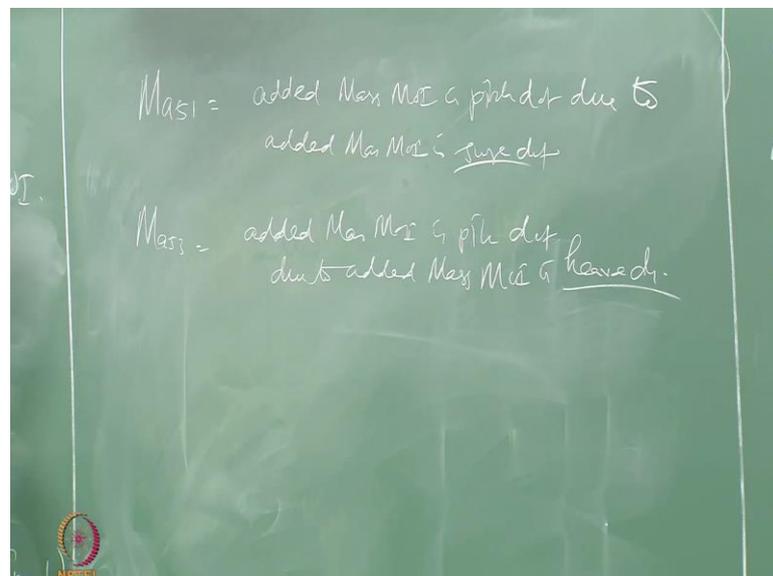
Similarly, m_{55} will be $m r_y y$ square and m_{66} will be $m r_z z$ square and we all agree the value of radius of variation it can be also experimentally obtained. Why it is easy in this case because using a simple flotation statistics of this problem can always find the radius of variation for the system. One can easily obtain these values. So, I have always the values ready for mass matrix. Now the question is, when I have all the values ready I will have added mass also; I will have added mass because the body is submerging partially I will have added mass because the wave is not as horizontal line as you see. Wave is actually a crest and trough there will be some added mass coming because of the crest same as the added mass deleting because of the trough. And so I have to account for that.

So, I will put that as m_{11} plus m_{a11} added mass and the first value which will be πd^2 square by 4, d stands for the diameter of the column member $c m - 1$. Why $c m - 1$? I am taking this as added mass in the left hand side actually added mass is a force coming from the right hand side so $c m - 1$ into ρ of course that is the density. Since we are looking for added mass moment of inertia I must say $x \ddot{}$ surge.

If you really wanted to find out m_{a22} ; there is no m_{a22} there is m_{a33} for a unidirectional wave I am talking about unidirectional wave only, so there is no sway but surge and heave have coupled already we have seen that set down and offset therefore m_{a33} will be there which will be equal to again πd^2 square by 4 $c m - 1 \rho x \ddot{}$ heave. So, I get these terms. Here, I will add m_{a33} star and star, star are nothing but the values what I am getting from here which are added mass terms.

Just now we saw heave is coupled to other degrees of freedom. In fact, heave is coupled all degrees of freedom we saw that in stiffness matrix also. Therefore, whenever I have an added mass in heave I must have this coupling in pitch. Whenever I added mass in surge I must have coupling in pitch also because surge and pitch are coupled. And heave is anyway coupled with surge. Now in with pitch is coupled with all degrees of freedom, so I must have the reflection of these values on m_{51} and m_{53} , where heave and pitch are anywhere coupled. And pitch and surge however coupled we know that. So, remaining all terms will be 0.

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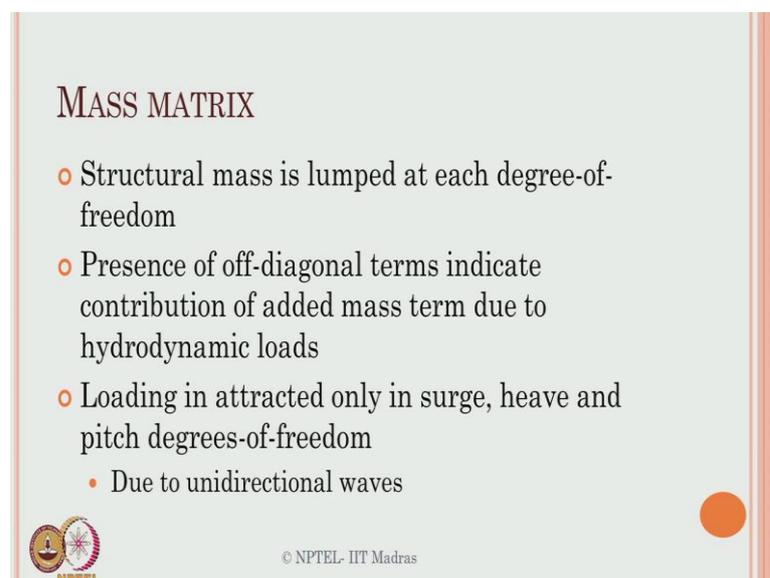
So, let us get these terms first m_{51} and m_{53} . If we really wanted to find out the additional mass moment of inertia because of m_{51} which is nothing but, added mass moment of inertia will pitch degree due to added mass moment of inertia in surge degree. How do you get that? How do you get this term m_{51} ? If we look back in module we read the problem I wanted to actually derive a stiffness mass matrix for a beam like this we gave unit acceleration here we found out the mass terms, similarly you can find this as well if you unit acceleration in one degree and find the force for unit acceleration. Instead of unity if it is the value given to you from here multiply this by this you will be able to get m_{51} .

Similarly, m_{53} will be added mass moment of inertia in pitch degree-of-freedom due to added mass moment of inertia in heave degree. So, here it is heave here it is pitch. So, we can easily find out this. Now you have got a classical problem in mass matrix. Mass matrix will be diagonally dominant as generally in all other mass matrices, but the off-diagonal terms present in mass matrix will make the mass matrix un-symmetric. Therefore, mass matrix cannot be diagonalized, because then you will lose these terms.

So, you have to handle mass matrix full, you have to handle stiffness matrix full because it is asymmetric you cannot band it. Therefore they are solving equation of motion with having full mass and full stiffness matrix which are response dependent. How, m_{51} , m_{53} will have a variable buoyancy submergence effect which is again dependent on the wave condition. So there is some r h s dependence on this, right hand side dependence on this in the mass matrix.

Now, the question comes how do we start with the problem, because k is response dependent I do not know k . Mass is having r h s right hand side equation of motion dependent therefore do not have mass of the complete. How do we start with the solution of equation of motion? So, look at the equation of motion here $m \ddot{x} + c \dot{x} + kx = f(t)$, we derived m we derived k we will now discuss about quickly c .

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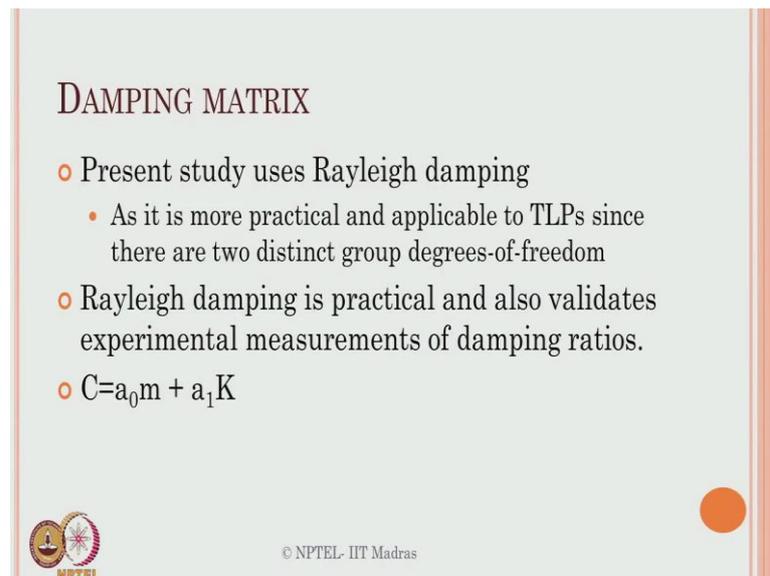
MASS MATRIX

- Structural mass is lumped at each degree-of-freedom
- Presence of off-diagonal terms indicate contribution of added mass term due to hydrodynamic loads
- Loading is attracted only in surge, heave and pitch degrees-of-freedom
 - Due to unidirectional waves

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Quickly c will be the rally dependent matrix anyways the mass matrix is lumped at each degree-of-freedom. The presence off-diagonal term will show that contribution of added mass term due to hydrodynamic loading coming from that. Loading is attracted only in surge heave and pitch degrees of freedom being unidirectional way we already know that.

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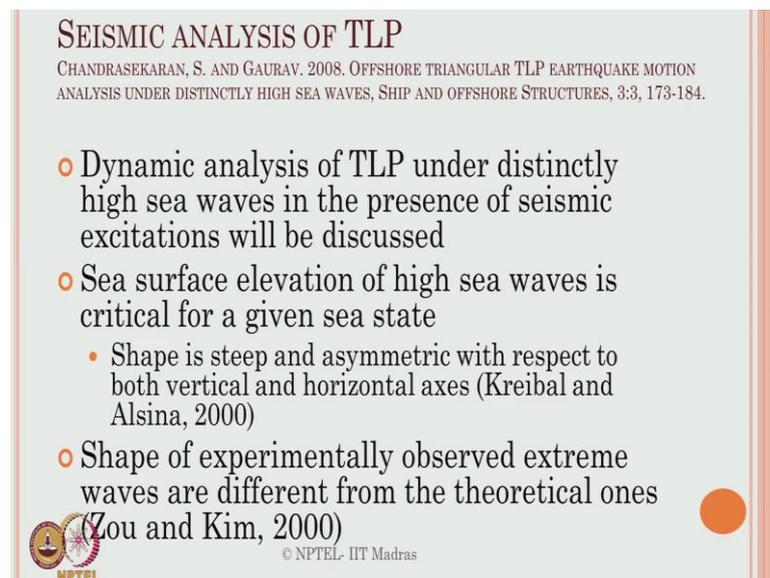
The slide is titled "DAMPING MATRIX" in a serif font. It contains three bullet points: "Present study uses Rayleigh damping" with a sub-bullet "As it is more practical and applicable to TLPs since there are two distinct group degrees-of-freedom"; "Rayleigh damping is practical and also validates experimental measurements of damping ratios."; and the equation $C = a_0 m + a_1 K$. In the bottom left corner is the NPTEL logo, and in the bottom center is the text "© NPTEL- IIT Madras". A solid orange circle is located in the bottom right corner of the slide area.

The damping matrix what we generally use in TLP is the Rayleigh damping which is proportional to mass and k both. One can ask me a question why I should take mass and k both proportional in this case, because you know either k or m will not represent the two behavior of the uniformed distribution of the model participation all the degrees of freedom.

One can ask me a question do you have to truncate the degrees of freedom. All of them are important because in this case we are having strong coupling between the one on another. So, I must have all omegas and all periods and all mode shapes and all frequencies, so there is no question of truncation coming though there are only six in number, but still there is nothing like a fundamental frequency in TLP, because the design is in such a manner where the frequencies are two sets actually. So, we must have all the values.

In that case I must use uniform distribution of damping so I should recommend Rayleigh damping. We already have enough idea about Rayleigh damping, we already said so that one can compute the Rayleigh damping ratios easily and solve this problem.

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SEISMIC ANALYSIS OF TLP
CHANDRASEKARAN, S. AND GAURAV. 2008. OFFSHORE TRIANGULAR TLP EARTHQUAKE MOTION ANALYSIS UNDER DISTINCTLY HIGH SEA WAVES, SHIP AND OFFSHORE STRUCTURES, 3:3, 173-184.

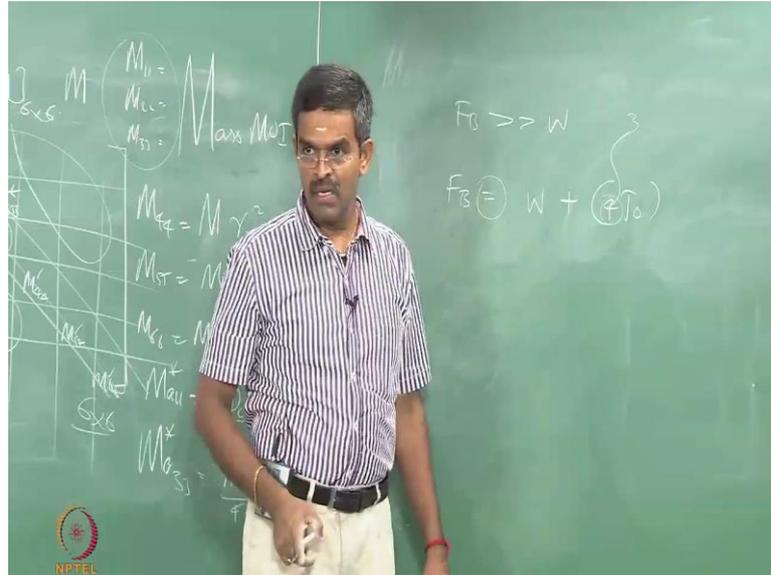
- Dynamic analysis of TLP under distinctly high sea waves in the presence of seismic excitations will be discussed
- Sea surface elevation of high sea waves is critical for a given sea state
 - Shape is steep and asymmetric with respect to both vertical and horizontal axes (Kreibal and Alsina, 2000)
- Shape of experimentally observed extreme waves are different from the theoretical ones (Zou and Kim, 2000)

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So, when we talk about solution of TLP or any other dynamic system using (Refer Time: 29:56) beta integration method I will come to that particular point later. Let us now talk about one interesting problem where, how TLP's will otherwise fail? Because now you see if the system is designed in a such a manner if the tether is full of they will have float if they are got to be let us say positioned restrained then you velocity and de velocity.

So, commuting or let say transporting or erection, commissioning, decommissioning becomes very simple except that it is very massive system of 90 meter 100 meter size you need large vessels and boats to tow them from location a to b. However, it was comparably easy with respects to that of other structures are fixed more or less, so that is a idea here.

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But the very serious problem in this case is since the static equilibrium always challenged one must understand the basic problem in TLP is that, buoyancy exceeds weight by a very large amount that is a designed philosophy here. To equate this we say the buoyance force it should be equal to weight plus some good amount of value which essentially come from only T naught or let us say 4 T naught to be very specific.

Now, instead of 4 if for example you have only 3 tethers one is cut off. You will have a terrible imbalance in static equilibrium so the problem with TLP is not failure, but it loose it is stability, it loses equilibrium actually. Because it is dependent on an external agency which is response dependent you have no control on T 0, because T 0 has got delta T naught also which is dependent on x, y, z, etcetera which we saw this in stiffness matrix. You really do not know if the surge displace is very high, very large then delta T naught will become very high. Therefore, this value can go into an unstable mode or non equilibrium set up, because you have no chance of changing w more or less w is prefixed.

And buoyance of course depends on response because the moment platform face an on offsets variable buoyance will change f b will be invoked automatically. If they get balanced automatically system will be in equilibrium. If they the moment they are not

getting balanced system will become not in the equilibrium. So, TLP's will fail only by that all, therefore here tether pull out becomes a very serious problem; tether pull out become a serious problem. Now, how to actually look at the tether pull out problems? Tether pull out will happen only when force exists in such manner the tether is getting cut. Tether will get cut only by two methods; one you give very excessive surge so that the cable is inextensible, cable gets pulled off.

The other is you have already have a massive weight in heave direction by increasing this weight by marginal 5 to 10 percent suddenly it can invoke tether pull off. So, heave stability is becomes very important, whereas in the original design of TLP people thought that heave degree will not be challenge this table equilibrium at all. To look at this we are trying to impose the new kind of force at the seabed which is in seismic force, because seismic force has a capacity of moving the plane in all the three directions; x, y and z as well, whereas waves are generally unidirectional. And the force at the bottom when it goes is practically 0 and the members are very thin in diameter tethers are very very thin compared to that of d c and pontoons they do not attract any forces at all.

Therefore, waves will not be able to cause a hypothetical pull out of the tether except by accidents of vessels, whereas if we impose a force at the seabed itself this is a possibility that the tethers can be pulled off. So, a study was done on seismic analysis of TLP by myself and government 2008, the paper is published in shipped and offshore. I will quickly discuss in few minutes that how you have a full access to this paper. Please look at the library intranet and look at this paper and try to read them. But I want to really make you to get through the paper quickly why this study is very interesting and different we will talk about this it is not self propaganda, but it is very important study.

In sense that how actually TLP is failed and what is the reality of this failure that was a real case I will show you seismic earthquake on seismic signal on mass TLP here. So, dynamic analysis is done for a TLP under distinctly high sea waves. One can ask me a question why distinctly high sea wave should be combined this seismic force. It is a very natural geographical phenomenon seismic waves are created only when the sea waves are distinctly high. There is a geographical inter connectivity between these two. When the sea is calm there is no earthquake, it is a very, it is a very interesting inter

connectivity. Therefore, the worse situation is distinctly high sea wave comes and earthquakes also come, what happens to TLP? That is a worse scenario because I am looking for a pull out of the tether what happens, so it has been considered.

Now, one immediately ask me question what do you mean by distinctly high sea wave it is an amplitude is it a frequency or what. So, Kreibal and Alsina really said and described in literature what do we called by a distinctly high sea surface elevation. It says it should have an asymmetric city both in vertical and horizontal access. So, you have to generate the sea wave schematically they are signal. It has been experimentally measured by Zow and Kim that one can always compare this extensive sea waves in the lab scale and it has been envisaged in one of the platforms in reality in mass TLP where there was an efficient of earthquake happened, I will show you the data now.

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- No systematic model is available to categorize such steep, irregular and asymmetric waves
- Hence, in the present study, distinctly high sea waves are generated using nonlinear kinematic wave theory
- Water particle kinematics obtained using Airy's wave theory from a randomly generated sea-surface elevation using a modified P-M spectrum with one-parameter equation (Pilotto et al., 2002, 2003; Michel, 1999)

$$S_{\eta\eta}(\omega) = \frac{8.1 \times 10^{-3} g^2}{\omega^5} \exp \left[-1.25 \left(\frac{\omega_m}{\omega} \right)^4 \right]$$

ω_m is the modal frequency (=0.46 rad/s) and $S_{\eta\eta}$ is the PSDF of the wave




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So, unfortunately no systematic model of this study was available in the literature, because people never attempted to study tether pull off in heave direction can also be a problem to a TLP's equilibrium. Hence, in the present study distinct high sea waves using non-linear kinematic wave theory was created signally artificially. The 11 parameter equation of P-M spectrum was modified which was suggested by Pilotto and Michel in 1999 and 2002-03 respectively. I will give you all the references at the end. So,

now the one parameter P-M (Refer Time: 36:09) spectrum modified given by Pilotto is shown in the screen now.

So, I have this equation now where the one parameter is nothing but omega. Where omega is what is called as a modal frequency which is in this study taken as 0.46 radian per second and $S_{\eta\eta}$ is the power spectral density function of the wave height given by p s and (Refer Time: 36:31) spectrum. Once I generate this let us see how the spectrum look likes.

(Refer Slide Time: 36:36)

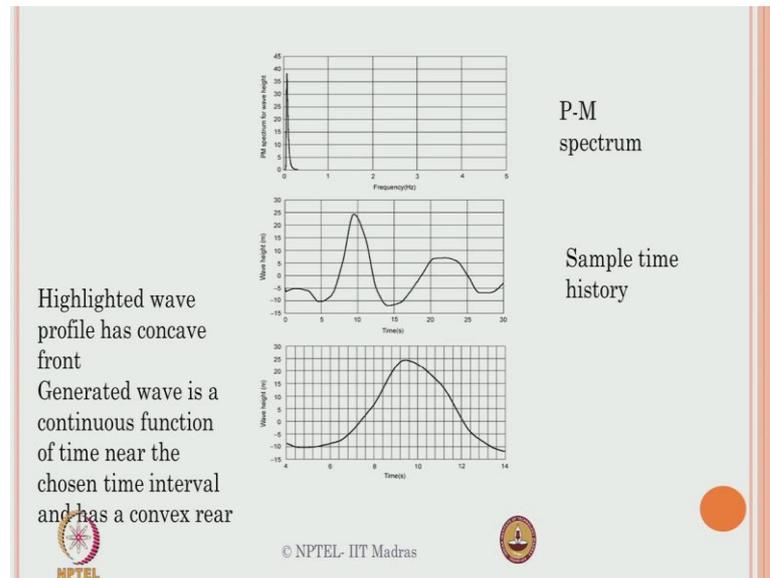
- Wave elevation is realized as discrete sum of many sinusoidal functions with different angular frequencies and random phase angles
- where k_i is the wave number, ω_i are discrete sampling frequencies, $\Delta\omega_i = (\omega_i - \omega_{i-1})$, n is the data points and ϕ_i is the random phase angles
- The generated wave profile is designed to have a peak at a particular time (t_0) which will be distinctly high in comparison with other wave heights

$$\eta(t) = \sum_{i=1}^n \sqrt{2S_{\eta\eta}(\omega_i)\Delta\omega_i} \cos(k_i x - \omega_i t - \phi_i),$$


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Now I want to convert this particular spectra in to a realized sinusoidal of elevation because Kreibal said the sea (Refer Time: 36:44) sinusoidal elevation should have a acute front and obtuse back, then only the wave can be called it is a distinctly high sea wave. So, η of t was generated. When I generate η of t I need to have the phase angle ϕ_i or ϕ_i which is a random number; I need to have ϕ_i of ϕ_i . So, we have used $S_{\eta\eta}$ ϕ_i and ω_i I from the previous equation substituted here for $\Delta\omega_i$ intervals we generated signal we should now qualify a distinctly high sea wave.

(Refer Slide Time: 37:12)



Now, let us see the spectrum here that is a typical P-M spectrum which has been generated. The sample time is say t if it is been generated from that. Now if you enlarge one specific location of the t you will see that the highlighted wave profile is a concave front and it is convex rear. So, this qualifies that the wave generated task is a distinctly high sea wave. Usually this kind of wave does not occur, but in reality this is occurred, I will show you where.

(Refer Slide Time: 37:40)

For arriving at the equivalent triangular geometry, equations of equilibrium are applied at the static sea condition

$F_B = (4T_0) + W$ and $F_B = 3(T_0) + W$

By keeping the initial pretension in triangular TLP as that of the square, total tension in tethers is reduced

Weight is increased by keeping the buoyancy forces same

So, a TLP was taken interestingly. This TLP had 3 legs only. So, there is a geometric optimization here, so where is the optimization we will talk about now. Now we know the equation of equilibrium is F_B which is $4T$ naught plus w . On the other hand if we have got 3 legs only it is going to be $3T$ naught plus w . So, you can either keep initial P tension same as that of the square, therefore tension in the tethers will be reduced or you can keep the total tension same, so initial tension each cable will be increased.

So, two optimization is possible; without touching w . Why it is touching w because I want the same platform with a same top side in the both the cases. So, either I can keep total T naught same. So, T naught to individual cable will be different or individual T naught same total T naught will be different; two optimizations. So, that will change the buoyancy force.

(Refer Slide Time: 38:32)

Property	TLP ₁	TLP ₂	TLP ₃
Weight (kN)	209,500.00	330,000.00	370,000.00
F_B (kN)	334,000.00	520,000.00	625,500.00
T_0 (kN)	124,500.00	190,000.00	255,500.00
Tether length, l (m)	471.00	568.00	1,166.00
Water depth, d (m)	500.00	600.00	1,200.00
CG above keel (m)	26.60	28.50	30.31
AE/ l (kN/m)	58,060.00	82,000.00	45,080.00
Plan dimension (m)	92.50	78.50	83.50
D (m)	14.20	17.00	18.80
r_x and r_y (m)	29.15	35.10	35.10
r_z (m)	32.10	42.40	42.40

○ Geometric properties of square TLPs, based on which configuration of triangular TLPs is arrived is shown in the above Table (Chandrasekaran and Jain 2002b)



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So, the study was done and 3 TLP's at 500, 600 and 1.2 kilometer depth were attempted and these all are real TLPs available in literature in (Refer Time: 38:44).

(Refer Slide Time: 38:52)

TLP	Natural period (s)			Natural frequency (Hz)		
	Surge	Heave	Pitch	Surge	Heave	Pitch
TLP ₁	83.33	1.92	1.96	0.012	0.52	0.51
TLP ₂	97.09	1.92	2.06	0.010	0.52	0.49
TLP ₃	131.58	3.11	3.12	0.008	0.32	0.32

○ Table above shows the natural periods of the equivalent triangular TLPs
 ○ It can be seen that the heave frequency is closer to that of high sea waves (whose modal frequency is 0.46 Hz)
 ○ An expected near-resonance response is examined in heave (stiff degree-of-freedom)
 ○ Rayleigh damping is used in this example problem



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So, they have been taken and these TLPs are remodeled with 3 legs without the equivalence of geometry and then they have been attempted to study with that of the p s

n (Refer Time: 38:55) spectrum and eta e eta t. And natural periods if you look at now I want you to pay attention to the natural frequency of heave period which is 0.25, whereas if you look at omega m which has been taken as an input frequency it is 0.46. So, I am trying to near resonate a case where the excitation frequency is very close to heave and when the heave is excited it will result in tether pull out.

So, I am indirectly generating a condition where the frequency of excitation is very close to one of the natural degrees of freedom which are stiff in nature, which was not attempted earlier because people thought heave degree freedom will not be compromised at all by any chance. So, near resonance was attempted in heave degree-of-freedom it is considered to be very stiff and Rayleigh damping was used in this problem.

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FORCE VECTOR

- Hydrodynamic forces arising from distinctly high sea waves are computed using Morison equation
- In addition, dynamic tether tension variations caused by horizontal and vertical seismic excitations are also accounted for
- Change in tether tension is given by:
- Where, $x(t)$ is instantaneous response, $\Delta T = \frac{AE}{\ell}(x(t) - x_g(t))$

$$x_g(t) = \{x_{1g}(t), 0, x_{3g}(t), 0, 0, 0\}^T$$

Where x_{1g} and x_{3g} will be ground displacements in surge and heave degrees-of-freedom respectively.



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So, force spectral was generated, but there is only one change here the delta T change in tension we will have two components; one is the except t what you already have minus x_g of f t where x_g of t is a vector which has got contributions from the heave and the surge that is x and z components of the earthquake motion. Now it has been put in the vector back and I get delta T once I get change in delta T I will get my change in stiffness matrix. So, I am imposing earthquake signal indirectly on the platform by changing tether tension variation. One can ask me a question the platform is about 1.2 kilometer

away from the sea bed what is the influence of the earthquake on the platform lying at the top.

So, there is no direct influence, but the influence of the earthquake changes delta T delta T changes k and k affects my system stability or equilibrium. Therefore, I am imposing earthquake force indirectly on the super structure by this mechanism.

(Refer Slide Time: 40:43)

○ Ground motion is generated using Kanai-Tajimi power spectrum

$$S_{\xi, \ddot{x}_g}(\omega) = \frac{\omega_g^4 + 4\zeta_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\zeta_g^2 \omega_g^2 \omega^2} S_0, \quad S_0 = \frac{2\zeta_g \sigma_g^2}{\pi \omega_g (1 + 4\zeta_g^2)}$$

- Three parameters namely: ω_g , ζ_g , S_0 are to be carefully chosen to match a realistic earthquake record
- On 10th Sep 2006, at 14:56:07 (coordinated universal time), an earthquake was epicentered at 26.34N, 86.57W in GoM, 250 miles away from WSW of Anna Maria, FL
- Actual EQ was measured as M5.8 with peak ground acceleration and velocity as 0.25g and 0.29m/s
 - Generated EQ signal has 0.25-0.39g and 0.2-0.3 m/s
- Interestingly, MARS TLP, is also operating in GoM in the near vicinity at Mississippi Canyon Block

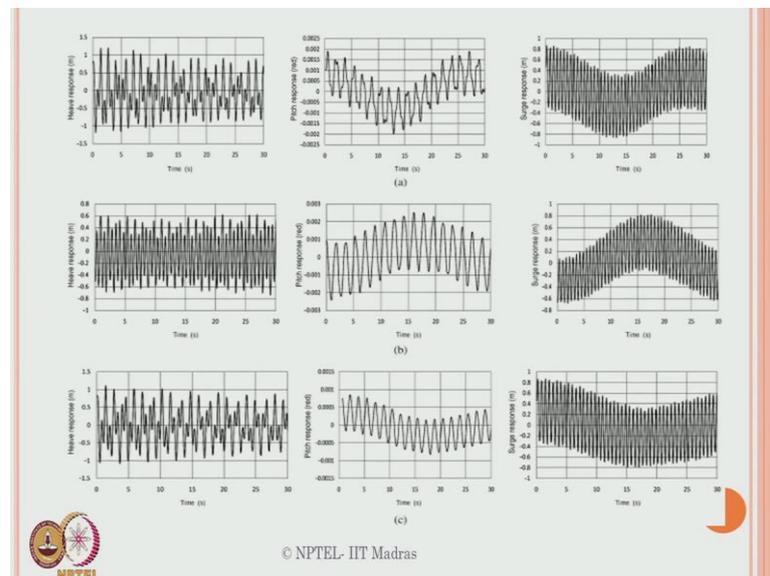
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Once we do this the ground motion was generating using Kanai-Tajimi we already have the spectrum already we have we know the. So, omega zeta g in s naught where the real time problem parameter chosen for the record one can ask me where you have taken this from on 10th September 2006 at 1456 coordinates given there. Gulf of Mexico 250 miles away in Anna Maria Florida there was an earthquake. And instantaneously this is the place where the MARS TLP are located.

So, it is the real time signal generated and that signal was having peak ground acceleration velocity which was 0.25 g and 0.29 meter per second which was measured there, but the signal which is generated to this Kanai-Tajimi spectrum gave us 0.25 to 0.39 g which was very close and the velocity of 0.2 to 0.3 meter per second.

So, there is a close resemblance of the generated signal with that of the recorded signal of the actual earthquake which happened in the a p center where MARS CLP is located, but unfortunately or fortunately MARS TLP's was not failed with earthquake. That is a different story, but has would I have failed how the failure would have been that is we are trying to look at here. So, one (Refer Time: 41:48) is done then now you see power spectral density of the ground acceleration. Now the equal at time history generated from Kanai-Tajimi spectrum (Refer Time: 41:57) of motion of solid using numas beta integration method, all non-linearities were considered in the analysis.

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The results are like this, so TLP 1, TLP 2, TLP 3, a b and c respectively because there are two TLP's 500 600 1200 meters depth all of them have seen. I will show the power spectral density functions which are the frequency domain approach.

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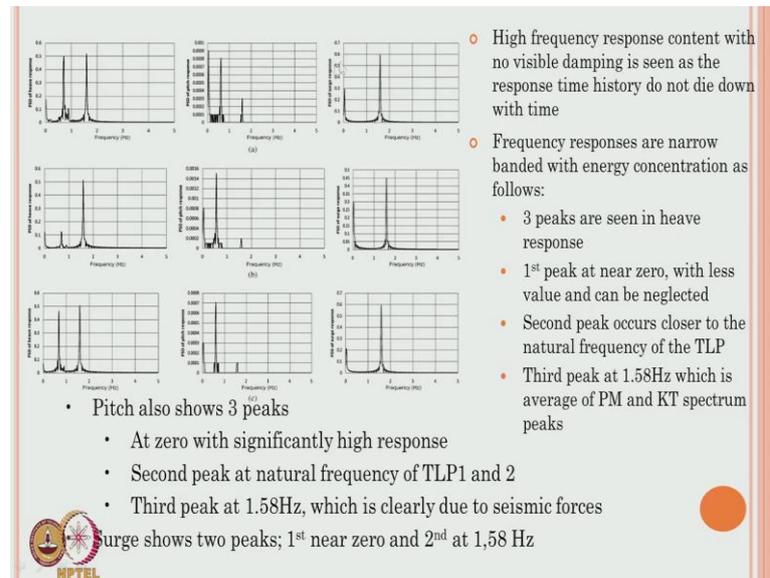
Description	TLP ₁	TLP ₂	TLP ₃
Time history response			
Heave (m)	1.2034	0.7384	1.1139
Pitch (rad)	0.002	0.0025	0.0011
Surge (m)	0.8683	0.8211	0.8655
PSD peaks at			
Heave (Hz)	~0.0, 0.706, 1.588	~0.0, 0.686, 1.588	~0.0, 0.686, 1.588
Pitch (Hz)	~0.039, 0.608, 1.588	~0.039, 0.569, 1.588	~0.02, 0.608, 1.588
Surge (Hz)	~0.039, 1.588	~0.039, 1.588	~0.02, 1.588
Dynamic tether tension variation (kN)	81,455.95	79,051.85	51,898.73
Change in tether tension (%)	65.43	41.606	20.313
Strain in tether (%)	0.298	0.146	0.109



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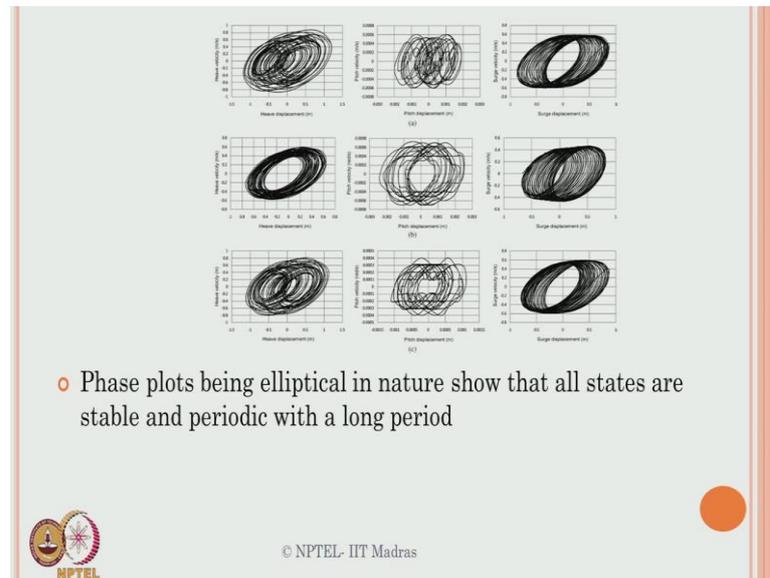
If you look at the TLP's you will see that the power spectral density function peaks at heave pitch and surge are located as specific values of 0 very close 0.7 hertz and 1.58 tethers this is. Very interesting that one of the frequencies is very close to natural frequency in the system peaks the 1.588 is very close to some frequency of earthquake and wave, so if the system as excited at resonance period or resonance frequency at two locations.

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You see the peaks here. So, if you look at the let say the top one is this is a heave power spectral density. If you look at the high frequency response this is actually pitch and this is surge, so if you look at the pitch response it shows me 3 peaks; one is closer to 0 which is very very high, the second peak is natural frequency, third peak is 1.58 which is very close to the seismic forces. Similarly, heave also the second peak and the third peak are very high one is occurring in the natural frequency of the system and third one is occurring close to the average of P-M spectrum and Kanai-Tajimi spectrum k t sense or Kanai-Tajimi. So, when they are super imposed you will see that heave is getting resonated in frequency content.

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Now, one is to analyze how the platform is not failed. We did a stability analysis one can plot the phase plots. If the phase plots are elliptical it shows by enlarge that the platform is stable for a long period waves. So, the periods waves come are very long, therefore the platform remains stable equilibrium platforms have not failed. That is the reason why it has not failed.

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CONCLUSIONS

- The example problem illustrates the complete dynamic analysis of TLP under distinctly high sea waves and seismic excitations
- It is seen that even stiff degrees-of-freedom like heave is getting excited due to seismic forces
- A new geometric form, triangular configuration of TLP is attempted
 - Geometric form is derived from the basis of equivalent square TLPs

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But, however importantly the tether tension variation in cables crossed more than 45 percent 1.45 times which was beyond the acceptable limit, but tethers still did not pull out because they were having stability in terms of a geometric. So, the platform stayed that is the reason.

(Refer Slide Time: 44:15)

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So, these are quick references for this particular study. The paper is available in NPTEL; reference of course, you can download from your library intranet and read the paper you will have more information on this. I can pack up only this much in few minutes to understand. To make you to appreciate the how dynamic analysis of fundamentals on TLP can be applied on a real time problem and show how non-frequent forces can cause extraordinary responses in research perspective. It is dynamic analysis can be used to understand the basics of the geometric design, to understand the optimization of the geometric design and also if we look at the real time failure of a real problem from the basic dynamic analysis.

So, dynamic analysis can be applied in different segments. Dynamic analysis does not stop only, but taking omega and phi's and tried to say the mode shape truncation etcetera, I can extend dynamic analysis and I design perspective also. So, I can also show from studies that the platform if it has failed how it has failed, if it has not failed why it has not failed. So, I can always understand post accident studies also partly from dynamic analysis if my basics on dynamic analysis are very good. That is a very interesting approach on research perspective for this kind of problem.

We have any questions I will have to answer otherwise I will take the next problem on they are very interesting approach on springing and ringing failures of TLP's. It is a very interesting area where people have not studied it is very seldom, but there is another application what you will have to show you on TLPs. Because why I am talking about TLP's are as I said from 80's till today TLP's are considered to be one of the promising platforms for depotters and ultradepotters. So, I would like to see whether they are really safe for all kind of natural phenomenons which are very rare in events occurred. And these are actually research based problems, because this is where people invest in oil companies just to know to be very certain that may platform geometric exist in survey in all odd conditions. In all design conditions it is always safe we already said that, but in all odd conditions (Refer Time: 46:12) really stay which is stable.

One would like to know this, is the research perspective, the design perspective one would like to know. So, these are all very good examples of application. Next class we will talk about springing and ringing which will happen after the break of the holidays.